Summer Packet for students entering AP Calculus AB or BC

Welcome to AP Calculus. AP Calculus is a demanding course that relies heavily upon a student's algebra, geometry and precalculus skills. You are expected to have a strong background in the skills reviewed in this packet. Resource links are listed below and embedded within each section throughout the packet. This packet will be checked for completion and **all material on this packet will be assessed.**

DUE: 1st week of school.

ASSESSED: 2nd week of school.

Digital Resources:

Khan Academy Wolfram Alpha Paul's Online Math Notes Purple Math Just Math Tutorials

YouTube:

Recommended YouTube searches: Organic Chemistry Tutor, Black Pen Red Pen, 3 Blue 1 Brown

FACTORING POLYNOMIALS AND RATIONALS: Factoring

Factor completely:

1) $x^3(x+4)^5 - 2x^2(x+4)^6$

2) $2x^{\left(\frac{1}{2}\right)}(x+2)^2 + 2x^{\left(\frac{3}{2}\right)}(x+2)^3$

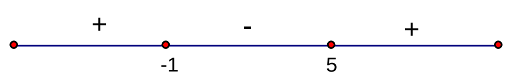
3) $\frac{8x(x+5)^3 - 4x^2(x+5)^2}{16x^2+80x}$

4) $\frac{3(2x^2-8)+12(x+2)^2}{12x^2+6x-36}$

SOLVING POLYNOMIAL INEQUALITIES: Polynomial Inequalities

Example 1: $x^2 - 4x - 5 < 0$

Solution: $(x+1)(x-5) < 0$ *factor and determine critical values: x = -1, 5*



Answer: (-1, 5) *mark the zeros; pick a test point to determine the sign of the polynomial in each interval- this is called a SIGN CHART!*

Solve, include a Sign Chart

5) $x^3 + 7x^2 + 10x > 0$

6) $3x(x+2)^3 + 9x^2(x+2)^2 \le 0$

CONICS: Parabola Circle Ellipse

Identify they type of conic, write in standard form and graph. Label key features.

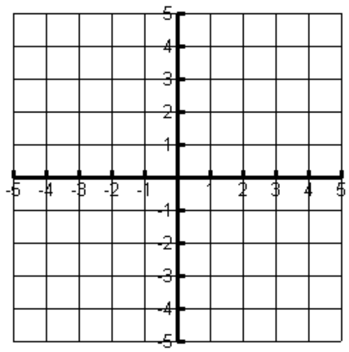
1. $4y^2 + 8y - 4x + 8 = 0$

2. $4x^2 + 36y - 32x + 9y^2 + 64 = 0$

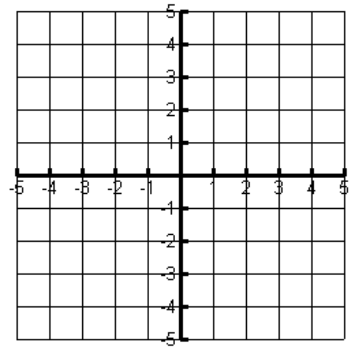
3. $x^2 + y^2 - 6x + 8y = 0$

Parent Functions that all graduating Pre-Calculus students should be able to sketch without the use of a calculator. Determine any key characteristics, domain, interval of continuity, symmetry, end behavior and type and location of any discontinuities.

