

# Incarnate Word Academy

## Summer Packet: AP Precalculus 2024

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### **Note:**

The 2024-2025 school year will be the second year that College Board offers AP Precalculus. This summer packet is a collection of math skills that you have been taught. In AP Precalculus we will expand on these concepts for a deeper understanding.

### **Instructions:**

The purpose of this packet is to keep your skills fresh throughout the summer, so that you will be fully ready to start AP Precalculus. All problems can be done without a calculator. I suggest working a minimum of 2 pages per week. Flag any problems that you do not understand. While this packet is not required, it is highly recommended. We will only spend one day reviewing, before starting Chapter 1. Enjoy your summer! I look forward to seeing you next school year.

## AP Precalculus Summer Packet Part A

**Describe the end behavior of each function.**

1)  $f(x) = -x^2 + 6$

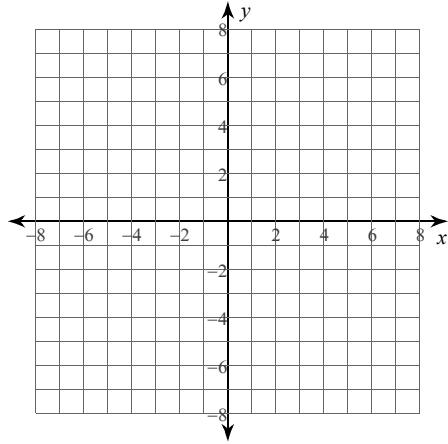
2)  $f(x) = -x^3 + 3x^2 - 2$

3)  $f(x) = -x^4 + 2x^2 + 1$

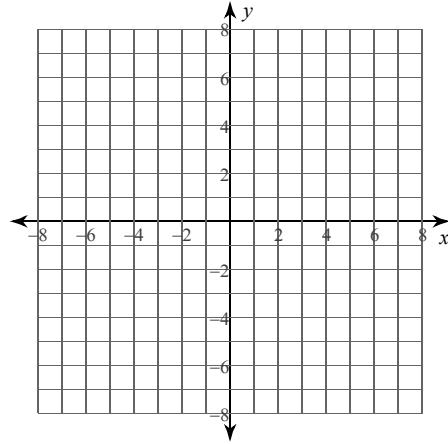
4)  $f(x) = -x^5 + 3x^3 - 2x - 2$

**Identify the holes, vertical asymptotes, x-intercepts, horizontal asymptote, and domain of each. Then sketch the graph.**

