AP PHYSICS C

SUMMER ASSIGNMENT
2024

AP PHYSICS C SUMMER ASSIGNMENT DIRECTIONS:

The summer assignment for AP Physics C includes a review of vector algebra in two dimensions and a review of calculus concepts involving basic derivatives. Answer keys are included so you can check your solutions.

For the vector (free response) problems, solutions should include any and all equations used, substitution steps and units on <u>all</u> steps.

For the calculus problems, all steps in determining the derivative should be shown, not just the final answers. Answers should be simplified and reduced, if possible. For problems that involve evaluating the derivative at a specific value of x, include the substitution step in your solution.

The summer assignment will **not** be collected or graded! The purpose of the summer assignment is to provide practice problems in vector physics and calculus that you are already familiar with to ensure we are ready to tackle more complex problems when school starts. You can do as much or as little of the summer assignment as you want but keep in mind that quizzes will be given during the first week to assess each student's understanding of the material in the summer assignment.

It is definitely advisable to work on the summer assignment in groups so that you may learn from each other and help teach each other as well. However, there is a difference between "working together" and copying someone else's work. Since all students will be evaluated independently once classes begin, it is advisable that everyone work to the best of his/her abilities on the summer assignment (and during the course of the year).

Enjoy the summer!

Sincerely,
Ms. Engelhardt (**If you have any questions, you can email me at physixchick@aol.com)
&
Ms. Christiansen

VECTORS AND SCALARS

VOCABULARY:

SCALAR- quantity expressed in terms of magnitude/ size only, with units.

EX) time	work
mass	power
distance	energy
speed	

VECTOR- quantity expressed in terms of magnitude (with units) and direction.

EX) displacement	momentum
velocity	impulse
force	torque
acceleration	

RESULTANT- single vector which equals the effect of all the individual vectors combined.

TIP-TO-TAIL RULE- to determine the direction of the resultant, arrange all vectors (without changing direction) so that the tip of one vector touches the tail of another. The resultant is then drawn from the free tail to the free tip.

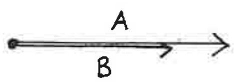
EQUILIBRIUM- state of BALANCE in which the net/resultant force acting on an object equals zero.

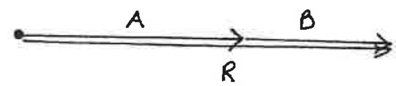
EQUILIBRANT- force which is equal in magnitude but opposite in direction to the resultant force; produces equilibrium.

VECTOR ALGEBRA REVIEW

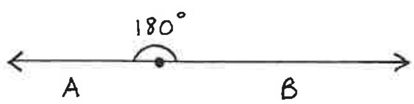
RULES:

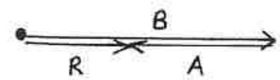
1. Vectors at 0° = same direction: ADD (**maximum** resultant)





2. Vectors at 180° = opposite direction/ antiparallel: SUBTRACT (**minimum** resultant)





The angles of 0° and 180° are very important because since they represent the angles at which the largest (maximum) and smallest (minimum) resultants occur, all other angles must yield resultants which fall between the maximum and minimum values.

3. Vectors at right angles (90°): use the pythagorean theorem & SOHCAHTOA

$$a^2 + b^2 = c^2$$

$$\cos\theta = \frac{ADJ}{HYP}$$

$$\sin\theta = \frac{OPP}{HYP}$$

$$\tan\theta = \frac{OPP}{ADJ}$$

4. Vectors at "other" angles: use Laws of sines and cosines:

$$c^2 = a^2 + b^2 - (2ab\cos C)$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

COMPONENTS METHOD CHART:

θ is measured counterclockwise
from the positive x axis!