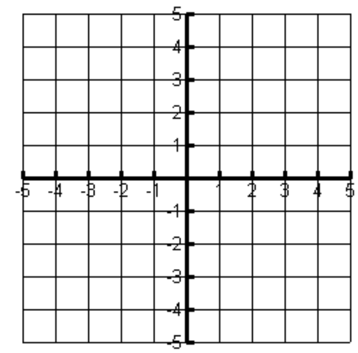
Linear: $y = x$



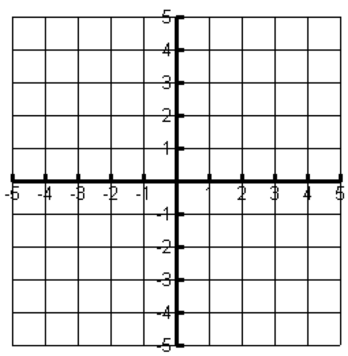
Quadratic: $y = x^2$



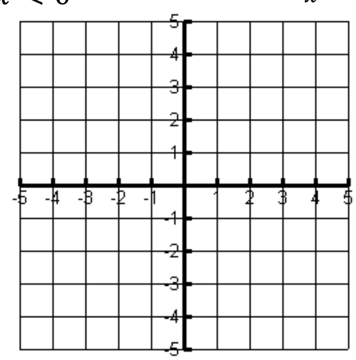
Cubic: $y = x^3$



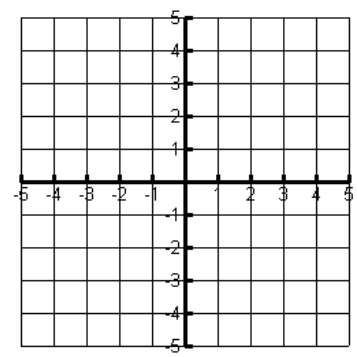
Absolute Value: $y = |x| = \begin{cases} x, if\ x \geq 0 \\ -x, if\ x < 0 \end{cases}$



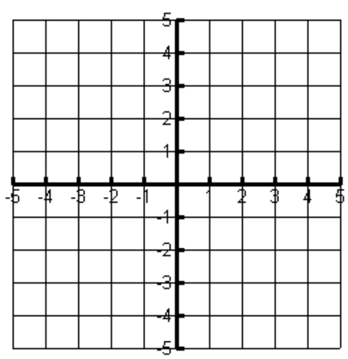
Rational: $y = \frac{1}{x}$



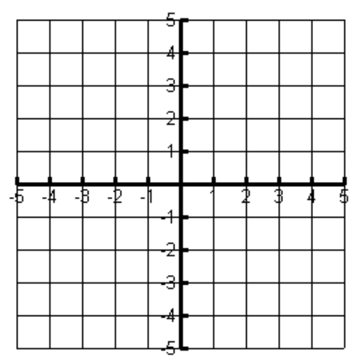
Root: $y = \sqrt{x}$



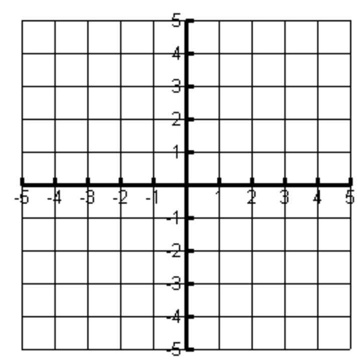
Exponential Growth: $y = e^x$



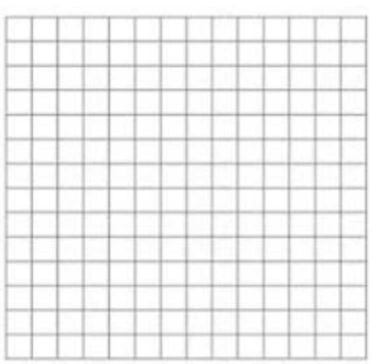
Exponential Decay: $y = e^x$



Logarithmic Growth: $y = \ln x$



Sine: $y = \sin(x)$



Cosine: $y = \cos(x)$

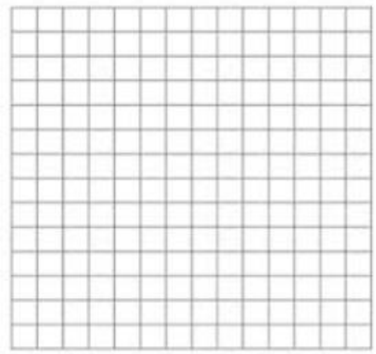
Tangent: $y = \tan(x)$

Absolute Value Functions: Absolute Value Functions

Write the absolute value function as a piecewise function then identify the vertex and sketch the graph.

1. $f(x) = 2|x-3| + 1$

2. $f(x) = |1-x|$



Rational Expressions and Functions: Graphing Operations

Domain: excludes all zeros of the denominator – both removable and non-removable.

Range: effected by any horizontal asymptotes. Recall, a function may cross a horizontal asymptote.

End Behavior (limits at infinity): $\lim_{x \to \pm\infty} f(x)$ determined by ratio of leading terms. If value that is HA if expression with x it follows the end behavior of that function.

Discontinuity: occur at the zeros of the denominator:

1. Zeros of the denominator that CANNOT be simplified: infinite discontinuities graph as vertical Asymptotes.
2. Zeros of the denominator that CAN be simplified: removable discontinuities graph as holes.