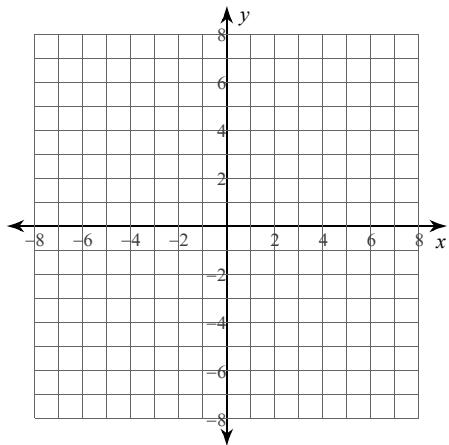
5)  $f(x) = \frac{3}{x+1} + 2$



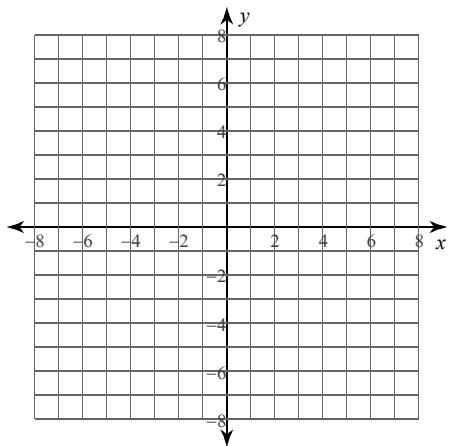
6)  $f(x) = -\frac{3}{x-1} - 2$



7)  $f(x) = \frac{-3x^2 + 3x + 6}{x^2 - x - 6}$



8)  $f(x) = \frac{-2x^2 + 12x - 16}{x^2 - 3x}$



**Find the discriminant of each quadratic equation then state the number and type of solutions.**

9)  $9k^2 - 8k - 2 = -2$

10)  $-4k^2 + 4k - 10 = -6$

11)  $4k^2 - 4k - 4 = -10$

12)  $-6x^2 + 4x - 2 = 3$

**Factor each and find all roots.**

$$13) \ x^2 - 16 = 0$$

$$14) \ x^2 - x - 6 = 0$$

$$15) \ x^3 - 6x^2 + 13x = 0$$

$$16) \ x^2 + 4x + 13 = 0$$

$$17) \ x^3 + x^2 + x + 1 = 0$$

$$18) \ x^3 - x^2 + 4x - 4 = 0$$

$$19) \ x^4 + 14x^2 + 48 = 0$$

$$20) \ x^4 + 11x^2 + 28 = 0$$

$$21) \ x^4 - 125x = 0$$

$$22) \ x^4 + 27x = 0$$

**Solve each equation by taking square roots.**

$$23) \ 10 - 3p^2 = -5$$

$$24) \ 16n^2 - 1 = 80$$

**Solve each equation with the quadratic formula.**

$$25) \ 12r^2 = -12r + 21$$

$$26) \ 10a^2 + 12a = -4$$

**Find the value that completes the square and then rewrite as a perfect square.**

$$27) \ y^2 - 24y + \underline{\quad}$$

$$28) \ x^2 - 12x + \underline{\quad}$$

**Solve each equation by completing the square.**

$$29) \ a^2 + 16a + 63 = 8$$

$$30) \ m^2 + 16m + 61 = 4$$

$$31) \quad 8n^2 - 16n + 84 = 6$$

$$32) \quad 6a^2 + 12a - 40 = 8$$

**Simplify.**

$$33) \quad \frac{5+4i}{3-10i}$$

$$34) \quad \frac{-2-4i}{-5+4i}$$

$$35) \quad \frac{\sqrt{2}}{4\sqrt{32}}$$

$$36) \quad \frac{4\sqrt{3}}{\sqrt{12}}$$

$$37) \quad \frac{\sqrt{5}}{3\sqrt{3} + \sqrt{5}}$$

$$38) \quad \frac{2}{3 - 4\sqrt{3}}$$

$$39) \quad \frac{3 + \sqrt{2x^2}}{3\sqrt{19x^4}}$$

$$40) \quad \frac{2 + \sqrt{2n}}{\sqrt{11n^3}}$$

$$41) \quad (-5 - 8i) - (-5 + 6i)$$

$$42) \quad (3 + i) + (-4 - 6i)$$

$$43) \quad (-4 + i) - (-8 + 4i)$$

