Name__________________

Advanced Placement Calculus BC Summer Assignment

The following is the required AP Summer Assignment for AP Calculus BC. Most questions in this packet were included because they involve skills and concepts that will be used extensively in BC calculus next year, and not retaught. Others are included NOT because they are frequently used skills, but because being able to solve them indicates a strong grasp of important mathematical concepts, and more importantly the ability to problem solve. BC Calculus is a challenging, demanding, and rewarding math course!

Therefore, this will be **collected the first day** of school in September and be used as the first quiz grade of the school year. Students who do not complete or arrive at school without this assignment in September will receive a zero (0/20) for the first quiz. Since most of the questions cannot be done without work, a ten (10/20) will be the score if the supporting work for each problem is not submitted. Simply completing the assignment, and submitting all supporting work will earn a score of twenty (20/20). **All work must be neatly organized. Calculator use is not permitted.** If you have any questions about this assignment, they may be directed to Mrs. Covington in Room 439 on or before June. Enjoy your summer and see you in September!!

A: Basic Algebra Skills

A1. True or false. If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

g. If $(x+3)(x-10) = \underline{2}$, then $x+3 = \underline{2}$ or $x-10 = \underline{2}$. T F

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T: Trigonometry

You should be able to answer these quickly, *without* using calculator and without referring to (or drawing) a unit circle.

T1. Evaluate Trig Functions without a calculator:

1. $\cos\pi$
2. $\sin\dfrac{\pi}{6}$
3. $\sec 210^\circ$
4. $\tan 90^\circ$
5. $\csc(-150)$
6. $\csc\dfrac{3\pi}{2}$
7. $\cos 0$
8. $\sin^{-1}\dfrac{-1}{2}$
9. $\mathrm{Cos}^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$
10. $\tan^{-1} 1$
11. $\arcsin 0$
12. $\mathrm{Tan}^{-1}\left(-\sqrt{3}\right)$
13. $\sin\dfrac{2\pi}{3}$
14. $\mathrm{Sin}^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$
15. $\arctan 0$

T2. Find the value of each expression, in exact form.

a. $\sin\frac{2\pi}{3}$ **b.** $\cos\frac{11\pi}{6}$

c. $\tan\frac{3\pi}{4}$ **d.** $\sec\frac{5\pi}{3}$

e. $\csc\frac{7\pi}{4}$ **f.** $\cot\frac{5\pi}{6}$

Note: You will need to know your trig identities, Sum & Difference & Double Angle Formulas:

Memorize the following Trig Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\quad$ $\sec^2\theta = 1 + \tan^2\theta$ $\quad$ $\csc^2\theta = 1 + \cot^2\theta$

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = \cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ $\quad$ $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

T3 Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\cos x = -1$

c. $\tan x = \sqrt{3}$

d. $\sec x = -2$

e. $\csc x$ is undefined

f. $\cot x = 1$

T4. Solve the equation. Give *all* real solutions, if any.

a. $\sin 3x = 1$

b. $2\sqrt{3}\cos(\pi x) = 3$

c. $\tan 2x = 0$

d. $4\sec x + 1 = 9$

e. $\csc(4x+3) = 0$

f. $3\cot 6x + \sqrt{3} = 0$

T5. Solve by factoring. Give *all* real solutions, if any.

a. $4\sin^2 x + 4\sin x + 1 = 0$

b. $\cos^2 x - \cos x = 0$

c. $\sin x\cos x - \sin^2 x = 0$

d. $x\tan x + 3\tan x = x + 3$

T6. Graph each function, identifying x- and y-intercepts, if any, and asymptotes, if any.

a. $y = -\sin(2x)$

b. $y = 4 + \cos x$

c. $y = \tan x - 1$

d. $y = \sec x + 1$

e. $y = \csc(\pi x)$

f. $y = 2\cot x$

S: Solving

S1. Solve by factoring.

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

S2. Solve by factoring. You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5(x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2(3x - 4)^2$

S3. Solve. (*Hint*: Each question *can* be solved by factoring, but there are other methods, too)

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

d. $\dfrac{6}{(2x + 1)^2} + \dfrac{3}{2x + 1} = 1 + \dfrac{2}{2x + 1}$

S4. Solving Inequalities: *Solve and graph the solution*

a. $|x - 3| > 12$

b. $|x - 3| \leq 4$

c. $|10x + 8| > 2$

d. $x^2 - 16 < 0$

e. $x^2 + 6x - 16 \leq 0$

f. $x^2 - 3x \geq 10$

L: Logarithms and Exponential Functions

L1. Evaluate Logarithms and Exponentials without a calculator

a. $\log_4 64$ **b.** $\log_3 \frac{1}{9}$ **c.** $\log 10$ **d.** $\ln e$

e. $\ln 1$ **f.** $\ln e^3$ **g.** $3^{\log_3 7}$ **h.** $4^{\log_4 \sin x}$

L2. Expand as much as possible.