Simplifying: factor out and cancel common factors, simplify complex fractions by multiplying through by the LCD over itself.

Solving: relate terms to zero, combine all terms, determine the zeros of the numerator and denominator (called critical values) and create a sign chart to determine the signs before, between and after all critical values.

1. Determine the following for the function $f(x) = \frac{3x^2+2x-1}{2x^2+x-1}$

Domain:

Range:

x-intercept(s):

y-intercept(s):

Asymptote(s):

End Behavior:

$\lim_{x \to -\infty} f(x) =$

$\lim_{x \to \infty} f(x) =$

Interval of Continuity:

Name and Location of Discontinuities:

2. Evaluating and Simplifying Functions. Determine $f(x+h)$, $f(x+h)-f(x)$ and $\frac{f(x+h)-f(x)}{h}$ for each of the following functions.

$f(x) = 3x^2 + 1$

$f(x) = \frac{1}{x-1}$

FUNCTION OPERATIONS AND COMPOSITION; INVERSE FUNCTIONS Composition Inverses

Composition of a function g with a function f is defined as:

$$h(x) = g\big(f(x)\big)$$

The domain of h is the set of all x-values such that x is in the domain of f and $f(x)$ is in the domain of g.

Inverses: Functions f and g are inverses of each other provided:

$$f\big(g(x)\big) = x \text{ and } g\big(f(x)\big) = x$$

The function g is denoted as f^{-1}, read as "f inverse" and $f^{-1}(x) \neq \frac{1}{f(x)}$

Horizontal Line Test: If any horizontal line which is drawn through the graph of a function f intersects the graph no more than once, then f is said to be a **one-to-one** function and has an inverse.

1. Let f and g be functions whose values are given by the table below. Assume g is one-to-one.

x	$f(x)$	$g(x)$
1	6	2
2	9	3
3	10	4
4	-1	6

a. $f\big(g(3)\big)$

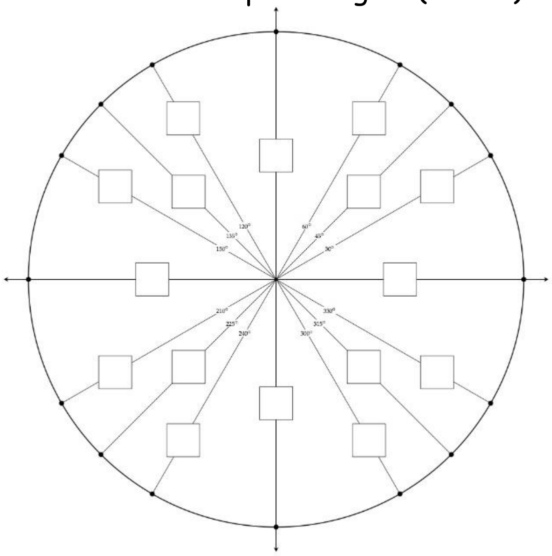
b. $g^{-1}(4)$

c. $f(g^{-1}(6))$

d. $f^{-1}\Big(f\big(g(2)\big)\Big)$

<u>TRIGONOMETRY</u> Unit Circle Solving

UNIT CIRCLE: Complete angles (radian) and Sine/Cosine



Key Trig Identities – formulas you should know:

$$\sin^2 x + \cos^2 x = 1$$
$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = 1 - 2\sin^2 x$$
$$1 + \tan^2 x = \sec^2 x$$
$$\sin(-x) = -\sin x \quad ODD$$
$$\cos(-x) = \cos x \quad EVEN$$
$$\tan(-x) = \tan x \quad ODD$$

15) Without using a calculator or table, find each value:

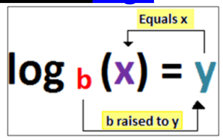
a. $\cos\left(-\frac{\pi}{3}\right)$ b. $\sec\left(\frac{11\pi}{6}\right)$ c. $\tan^{-1}(-1)$ d. $\sec^{-1}(-2)$

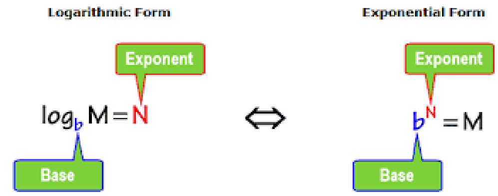
16) Solve the trigonometric equations algebraically by using identities and **without** the use of a calculator. Find all solutions in the interval $0 \le \theta \le 2\pi$.

a. $2\sin^2\theta - 1 = 0$ b. $\sin(2x) = \cos x$

EXPONENTIAL AND LOGARITHMIC FUNCTIONS: Logs

Logarithm: Inverse operation of exponential.