$$44) \quad (-3i) + (4i) + (-3 + 3i)$$

$$45) \quad -7(-8i)(3 - 3i)$$

$$46) \quad (-1 - 5i)(-4 - 6i)$$

**Simplify each expression.**

$$47) \frac{2}{5} \cdot \frac{3a}{8a}$$

$$48) \frac{8}{3a} \cdot \frac{13a}{20a}$$

$$49) \frac{5(r-9)}{8(r-9)} \cdot \frac{8(r-3)}{5}$$

$$50) \frac{7(5r+8)}{2(5r+8)} \cdot \frac{2(r+4)}{r+4}$$

$$51) \frac{a^2 - 12a + 36}{a-6} \cdot \frac{1}{a+9}$$

$$52) \frac{4x^3 + 36x^2}{x+9} \cdot \frac{9}{4x^2}$$

$$53) \frac{x-5}{x^2 - 7x + 10} \div \frac{1}{x+2}$$

$$54) \frac{k^2 - 7k + 12}{k^2 + 3k - 28} \div \frac{k-3}{6}$$

$$55) \frac{7}{7(m-6)} \div \frac{1}{6m}$$

$$56) \frac{8m^2}{8m^2(m+2)} \div \frac{1}{3(m+2)}$$

$$57) \frac{\frac{4}{x^2}}{\frac{2}{3}}$$

$$58) \frac{\frac{4}{m} - \frac{1}{2}}{\frac{m^2}{m+1}}$$

$$59) \frac{\frac{1}{5} - \frac{x}{5}}{\frac{x}{5}}$$

$$60) \frac{\frac{m^2}{2m+1} - \frac{15}{2m+1}}{\frac{1}{5} - \frac{5}{m^2}}$$

**Write the slope-intercept form of the equation of the line through the given point with the given slope.**

61) through:  $(1, -4)$ , slope =  $-6$

62) through:  $(-4, -2)$ , slope =  $\frac{3}{2}$

**Write the slope-intercept form of the equation of the line described.**

63) through:  $(3, -1)$ , parallel to  $y = -\frac{4}{3}x$

64) through:  $(2, 3)$ , parallel to  $y = -\frac{1}{2}x$

65) through:  $(3, -4)$ , perp. to  $y = \frac{1}{3}x - 3$

66) through:  $(2, -4)$ , perp. to  $y = x + 3$

**Write the slope-intercept form of the equation of the line through the given points.**

67) through:  $(0, 1)$  and  $(1, 0)$

68) through:  $(-5, 0)$  and  $(5, 5)$

**Solve each equation.**

69)  $-3(1 + 7x) - 3(-6x + 6) = x - 5 + 8$

70)  $-2 + 8(3 + x) = -7(x - 1)$

**Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.**

71) 
$$\frac{\left(\frac{1}{x^4}y^{\frac{1}{2}}\right)^{-\frac{3}{4}}}{x^{\frac{3}{4}}y^{-\frac{1}{4}} \cdot y}$$