		$A\cos\theta$	$A\sin\theta$
Vector	**θ**	**X**	**Y**
A		A_x	A_y
B		B_x	B_y
C		C_x	C_y
D		D_x	D_y
		R_x	R_y

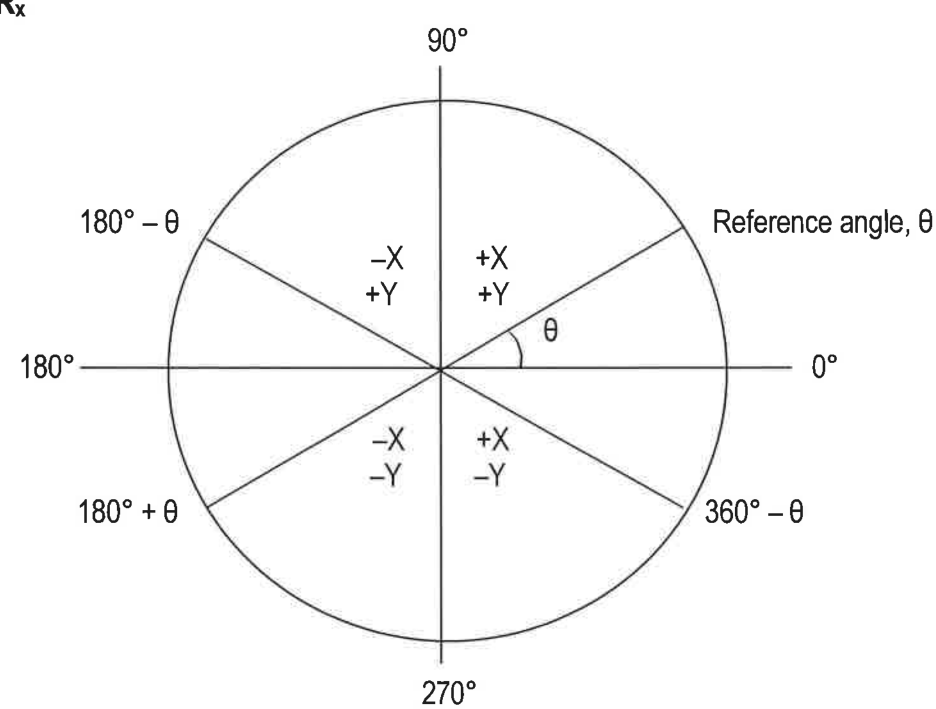
$R_x = A_x + B_x + C_x + \ldots$

$R_y = A_y + B_y + C_y + \ldots$

To find the resultant (or an unknown vector) use the Pythagorean theorem and tangent:

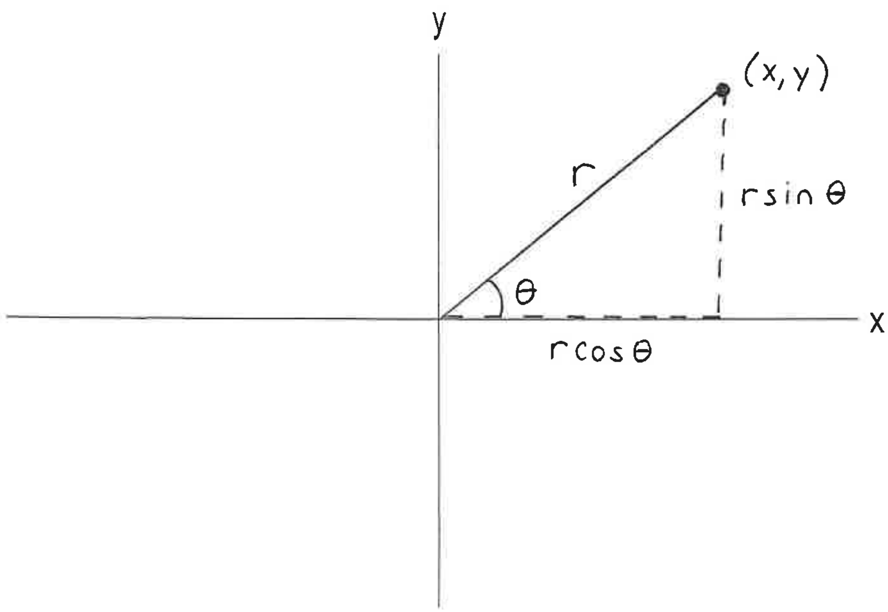
$R = \sqrt{R_x{}^2 + R_y{}^2}$ (or $a^2 + b^2 = c^2$)