Properties of Logarithms (for $x, y, b > 0, b \neq 1$)

Exponential Properties:	Changing Between Forms	Logarithmic Properties:
• $b^x \cdot b^y = b^{x+y}$	⟷	$\log_b(x \cdot y) = \log_b x + \log_b y$
• $\frac{b^x}{b^y} = b^{x-y}$	⟷	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
• $(b^x)^y = b^{xy}$	⟷	$\log_b(x)^y = y \cdot \log_b x$
• $b^0 = 1$	⟷	$\log_b 1 = 0$ or $\ln 1 = 0$
Point on Exponential Graph: (0, 1)!		Point on Logarithmic Graph: (1, 0)!
• $b^1 = b$	⟷	$\log_b b = 1$ or $\ln e = 1$
• $b^x = b^x$	⟷	$\log_b b^x = x$
• $b^{\log_b x} = x, x > 0$	⟷	$\log_b b^x = \log_b b^x$

Simplify.

1. $\log_5 \frac{1}{125}$

2. $\log_4 \sqrt[8]{16}$

3. $2 \ln \frac{1}{e^3}$

4. $\frac{1}{t} \ln e^t$

5. $e^{3\ln(x+5)}$

6. e^{-lnx}

7. $4 \ln e^{x^2}$

8. e^{3x+ln5}

9. $e^{3(\ln(x)+\ln 5)}$

10. Write $5\,ln\,(x) - 3\,ln\,(y) + 2\,ln\,(4x) - 6\,ln\,(y^2)$ as a a single logarithm.

11. Expand $\ln\left(\frac{3x^2}{2\sqrt{y^2-4}}\right)$.

12. Evaluate:

a. $\ln 1$

b. $\ln 3e$

c. $\ln e$

d. $\ln 0$

e. $\ln(\ln e)$

Limits: $\lim_{x \to c} f(x) = N$ "Limit as x approaches c of f(x) equals N" [Limits]

LIMIT ≠ CONTINUITY

Understanding the difference between limits and continuity: Limit is the value (y) that the function APPROACHES as you get close to c (either from one side or from both sides). Continuity implies that the limit exists (from both sides) and that the limit = the value at c!

Limit Exists: $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) \;\therefore\; \lim_{x \to c} f(x)\ exists$ and will equal all the same value or $\pm\infty$.

Continuous: $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)\ AND = f(c) = A\ (where\ A\ is\ a\ constant\ not\ \pm\infty)$ **MUST SHOW ALL 3!!**

One-Sided limit: exists if graph approaches a value from one side: $+\ (right)$ or $-\ (left)$

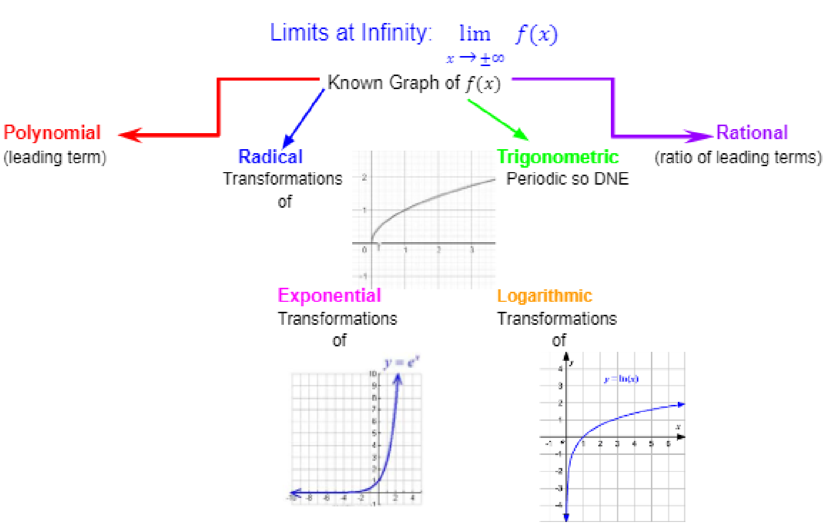
$\lim_{x \to a} c = c$ $\quad \lim_{x \to a}[f(x) \pm g(x)] = L \pm M$ $\quad \lim_{x \to a} kf(x) = k \lim_{x \to a} f(x) = kL$ $\quad \lim_{x \to a}[f(x) \cdot g(x)] = L \cdot M$ $\quad \lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{M}$