72) 
$$\left(\frac{u^2 \cdot vu^0}{u^{-1}v^{-\frac{1}{3}}}\right)^{-2}$$

**Find the inverse of each function.**

$$73) \ f(x) = x - 5$$

$$74) \ g(x) = -\frac{1}{2}x + 2$$

$$75) \ g(x) = \frac{-3x + 9}{8}$$

$$76) \ f(n) = -\frac{2}{n+1} - 3$$

$$77) \ y = \log_3 x - 2$$

$$78) \ y = \log(x - 1)$$

$$79) \ y = 2^{\frac{x}{5}}$$

$$80) \ y = -\frac{5^x}{2}$$

**Perform the indicated operation.**

$$81) \ h(x) = 2x - 2$$
$$g(x) = x^2 - 2x$$

Find  $(h - g)(x)$

$$82) \ f(a) = -2a + 3$$
$$g(a) = a - 5$$

Find  $(f + g)(a)$

$$83) \ h(x) = 4x + 5$$
$$g(x) = 4x$$

Find  $(h - g)(0)$

$$84) \ h(n) = n^2 + 2n$$
$$g(n) = 3n - 2$$

Find  $h(0) - g(0)$

$$85) \ f(a) = a^3 + 5$$
$$g(a) = -a + 3$$

Find  $(f - g)(-a)$

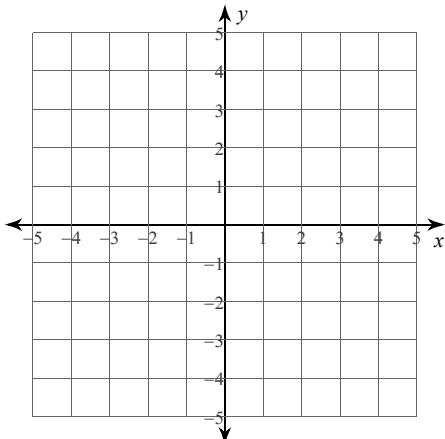
$$86) \ f(n) = -n$$
$$g(n) = 3n - 5$$

Find  $\left(\frac{f}{g}\right)(4n)$

**Sketch the solution to each system of inequalities.**

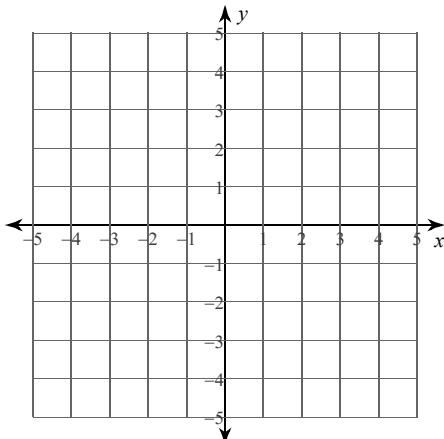
87)  $y > -\frac{2}{3}x - 1$

$$y \leq \frac{1}{3}x + 2$$



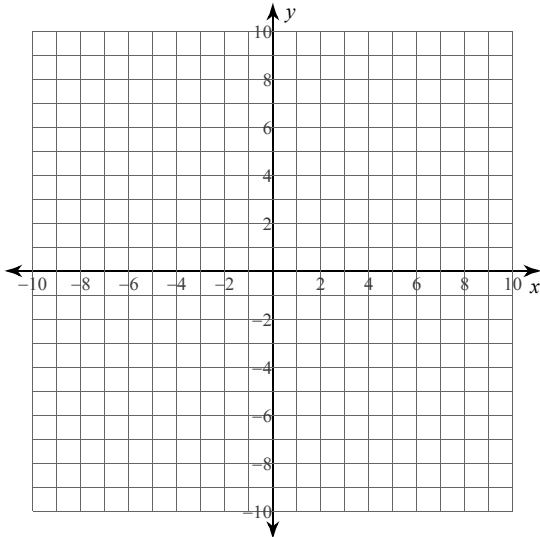
88)  $y \leq 2x - 3$

$$y < -4x + 3$$



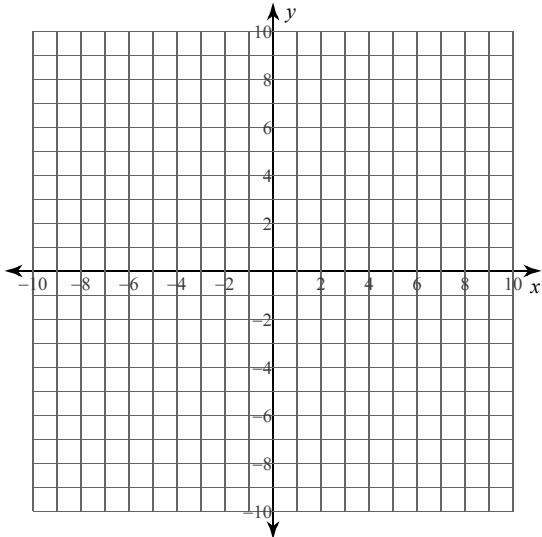
89)  $11x - 5y \geq -40$

$$x + y > -8$$



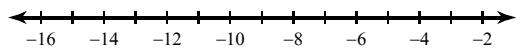
90)  $x \leq -7$

$$3x - 7y > 21$$

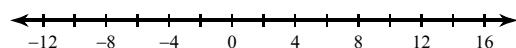


**Solve each inequality and graph its solution.**

91)  $\frac{|9+x|}{4} \leq 1$

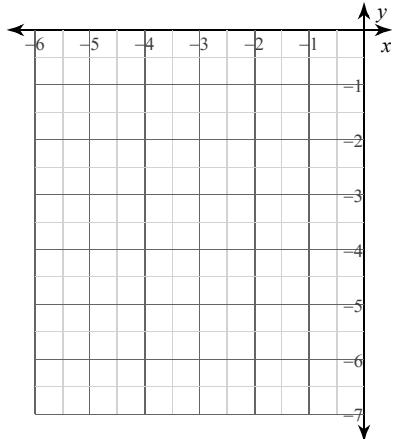


92)  $-2|3x - 4| \leq -62$

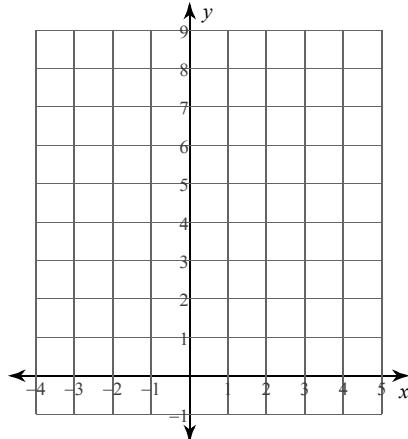