a. $\ln x^2y^3$

b. $\ln \frac{x+3}{4y}$

c. $\ln 3\sqrt{x}$

d. $\ln 4xy$

L3. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$

b. $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$

d. $\frac{\ln x}{\ln 2}$

(contrast with part **c**)

L4. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $\ln (x+3) = 2$

b. $\ln x + \ln 4 = 1$

c. $\ln x + \ln (x+2) = \ln 3$

d. $\ln (x+1) - \ln (2x-3) = \ln 2$

L5. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $e^{4x+5} = 1$

b. $2^x = 8^{4x-1}$

c. $100e^{x\ln 4} = 50$

d. $2^x = 3^{x-1}$

(need rounded answer only in **d**)

L6. Round final answers to 3 decimal places.

a. At $t = 0$ there were 140 million bacteria cells in a petri dish. After 6 hours, there were 320 million cells. If the population grew exponentially for $t \geq 0$...

...how many cells were in the dish 11 hours after the experiment began?

...after how many hours will there be 1 billion cells?

b. The *half-life* of a substance is the time it takes for half of the substance to decay. The *half-life* of Carbon-14 is 5568 years. If the decay is exponential...

...what percentage of a Carbon-14 specimen decays in 100 years?

...how many years does it take for 90% of a Carbon-14 specimen to decay?

F: FUNCTIONS

Graph each of the following Parent Functions and be familiar with these graphs

1. $f(x)=x$	2. $f(x)=x^2$	3. $f(x)=x^3$
4. $f(x)=\|x\|$	5. $f(x)=\sqrt{x}$	6. $f(x)=\frac{1}{x}$
7. $f(x)=\frac{1}{x^2}$	8. $f(x)=e^x$	9. $f(x)=\ln x$
10. $f(x)=\sin x$	11. $f(x)=\cos\ x$	12. $f(x)=\tan x$
13. $f(x)=\tan^{-1} x$	14. $f(x)=x^{\frac{2}{3}}$	15. $f(x)=\frac{1}{1+x^2}$
16. $f(x)=[x]$	17. $f(x)=\sqrt{1-x^2}$	18. $f(x)=\frac{\|x\|}{x}$

Analyzing Functions

F1. Increasing/Decreasing

Determine the interval(s) over which f(x) is:

a. Increasing ____________________

b. Decreasing ____________________

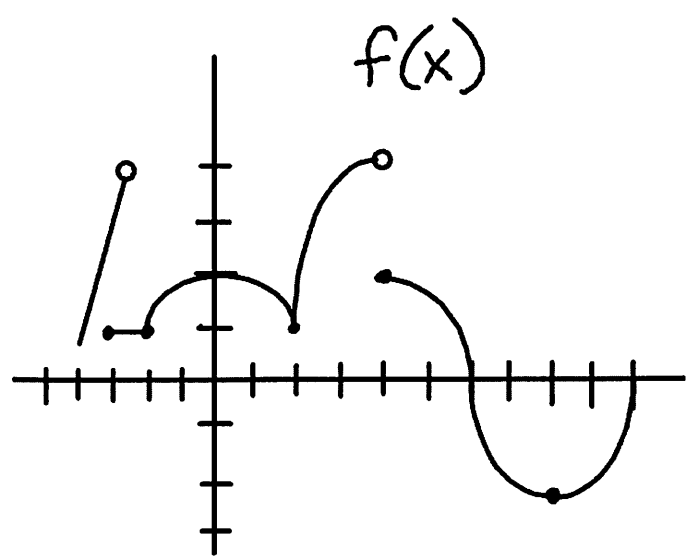
c. Constant ____________________

d. Linear ____________________

e. Concave Up ____________________

f. What are the zeros of f? ____________

g. For what values of x is f(x) discontinuous? ________________



F2. Compositions

1. Let $f(x)=3x^2$ and $g(x)=\frac{x-9}{x+1}$, find the following:

a. $f(g(x))$ b. $g(f(x))$ c. $f^{-1}(x)$

d. Domain, Range, and Zeros of f(x) e. Domain, Range, and Zeros of g(x)

Find f^{-1} and verify that $(f\circ f^{-1})(x)=(f^{-1}\circ f)(x)=x$.

2. $f(x)=2x+3$ 3. $f(x)=x^3-1$

F3. Piecewise Functions:

Graph and then evaluate the function at the indicated points.