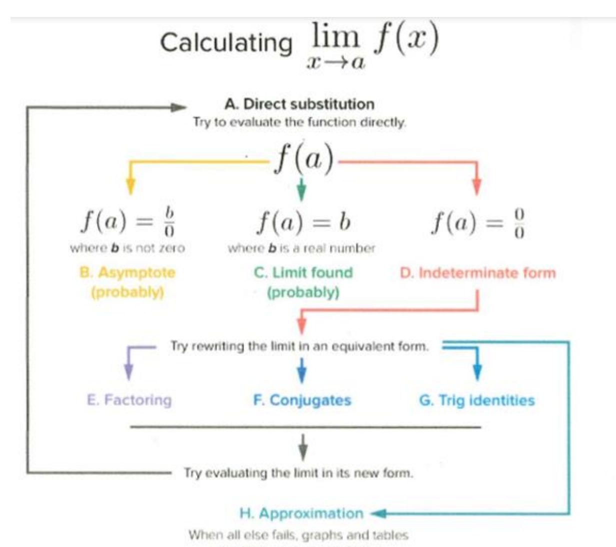
Evaluating Limits:

1. **Tables:** either from table or by graphing and viewing table looking for values of y approaching the same number. Check one sided limits first and if they are equal the overall limit exists.
2. **Graphically:** Check one sided limits (again – value of limit is the y-value or the height of the graph) and if they are approaching the same height, then the overall limit exists.
3. **Algebraically – Direct substitution:** plug in c. If you get a constant then limit exists. Rational functions may need to be simplified first!
   When Direct substitution results in:
   - $\frac{0}{number}$ then the limit is equal to 0
   - $\frac{0}{0}$ most likely a removable discontinuity, try simplifying or conjugate.
   - $\frac{number}{0}$ most likely an infinite discontinuity, look at each side and evaluate one sided limit or if asking for two sided limit make sure the function is approaching the same height from both sides.

$\lim_{x \to \pm\infty} f(x)$: **End Behavior:**

- Polynomial: use degree (even or odd) and sign of leading coefficient.
- Rational: analyze the ratio of the leading terms.
- Other "Known" Graphs: know the graphs and consider transformations.





Examples: Evaluate the limits. Justify any limits that do not exist.

29) $\lim_{x\to 3}(2x^2)$

30) $\lim_{x\to -2^+}\frac{1}{x+2}$

31) $\lim_{x\to \frac{\pi}{2}}(x\sin x)$

32) $\lim_{x\to 5}\frac{3}{x-5}$

33) $\lim_{x\to 0^+}\ln(x)$

34) $\lim_{x\to -\frac{\pi}{2}}\tan(x)$

35) $\lim_{x\to -\infty}3e^{2x}$

36) $\lim_{x\to 1}\frac{3x^2-2x-1}{x^2-1}$

37) $\lim_{x\to \infty}\frac{2x}{x-1}$

38) $\lim_{x\to 1}\frac{\frac{1}{x}-1}{x-1}$

39) $\lim_{x\to -\infty}2x(x-3)^2(x+1)$

40) $\lim_{x\to -\infty}\frac{3}{x-5}$

41) If $f(x)=\begin{cases}2x-3, & if\ x<-1\\ 2, & if\ x=-1\\ 5x, & if\ x>-1\end{cases}$, determine the following:

a. $\lim_{x\to -1^-}f(x)$

b. $\lim_{x\to -1^+}f(x)$

c. $\lim_{x\to -1}f(x)$

d. Is f continuous?

42) If $f(x)=|x+2|=\begin{cases}x+2, & x\geq -2\\ -(x+2), & x<-2\end{cases}$, determine the following:

a. $\lim_{x\to -2^-}f(x)$

b. $\lim_{x\to -2^+}f(x)$

c. $\lim_{x\to -2}f(x)$

d. Is f continuous?