**Sketch the graph of each inequality.**

93)  $y < -x^2 - 7x - 15$



94)  $y \leq 2x^2 - 6x + 5$



**Solve each system of equations.**

95)  $x^2 + 2y^2 + 37x - y + 36 = 0$   
 $3x + y + 3 = 0$

96)  $-3x^2 + y^2 + 32x + 2y - 101 = 0$   
 $x + y - 1 = 0$

97)  $x^2 + 2y^2 + 10x - 7y + 16 = 0$   
 $x^2 - 10y^2 + 10x - 7y + 16 = 0$

98)  $x^2 + y^2 - 4x + 6y - 61 = 0$   
 $x^2 + y^2 - 11x + 6y + 2 = 0$

**State if the point given is a solution to the system of equations.**

99)  $x^2 - 9x - y + 12 = 0$   
 $x + y + 4 = 0$   
Point:  $(4, -8)$

100)  $x^2 + y^2 + 16x - 2y + 49 = 0$   
 $x^2 + y^2 + 16x - 19y + 134 = 0$   
Point:  $(5, 10)$

**Solve each system by elimination.**

101)  $a + 6b + c = -21$   
 $4a + b + 4c = 8$   
 $-a - 3b - 2c = 10$

102)  $-3r - 3s + 6t = 15$   
 $2r - 3s - 3t = -9$   
 $-4r - 6s + 3t = -12$

## AP Precalculus Summer Packet Part B

For each problem, find the average rate of change of the function over the given interval.

1)  $f(x) = x^2 + 1; [1, \frac{5}{4}]$

2)  $f(x) = 2x^2 - 2x - 2; [0, \frac{1}{3}]$

3)  $f(x) = \frac{1}{x+3}; [-1, 0]$

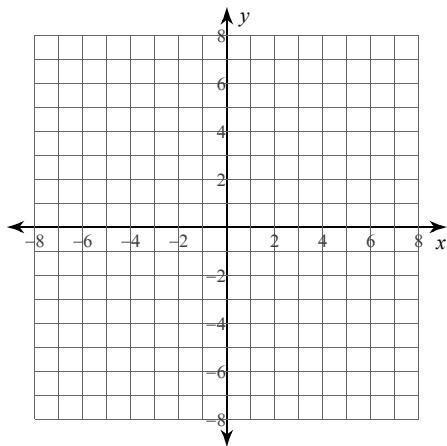
4)  $f(x) = \frac{1}{x-2}; [-4, -2]$

5)  $f(x) = 2x^2 + 2; [-1, -\frac{2}{3}]$

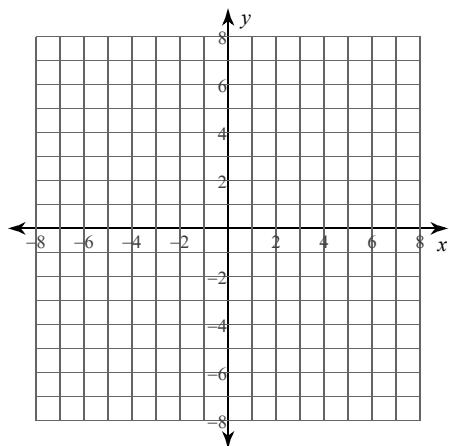
6)  $f(x) = 2x^2 + 1; [-1, -\frac{2}{3}]$

Sketch the graph of each function.

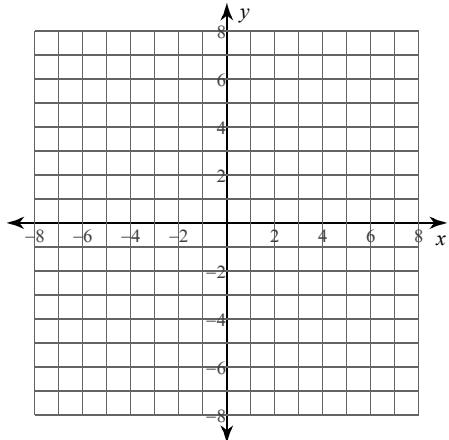
7)  $w(x) = \begin{cases} -4 + \sqrt{x}, & x < 1 \\ -x - 1, & x \geq 1 \end{cases}$