$\tan\theta = R_y/R_x$



POLAR COORDINATE SYSTEMS

To represent the location of a point in space, one can use polar coordinates **(r, θ)** where **r** is the distance from the origin to a point with rectangular/Cartesian coordinates **(x, y)** and **θ** is the angle between the line (from the origin to the point) and a fixed axis (usually the positive x axis, so angles could be measured counterclockwise from this axis).



EXAMPLES:
Determine the Cartesian coordinates of the following points expressed in polar coordinates:

a. (16, 160°)

$X = r\cos\theta$ $\quad$ $y = r\sin\theta$ $\quad$ $(x,y) = (-15.04, 5.5)$

$X = 16\cos 160°$ $\quad$ $y = 16\sin 160°$

$X = -15.04$ $\quad$ $y = 5.5$

b. (28, 70°)

$X = r\cos\theta$ $\quad$ $y = r\sin\theta$ $\quad$ $(x,y) = (9.6, 26.3)$

$X = 28\cos 70°$ $\quad$ $y = 28\sin 70°$

$X = 9.6$ $\quad$ $y = 26.3$

c. A point has Cartesian coordinates (-4.5, y) and polar coordinates (r, 150°). Determine the values of y and r.

$X = r\cos\theta$

$-4.5 = r\cos 150°$

$r = 5.196 \sim 5.2$

$y = r\sin\theta$

$y = 5.2 \sin 150°$

$y = 2.6$

d. A point located in the 4th quadrant of the xy plane has Cartesian coordinates (5, y) and polar coordinates (8.3, θ). Determine the values of y and θ.

$X = r\cos\theta$

$5 = 8.3 \cos\theta$

$\theta = 52.96° \sim 53° \rightarrow 4^{th}$ Quad: $\theta = 360° - 53° = 307°$

$y = r\sin\theta$

$y = 8.3 \sin 307°$

$y = -6.6$

PROPERTIES OF VECTORS

1. **Multiplying a vector by a scalar:**

   k $\vec{A}$ is a vector that has the same direction as $\vec{A}$, but has a magnitude multiplied by value "k".

2. **Negative Vectors:**

   A negative vector, $-\vec{A}$, has the same magnitude as vector $\vec{A}$, but is in the opposite direction.

3. **Vector Addition (Vector Sum):**

   $\vec{A} + \vec{B} = \vec{C}$ refers to the process in which a resultant (or unknown vector) is found by using the appropriate mathematical method (addition, Pythagorean theorem, law of sines/cosines etc) or solving graphically.

4. **Vector Subtraction:**

   Vector subtraction can be treated as vector addition with a negative vector.

   $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

5. **Commutative Law of Addition:**

   The vector sum of two vectors is independent of the order in which they are combined.

   $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

6. **Associative Law of Addition:**

   The vector sum of three or more vectors is independent of how the individual vectors are grouped together.

   $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

VECTOR REVIEW

1. A person hikes 5.6 Km at 38° north of west, then 6.0 Km at 10° north of east, then 13.5 Km at 52° south of west. The person ends up 4.3 Km, 25° south of east of her starting point. Determine the magnitude and direction (θ) of the fourth part of her hike.

2. Three forces act concurrently on an object:
   F_1 = 26 N, 18° west of north

   F_2 = 40 N, 35° south of east

   F_3 = 57 N, 20° south of west

   Determine the magnitude and direction (θ) of the resultant force and the equilibrant force, which will produce equilibrium.

3a. Three forces act concurrently on a particle. The magnitude and direction of each force is:

$\vec{A}$ = 35 N, 20° north of west

$\vec{B}$ = 18 N, 15° west of south

$\vec{C}$ = 21 N, 35° south of east

Determine the magnitude and direction (θ) of $\vec{A} - 3\vec{B} + \vec{C}$.

b. Determine the magnitude and direction (θ) of $-2\vec{A} + \vec{B} - \vec{C}$.