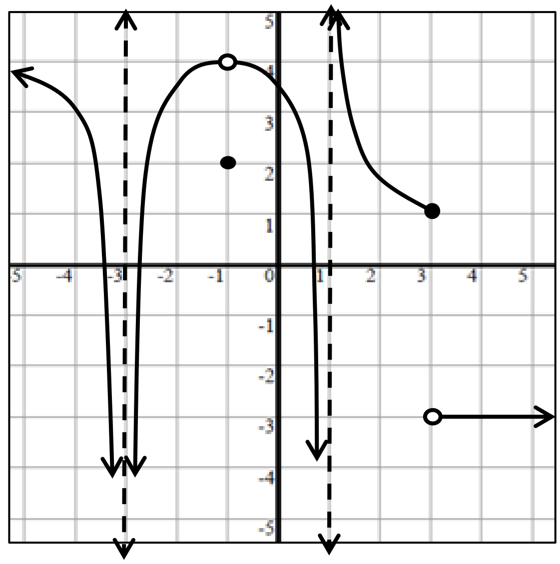
43) Determine whether $f(x)$ is continuous. Justify. Then, determine domain, interval of continuity and location and type of any discontinuities.

a. $f(x)=\begin{cases}3x^2, & x<0\\ 4, & x=0\\ 3\sin^2 x, & x>0\end{cases}$

b. $f(x)=\begin{cases}\ln x, & x<e^2\\ \sqrt{x}, & x\geq e^2\end{cases}$

c. $f(x)=\begin{cases}3x^2-1, & x\leq 2\\ 5x+1, & x>2\end{cases}$

44) Let f(x) be the graph below. Determine the following:



a. $\lim_{x\to -3^-} f(x) =$

b. $\lim_{x\to -3^+} f(x) =$

c. $\lim_{x\to -3} f(x) =$

d. $f(-3) =$

e. Is f continuous at -3?

f. $\lim_{x\to -1^-} f(x) =$

g. $\lim_{x\to -1^+} f(x) =$

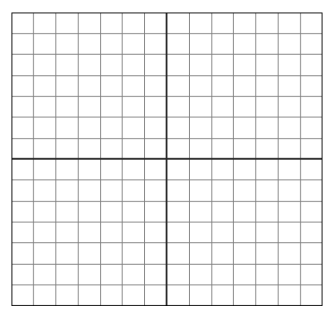
h. $\lim_{x\to -1} f(x) =$ i. $f(-1) =$ j. $\lim_{x\to 1^-} f(x) =$ k. $\lim_{x\to 1^+} f(x) =$ l. $\lim_{x\to 1} f(x) =$

m. $f(1) =$ n. $\lim_{x\to 3^-} f(x) =$ o. $\lim_{x\to 3^+} f(x) =$ p. $\lim_{x\to 3} f(x) =$ q. $f(3) =$

r. $\lim_{x\to -\infty} f(x) =$ s. $\lim_{x\to \infty} f(x) =$ t. Continuity Interval:

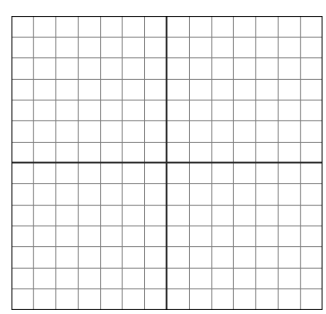
u. Location and type of each discontinuity: v. Domain:

45) Draw a graph given the following conditions:



- $f(0) = 0$
- $\lim_{x\to -\infty} f(x) = -1$
- $f(1) = 2$
- $\lim_{x\to \infty} f(x) = 1$
- $f(4) = -3$
- $\lim_{x\to 3^+} f(x) = -\infty$
- $\lim_{x\to 3^-} f(x) = \infty$

46) Draw a graph given the following conditions:



- $f(-2) = 0$
- $\lim_{x\to -\infty} f(x) = -\infty$
- $f(2) = 2$
- $\lim_{x\to \infty} f(x) = 5$
- $\lim_{x\to -2^-} f(x) = 4$
- $\lim_{x\to -2^+} f(x) = 0$