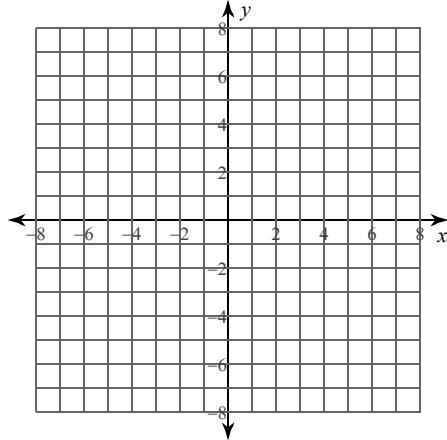
8)  $f(x) = \begin{cases} 2x - 3, & x \leq 1 \\ (x - 2)^2, & x > 1 \end{cases}$



9)  $f(x) = \begin{cases} \sqrt{-4x}, & x \neq -4 \\ -6, & x = -4 \end{cases}$



10)  $f(x) = \begin{cases} -|x|, & x \leq 1 \\ 5, & x > 1 \end{cases}$



**Describe the transformations necessary to transform the graph of  $f(x)$  into that of  $g(x)$ .**

11)  $f(x) = \sqrt{x}$   
 $g(x) = -\sqrt{-\frac{1}{2}(x+2)} - 1$

12)  $f(x) = x^3$   
 $g(x) = -(2(x-3))^3 + 2$

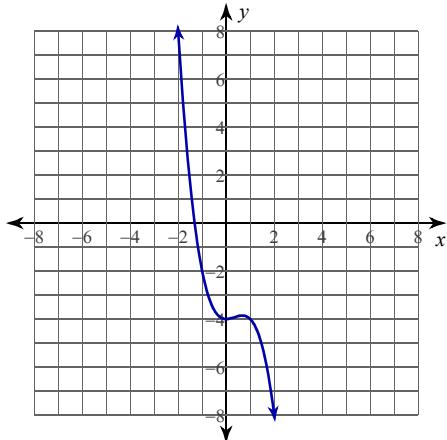
**Transform the given function  $f(x)$  as described and write the resulting function as an equation.**

13)  $f(x) = x^2$   
 expand vertically by a factor of 2  
 reflect across the x-axis  
 translate left 3 units  
 translate down 1 unit

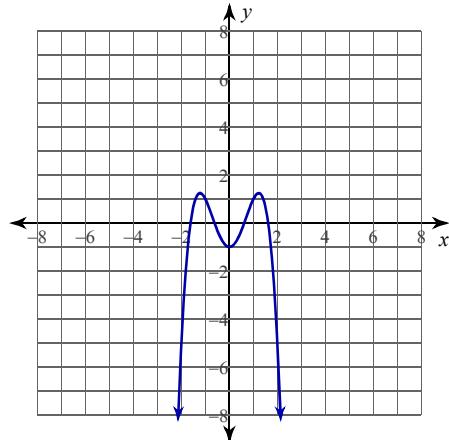
14)  $f(x) = \frac{1}{x}$   
 compress horizontally by a factor of 2  
 reflect across the x-axis  
 translate right 2 units  
 translate down 3 units

**Approximate the intervals where each function is increasing and decreasing.**

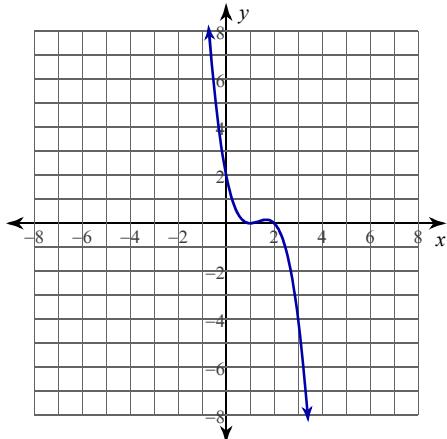
15)