4. Three forces are located in the xy plane and act concurrently on a proton. Force $\vec{\mathbf{A}}$ has a magnitude of 62.0 N and is directed at 30° above the -x axis. The magnitudes of forces $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ are unknown. Force $\vec{\mathbf{B}}$ is directed at 65° below the -x axis and force $\vec{\mathbf{C}}$ is directed at 7° above the +x axis. The three forces together result in the proton being in equilibrium. Algebraically determine the magnitudes of forces $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$.

VECTORS REVIEW

MULTIPLE CHOICE

1. A plane flies with a velocity of 410 Km/hr at 33° south of east. The southern component of the plane's velocity is approximately
   (A) 230 Km/hr
   (B) 344 Km/hr
   (C) 223 Km/hr
   (D) 255 Km/hr
   (E) 305 Km/hr

2. A 68 N force is applied to an object, at an angle θ with respect to the horizontal. If the vertical component of the applied force is 29 N, angle the force makes with the horizontal is
   (A) 25°
   (B) 35°
   (C) 45°
   (D) 55°
   (E) 65°

Questions 3 and 4 are based on the following information.
A crate whose mass is 27.4 Kg is held at rest on a frictionless incline, which makes a 40° angle with respect to the horizontal.

3. The magnitude of the crate's weight perpendicular to the incline is closest to
   (A) 195 N
   (B) 206 N
   (C) 173 N
   (D) 214 N
   (E) 186 N

4. If the angle of inclination is decreased, which of the following statements correctly describes what happens to the parallel and perpendicular components of the crate's weight?
   (A) Both the parallel and perpendicular components will increase.
   (B) Both the parallel and perpendicular components will decrease.
   (C) The parallel component will increase and the perpendicular component will decrease.
   (D) The parallel component will decrease and the perpendicular component will increase.
   (E) The parallel component will decrease and the perpendicular component will remain the same.

5. The x coordinate of the point with the following polar coordinates (14, 156°) is
   (A) +5.7
   (B) -12.8
   (C) -13.2
   (D) +6.4
   (E) -5.2

6. A point has rectangular coordinates (x, -16) and polar coordinates (r, 232°). The value of x is
   (A) +14.0
   (B) +20.3
   (C) -10.8
   (D) -14.2
   (E) -12.5

7. Point **P** is located in the 2nd quadrant and has rectangular coordinates (x, 9.2) and polar coordinates (16.7, θ). The value of θ is
   (A) 147°
   (B) 153°
   (C) 123°
   (D) 135°
   (E) 161°

8. When two collinear vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are added, the resultant has a magnitude equal to 15. If $\vec{\mathbf{A}}$ is subtracted from $\vec{\mathbf{B}}$, the resultant has a magnitude equal to 3. The magnitude of $\vec{\mathbf{A}}$ is
   (A) 3
   (B) 4
   (C) 6
   (D) 7
   (E) 9

9. When vector $\vec{\mathbf{C}}$ is added to vector $\vec{\mathbf{D}}$, the resultant vector has a magnitude three times that of $\vec{\mathbf{C}}$ and is perpendicular to $\vec{\mathbf{C}}$. If the magnitude of vector $\vec{\mathbf{D}}$ is 20, the magnitude of $\vec{\mathbf{C}}$ is closest to
   (A) $\sqrt{60}$ N
   (B) $\sqrt{40}$ N
   (C) $\sqrt{50}$ N
   (D) $\sqrt{45}$ N
   (E) $\sqrt{55}$ N

10. The y coordinate of the point with the following polar coordinates (26, 72°) is
(A) +22.8
(B) -26.0
(C) -8.0
(D) +24.7
(E) -10.6

11. The x coordinate of the point with the following polar coordinates (52, 312°) is
(A) +34.8
(B) +32.6
(C) -30.9
(D) +36.0
(E) -32.5