1. $f(x)=\begin{cases} 3x+2, & x>3 \\ -x+4, & x\leq 3 \end{cases}$

a. f(2) b. f(3) c. f(5)

2. $f(x)=\begin{cases} x^2\text{-}1, & x<-2 \\ 4, & -2\leq x\leq 1 \\ 3x+1, & 1<x<3 \\ x^2-1, & x>3 \end{cases}$

a. f(-3) b. f(-2) c. f(2) d. f(5) e) f(3)

F4. Even/Odd Functions

Show work to determine if the relation is even, odd, or neither.

a. $f(x)=2x^2-7$ **b.** $f(x)=-4x^3-2x$ **c.** $f(x)=4x^2-4x+4$

d. $f(x)=x-\frac{1}{x}$ **e.** $f(x)=|x|-x^2+1$ **f.** $f(x)=\sin x+x$

F5. Domains of Functions: Find the Domain of each.

a. $y=\frac{3x-2}{4x+1}$ **b.** $y=\frac{x^2-4}{2x+4}$ **c.** $y=\frac{x^2-5x-6}{x^2-3x-18}$

d. $y=\frac{2^{2-x}}{x}$ **e.** $y=\sqrt{x-3}-\sqrt{x+3}$ **f.** $y=\frac{\sqrt{2x-9}}{2x+9}$

F6. Asymptotes

Find the equation of both Horizontal and Vertical Asymptotes for the following functions. Find the coordinates of any holes.

a. $y=\frac{x}{x-3}$ **b.** $y=\frac{x+4}{x^2-1}$

c. $y=\frac{x^2-2x+1}{x^2-3x-4}$ **d.** $y=\frac{x^2-9}{x^3-3x^2-18x}$

R: Rational Expressions and Equations

R1.

	Function	Domain	Hole(s): (x, y) if any	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$				
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$				
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	skip	

R2. Write the equation of a function that has...

a. asymptotes $y = 4$ and $x = 1$, and a hole at (3, 5)

b. holes at (-2, 1) and (2, -1), an asymptote $x = 0$, and no horizontal asymptote

R3. Find the x-coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of x, if any, is the output of the function $f(x)=\dfrac{x^2+4}{e^{6x}-1}$

...equal to zero? ...undefined?

b. For what real value(s) of x, if any, is the output of $g(x)=\dfrac{\cos^2(\pi x)}{\sin x+2}$...

...equal to zero? ...undefined?

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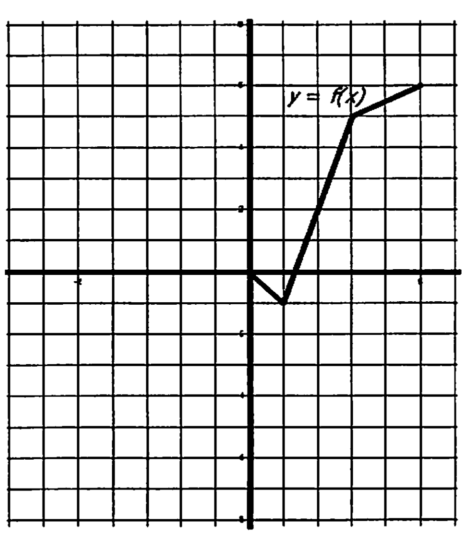
G: Graphing

G1. PART of the graph of f is given. Each gridline represents 1 unit.



a. Complete the graph to make f an EVEN function.

b. What are the domain and range of f_{even}?

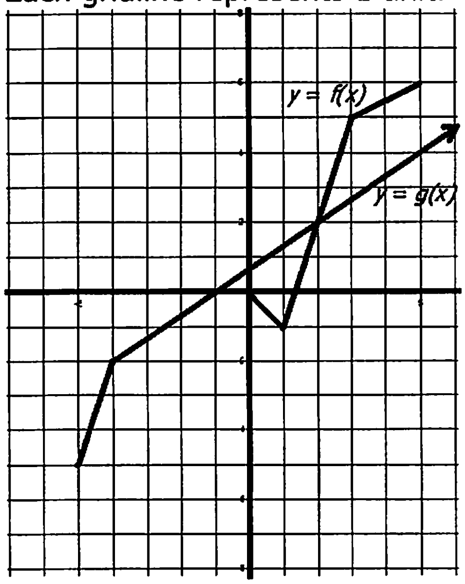
c. What is $f_{even}(-3)$?

d. Complete the graph to make f an ODD function.

e. What are the domain and range of f_{odd}?

f. What is $f_{odd}(-3)$?

G2. The graphs of f and g are given. Answer each question, if possible. If impossible, explain why. Each gridline represents 1 unit.



a. $f^{-1}(5) =$

b. $f(g(5)) =$

c. $(g \circ f)(3) =$

d. Solve for x: $f(g(x)) = 5$

e. Solve for x: $f(x) = g(x)$

For parts **f – i**, respond in interval notation.

f. For what values of x is $f(x)$ increasing?