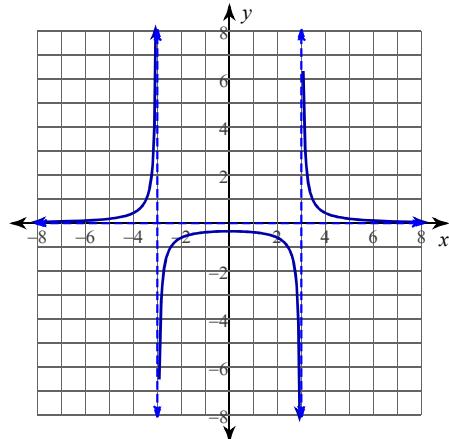
16)



17)



18)



**Solve each equation.**

19)  $8^{2p} = 4^{2p-3}$

20)  $4^{-v} = 1$

**Expand each logarithm.**

21)  $\log_6 (3 \cdot 11^6)^4$

22)  $\log_9 (c^3 \sqrt[3]{a})$

**Condense each expression to a single logarithm.**

23)  $5 \log_4 x + 3 \log_4 y$

24)  $4 \log u - 6 \log v$

**Solve each equation.**

25)  $10^{10k} - 10 = 61$

26)  $-10 \cdot 12^{2x} = -95$

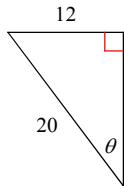
**Convert each degree measure into radians and each radian measure into degrees.**

27)  $-90^\circ$

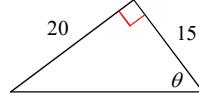
28)  $-320^\circ$

**Find the value of the trig function indicated.**

29)  $\cos \theta$



30)  $\tan \theta$



**In each triangle ABC, angle C is a right angle. Find the value of the trig function indicated.**

31) Find  $\sin A$  if  $a = 20$ ,  $c = 25$

32) Find  $\cos A$  if  $c = 13$ ,  $b = 12$

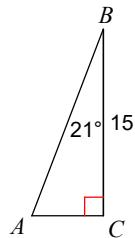
**Find the value of the trig function indicated.**

33) Find  $\sin \theta$  if  $\tan \theta = \frac{10\sqrt{21}}{21}$

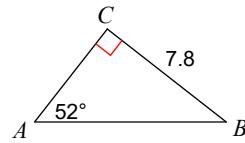
34) Find  $\sin \theta$  if  $\tan \theta = \frac{3}{4}$

**Solve each triangle. Round answers to the nearest tenth.**

35)

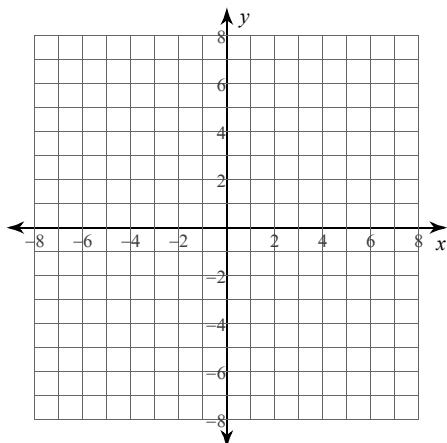


36)

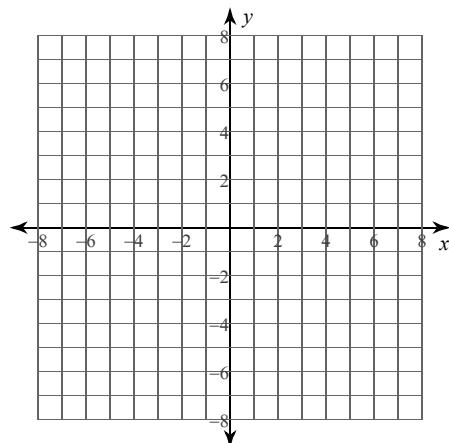


**Identify the center and radius of each. Then sketch the graph.**

37)  $(x + 1)^2 + (y + 3)^2 = 4$



38)  $(x + 4)^2 + (y + 4)^2 = 4$



**Use the information provided to write the standard form equation of each circle.**

39) Center:  $(-14, -12)$   
Radius: 4

40) Center:  $(3, -7)$   
Radius: 1

41) Center:  $(-3, -1)$   
Point on Circle:  $(12, 4)$

42) Center:  $(-16, 4)$   
Point on Circle:  $(-15, 3)$