12. Two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, lie along the ±x axis. The magnitude of and direction of vector $\vec{\mathbf{A}}$ is unknown. The magnitude and direction of vector $\vec{\mathbf{B}}$ is 18 N in the –x direction. The resultant of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ has a magnitude of 11 N and lies along the ±x axis. Which of the following is a possible magnitude and direction of vector $\vec{\mathbf{A}}$?
(A) 7 N in the -x direction
(B) 9 N in the –x direction
(C) 25 N in the –x direction
(D) 5 N in the + x direction
(E) 29 N in the +x direction

13. The minimum resultant of two concurrent forces has a magnitude of 48 N. If the magnitude of one of the forces is 23 N, the magnitude of the maximum resultant produced by these two forces is
(A) 88 N
(B) 94 N
(C) 86 N
(D) 92 N
(E) 96 N

14. Force $\vec{\mathbf{A}}$ has a magnitude of 15 N and is directed due west and force $\vec{\mathbf{B}}$ has a magnitude of 20 N and is directed due north. The magnitude of $\mathbf{2\vec{A} - 3\vec{B}}$ is approximately
(A) 57 N
(B) 73 N
(C) 67 N
(D) 62 N
(E) 77 N

15. Which combination of forces could produce equilibrium?
   (A) 12 N, 19 N and 25 N
   (B) 8 N, 18 N and 29 N
   (C) 6 N, 14 N and 7 N
   (D) 11 N, 28 N and 45 N
   (E) 32 N, 15 N and 12 N

16. Two forces have magnitudes of 7 N and 16 N. If the magnitude of their resultant is 19 N, the angle between the forces is
   (A) 0°
   (B) between 0° and 90°
   (C) 90°
   (D) between 90° and 180°
   (E) 180°

17. A vector located in the xy plane has an x component of -6 and a y component of +3. If this vector is rotated 90°counterclockwise, its x and y components will be
   (A) +6, +3
   (B) +6, -3
   (C) -3, +6
   (D) +3, +6
   (E) -3, -6

18. Vector $\vec{\mathbf{A}}$ has an x component of -5, an unknown y component and is directed at an angle of 127° counterclockwise from the +x axis. The y component of vector $\vec{\mathbf{A}}$ is
   (A) +3.8
   (B) -6.2
   (C) +5.8
   (D) +6.6
   (E) -5.5

VECTORS ANSWER KEY

1. 11.55 Km, θ = 22.0° north of east (or 22.0° cc)
2. R = 33.83 N, θ = 31.6° south of west (or 211.6° cc)

   E = 33.83 N, θ = 31.6° north of east (or 31.6° cc)

3a. 52.11 N, θ = 88.1° north of west (or 91.9° cc)

b. 52.79 N, θ = 33.7° south of east (or 326.3° cc)

4. B = 43.98 N, C = 72.79 N

MULTIPLE CHOICE ANSWER KEY

1. C	5. B	9. B	13. B	17. E
2. A	6. E	10. D	14. C	18. D
3. B	7. A	11. A	15. A	
4. D	8. C	12. E	16. B	

THE DERIVATIVE

On a graph, the slope of the **secant line** between two points represents an **average value** (such as average velocity, average acceleration). The **derivative** of a function represents the slope of the **tangent line** at a given point on the graph which represents an instantaneous value (such as instantaneous velocity or acceleration).

RULES FOR FINDING DERIVATIVES:

A. Power Rule

$y = Ax^n \qquad dy/dx = n\,A\,x^{n-1}$

B. Product Rule

$d/dx\ [\, f(x)\, g(x)\,] = f(x)\ d/dx\, [\, g(x)\,] + g(x)\, d/dx\, (f(x)]$

C. Quotient Rule

$$d/dx\ [f(x)/g(x)] = \frac{g(x)\ d/dx\,[f(x)] - f(x)\ d/dx\,[g(x)]}{[g(x)]^2}$$

D. Trig functions

$y = A \sin (a\,x) \qquad dy/dx = A\,a \cos (a\,x)$

$y = A \cos (a\,x) \qquad dy/dx = -A\,a \sin (a\,x)$

E. Chain Rule

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

F. Functions with e

$y = e^{ax} \qquad dy/dx = a\,e^{ax}$

G. Functions with ln

$f(x) = u \qquad y = A \ln u \qquad dy/dx = A\ (1/u)\, du/dx$

A. The Power Rule: $y = Ax^n$ $dy/dx = n A x^{n-1}$

For each of the following, find dy/dx (or y') and simplify/reduce answers, if possible.