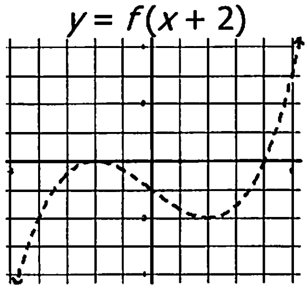
g. For what values of x is $g(x)$ positive?

h. Solve for x: $f(x) < 4$

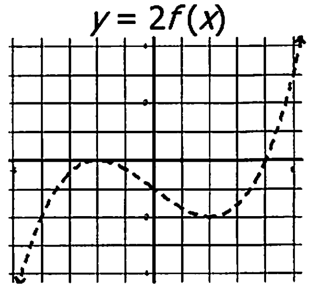
i. Solve for x: $f(x) \geq g(x)$

G3. Given the graph of $y = f(x)$ (dashed graph), sketch each transformed graph.

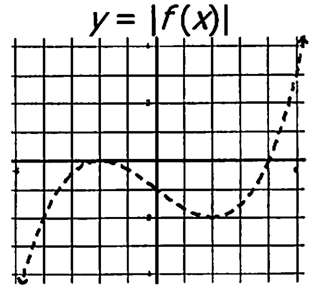
a. $y = f(x + 2)$



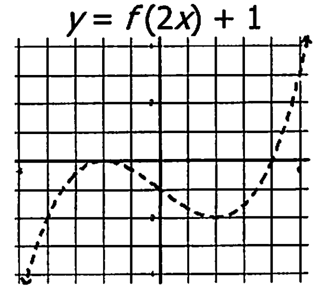
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = f(2x) + 1$



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PF: Partial Fractions

PF1. Find the partial fraction decomposition of

(a) $\dfrac{2x-1}{(x-2)(x-3)}$ (b) $\dfrac{x+7}{x^2-x-6}$ (c) $\dfrac{x^2+2}{(x-1)(x+2)(x-3)}$

SQ: Sequences

SQ 1. Determine if the sequence is arithmetic, geometric or neither. If the sequence is arithmetic, calculate the common difference and find the formula for the nth term. If it is geometric, calculate the common ratio and find the formula for the nth term.

A. 6, 24, 96, 384,... B. 1, 3, 7, 13, ... C. 4, 13, 22, 31,... D. $\frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \ldots$

SQ 2 Find the sum of the first 20 terms of the series.

A. 34 + 25 + 16 + 7 + ... B. 1 + 2 + 4 + 8 + ... C. $\sum_{i=1}^{\infty}\left(5\left(\frac{2}{3}\right)^{i-1}\right)$

SQ 3. Use sigma notation to write each sum.

A. $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots$ B. $2 + 5 + 8 + 11 + \ldots + 29$

C. $-1 + 2 + 7 + 14 + 23 + \ldots. + 62$ D. $-2 + 2 - 2 + 2 - 2 + \ldots.$

7. Let $y = 3\sin(2x - \pi) + \sqrt{2}$. Find the following and give exact values.

Amplitude: ____________ Period: ____________

Vertical shift: __________ Horizontal shift: ________ Range: ____________

8. Find the value for each without using a calculator:

A. $\sin\left(\cos^{-1}\frac{1}{2}\right)$ B. $\sec\left(\sin^{-1}\frac{12}{13}\right)$ C. $\sin\left(\arctan\frac{12}{5}\right)$

9. Find the solutions of the equation on $[0, 2\pi]$:

A. $\sin 2\theta + \sin\theta = 0$ B. $\cos(6\theta + \pi) = 0$ C. $2\sin^2\theta - \sin\theta = 1$

10. If $f(x) = \frac{50}{1 - e^{-x}}$, find $f^{-1}(x)$. Show all your work.

11. If x and y are real numbers, what is the domain of the function $y = \frac{x}{\sqrt{9 - x^2}}$?

12. For the function $f(x) = x^2 + 7$, evaluate and simplify the following:

A. $f(3a - 1)$ B. $\frac{f(x+h) - f(x)}{h}, h \neq 0.$

13. Simplify the following expressions and state any restrictions on the variables:

A. $\frac{v - v^{\frac{3}{2}}}{\sqrt{v}}$ B. $\frac{b^2 - 3b - 4}{b + 1}$ C. $\frac{(x^2y^2 - 1) + (xy - 1)}{(xy - 1)}$ D. $\frac{\frac{1}{x} + \frac{4}{x^2}}{3 - \frac{7}{x}}$

14. State the domain and range for:

A. $y = \arccos x$ Domain: ______________ Range: ______________

B. $y = \arctan x$ Domain: ______________ Range: ______________

C. $y = \log(x - 3)$ Domain: ______________ Range: ______________

D. $f(x) = \sqrt[4]{\frac{3 - x^2}{x - 5}}$ Domain: ______________ Range: ______________