1. $y = 8x^4 - 7x^2 + 3/x^3$

2. $y = 6/x^2 - 10x^3 - 4\sqrt[3]{x^2}$

3. $y = \sqrt{x^5} + 2x^{-4}$

4. If $y = 12x^3 + 8x^2 - 5x + 6$, evaluate dy/dx at $x = -4$.

5. If $y = -3x^5 + 4x^3 + 15x^2$, evaluate dy/dx at x = +3.

6. If $y = 2/3\ x^3 + 9/2\ x^2 - 35x$, at what value(s) of x is dy/dx equal to zero?

B. The Product Rule: d/dx [f(x) g(x)] = f(x) d/dx [g(x)] + g(x) d/dx (f(x)]

In other words, the first function times the derivative of the second function + the second function times the derivative of the first function.

For each of the following, find dy/dx (or y') and simplify/reduce answers, if possible.

1. $y = (x^3 + 4x)(2x^2 + 6)$

2. $y = (6x + 3)(2x^2 - 5x + 1)$

3. $y = (4x^2 - 10)(8\sqrt{x} + 2)$

4. If $y = (x + 8)(x^4 - 12)$, evaluate dy/dx at $x = -6$.

5. If $y = (-5x^2 + 3x)(3x^3 - 4)$, evaluate dy/dx at $x = -1/2$.

C. The Quotient Rule: $\frac{d}{dx}[f(x)/g(x)] = \dfrac{g(x)\ \frac{d}{dx}[f(x)] - f(x)\ \frac{d}{dx}[g(x)]}{[g(x)]^2}$

In other words, the bottom function times the derivative of the top function minus the top function times the derivative of the bottom function divided by the bottom function squared.

For each of the following, find dy/dx (or y') and simplify/reduce answers, if possible.

1. $y = \dfrac{-5x}{4x + 2}$

2. $y = \dfrac{3x^4 + 2}{3x^3 - 2}$

3. $y = \dfrac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

4. If $y = \dfrac{4x^2 - 6}{x + 3}$ evaluate dy/dx at x = 0.

5. If $y = \frac{x^3 - 10}{6x^2 - 5}$ evaluate dy/dx at x = -2.

D. Trig functions:

$y = A \sin(a x)$ $\qquad$ **$dy/dx = A a \cos(a x)$**

$y = A \cos(a x)$ $\qquad$ **$dy/dx = -A a \sin(a x)$**

For each of the following, find dy/dx (or y') and simplify/reduce answers, if possible.

1. $y = 4x^2 \cos 8x$

2. $y = \cos 5x \sin 3x$

3. $y = 1/2 \cos(6x + 12)$

4. $y = \dfrac{1 - \sin x}{x - \cos x}$

5. $y = \frac{1}{4} x^3 \sin 2x$

6. $y = \sin^2(9x)$

7. $y = \sqrt{2 + \sin (x/4)}$

E. The Chain Rule: used when quantities are raised to a power.

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$$

*** Let u equal the quantity in parenthesis.**

1. $y = (5 + 4x^3)^3$

2. $y = \sqrt[3]{6x^3 + 15x}$

3. $y = (12x + 8)^2 (1 - 2x)^3$

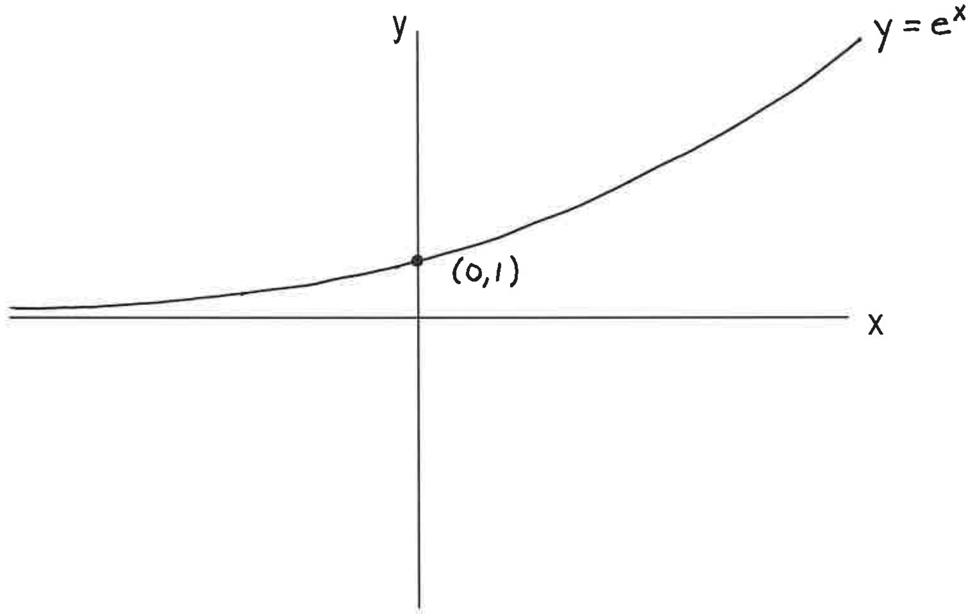
4. $y = \sqrt[3]{9 + 2x}$

5. $y = \dfrac{1}{\sqrt{2x^4 - 10}}$

6. If $y = (10x - 3x^3)^4$ evaluate dy/dx at x = -0.6.

7. If $y = (1/4\ x^2 - 2x - 10)^{-2}$ evaluate dy/dx at x = -2.

F. Functions with e:



$$y = e^{ax} \qquad \frac{dy}{dx} = a\,e^{ax} = e^{\text{function}} \cdot \text{derivative of function}$$

For each of the following, find dy/dx (or y') and simplify/reduce answers, if possible.

1. $y = 1/5\ e^{3x+8}$

2. $y = \dfrac{2e^x}{3x}$

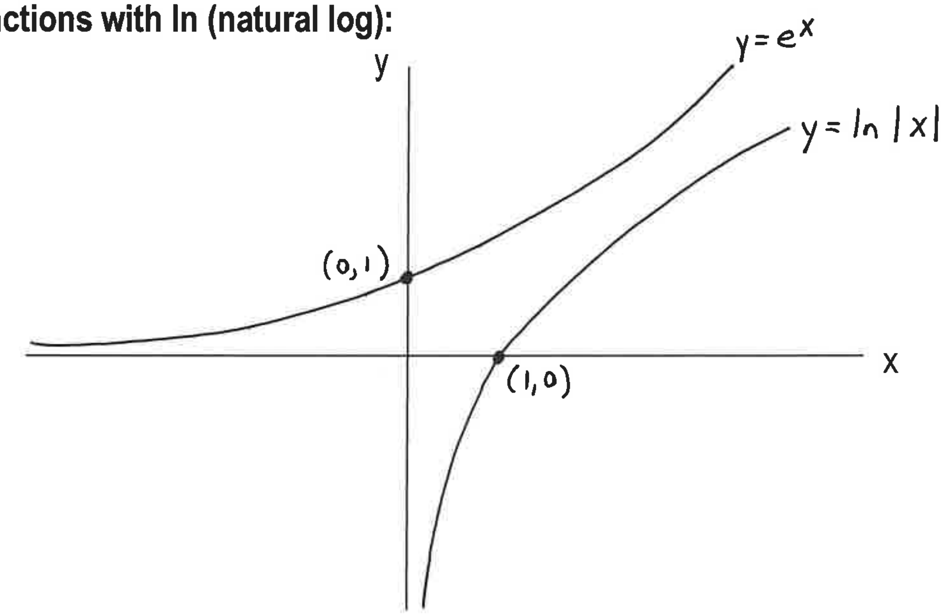
3. $y = (5 - e^{-2x})^3$

4. $y = 4e^{x^2} \cos (1 - 3x)$

5. $y = 5x\, e^{-2x}$

6. $y = \dfrac{3e^{-4x}}{x^3}$

G. Functions with ln (natural log):



f(x) = u **y = A ln u** **dy/dx = A (1/u) du/dx**

*** (1/function) · (derivative of the function)**

For each of the following, find dy/dx (or y') and simplify/reduce answers, if possible.

1. $y = \ln(x + \sin x)$

2. $y = \dfrac{\ln 5x}{2x}$

3. $y = -6x^3 \ln 2x$

4. $y = \ln(4x\, e^{2x-1})$

5. If $y = \ln(10x^3 + 3x^2)$, evaluate dy/dx at $x = -1$.

6. If $y = \ln(8 - 4x + 6x^2)$, evaluate dy/dx at $x = +5$.

DERIVATVES ANSWER KEY

A. Power Rule

1. $32x^3 - 14x - 9x^{-4}$
2. $-12x^{-3} - 30x^2 + 8/3\, x^{-1/3}$
3. $5/2\, x^{3/2} - 8x^{-5}$
4. 507
5. -1017
6. x = +5/2, -7

B. Product Rule

1. $10x^4 + 42x^2 + 24$
2. $36x^2 - 48x - 9$
3. $80x^{3/2} - 40x^{-1/2} + 16x$
4. -444
5. -41.1875

C. Quotient Rule

1. $\frac{-10}{(4x + 2)^2}$
2. $\frac{9x^6 - 24x^3 - 18x^2}{(3x^3 - 2)^2}$
3. $\frac{-10x^6 - 8x^3 - 60x^2}{(2x^4 - 4)^2}$
4. 2/3 or 0.66
5. -204/361 or -0.565

D. Trig. Rule

1. $-32 \sin 8x + 8x \cos 8x$
2. $3 \cos 5x \cos 3x - 5 \sin 3x \sin 5x$
3. $-3 \sin (6x + 12)$
4. $\frac{-x \cos x}{(x - \cos x)^2}$
5. $1/2\, x^3 \cos 2x + 3/4\, x^2 \sin 2x$
6. $18 \sin 9x \cos 9x$
7. $1/8\, (2 + \sin (x/4))^{-1/2} \cos (x/4)$

E. Chain Rule

1. $36 x^2 (5 + 4 x^3)^2$
2. $1/3\, (6 x^3 + 15 x)^{-2/3} (18 x^2 + 15)$
3. $-6 (12 x + 8)^2 (1 – 2 x)^2 + 24 (1 – 2 x)^3 (12 x + 8)$
4. $2/3\, (9 + 2 x)^{-2/3}$
5. $-4 x^3 (2x^4 – 10)^{-3/2}$
6. -4145.29
7. -6/125 or -0.048

F. "e" Rule

1. $3/5\, e^{3x+8}$
2. $\frac{2e^x (x – 1)}{3 x^2}$
3. $6e^{-2x} (5 – e^{-2x})^2$
4. $12e^{x^2} \sin (1 - 3 x) + 8x e^{x^2} \cos (1 - 3x)$
5. $-10 x e^{-2x} + 5 e^{-2x}$
6. $-12x^{-3} e^{-4x} - 9x^{-4} e^{-4x}$

G. ln Rule

1. $\frac{1 + \cos x}{x + \sin x}$

2. $\frac{1 - \ln 5x}{2x^2}$

3. $-6x^2 (1 + 3 \ln 2x)$

4. $2 + 1/x$

5. $24/-7 = -3.43$

6. $56/138 = 0.406$