

## Welcome to Honors Math Analysis!

I am excited to delve more deeply into many math topics with you next year!

In order to be best-prepared for our endeavors next year, I have included a number of worksheets for your summer homework. Reviewing these topics will help you to be ready to take on more advanced work next year.

I especially encourage you to diligently work through the problem sets if you are new to the honors level.

You will be expected to solve problems without a calculator throughout the year so now is the time to practice your skills. (Don't worry ... we also use our calculators throughout the year!! :)

I will post the answer keys to all worksheets in August so you can refer to them as needed. I know that checking your answers and figuring out your own mistakes is a useful strategy.

Complete the work at your own pace throughout the summer - I will collect it the first day of school. Your first points of the year!!

Have a great summer and I look forward to next year!

Mrs. Loeffler  
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You should be able to do all of these WITHOUT A CALCULATOR! PRACTICE!  
CIRCLE your answers clearly!

Adapted from <https://smaneworleans.com>

### Function Notation

#### A. Find the value of each in the space provided, showing all steps.

Given:  $f(x) = 3x - 7$  and  $g(x) = x^2 + 3$

$$1) \quad f(-1) = \qquad \qquad \qquad 2) \quad f(x + 3) = \qquad \qquad \qquad 3) \quad f(f(x)) =$$

$$4) \quad g(x + 2) - g(x) = \qquad \qquad \qquad 5) \quad f(g(2)) = \qquad \qquad \qquad 6) \quad g(f(2)) =$$

### Rules of Exponents

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$(a)^{\frac{p}{r}} = \sqrt[r]{a^p}$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

#### B. Simplify each, showing all steps. Answers should have only positive exponents. Do without a calculator.

$$7) \quad (5x^3y)^0(4xy) \qquad \qquad \qquad 8) \quad m^{-4}n^5m^6 \qquad \qquad \qquad 9) \quad \frac{5^{-7}5^8}{5^3}$$

$$10) \quad (3y)^{-3}(3x)^4(9y)$$

$$11) \quad \frac{(d^{-5}e^2f)^{-2}}{(de^{-4}f^3)^{-1}}$$

$$12) \quad \frac{2^48^216^{-5}}{32^{-2}}$$

$$13) \left(\frac{5a^2b}{2ab^2}\right)^2 \left(\frac{-3ab}{2a^2b}\right)^3$$

$$14) (4^{-1} + 5^{-1})^2$$

$$15) \frac{x^{-1} - 5x^{-2}}{3^{-2}}$$

$$16) 3x^{-\frac{1}{6}} \left(4x^{\frac{1}{6}} - 5x^{-\frac{5}{6}}\right)$$

$$17) \frac{(\sqrt{3a})^5}{(\sqrt{3a})^9}$$

$$18) \frac{(\sqrt[3]{2m})^2}{(\sqrt[6]{8m^5})}$$

## Factoring Strategies

Strategies to use:

1. Greatest Common Factor (GCF)
2. Difference of Squares
3. Trinomials
4. Sum and Difference of Cubes  
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$   
 $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
5. Grouping

Factor each completely

### C. Trinomials:

$$19) 2x^2 + 5x + 3$$

$$20) 3x^2 + 7x + 2$$

$$21) 5x^2 - 7x + 2$$

$$22) 6x^2 - 11x + 3$$

$$23) 6x^2 - 13x - 5$$

$$24) 4x^2 - 11x - 3$$

$$25) 7x^2 + 9x + 2$$

$$26) 7x^2 - 10x + 3$$

$$27) 2x^2 - 9x - 5$$

$$28) 2x^2 + xy - 6y^2$$

$$29) 3y^2 - 17xy - 6x^2$$

$$30) 8x^2 - 27x - 20$$

**D. Difference of perfect squares:**

31)  $25x^2 - 9y^2$

32)  $49x^4 - 225y^8$

33)  $81a^4 - 16b^8$

**E. Sums and differences of perfect cubes:**

34)  $125x^3 + 1$

35)  $27x^3 - 8$

36)  $y^6 + 27$

37)  $y^6 - 3$  (yes, factor this! :)

**F. Combinations of perfect squares and cubes:**

38)  $m^6 - 1$

39)  $m^6 + n^{12}$

40)  $m^{12} - n^{24}$

**G. Polynomials in quadratic form:**

41)  $16x^4 - 40x^2 + 25$

42)  $2x^9 + 10x^6 + 12x^3$

43)  $2x^8 - 128x^2$

44)  $9x^4 - 37x^2 + 4$

45)  $x^8 - 5x^4 - 36$

46)  $5x^4 - 79x^2 - 16$

**H. Factor by grouping:**

47)  $2ax + 6xc + ba + 3bc$

48)  $3my + 7x + 7m + 3xy$

49)  $a^2 - 2ab + a - 2b$

50)  $4ax - 14bx + 35by - 10ay$

51)  $x^3 + 2x^2 - x - 2$

52)  $x^2 + 6x + 9 - a^2$

53)  $n^2 + 2nx - 1 + x^2$

54)  $b^2 - y^2 - 2yp - p^2$

55)  $a^2 + 2ab + b^2 - 9$

56)  $x^3 + y^3 - x^2y - xy^2$

**Rational Expressions**

**I. Simplify the following expressions to a single fraction. You may leave your answer in factored form.**

57)  $\frac{2x^2+x-6}{x^2+4x-5} \cdot \frac{x^3-3x^2+2x}{4x^2-6x}$

$$58) \frac{x}{xy^2} - \frac{2x}{x^2}$$

$$59) \frac{x}{x-3} + \frac{2}{3x+4}$$

$$60) \frac{x^2-5x+6}{x-2}$$

$$61) \frac{1-x}{x-1}$$

$$62) \frac{\frac{x^2}{x-1}}{\frac{2x}{x-1}}$$

$$63) \frac{\frac{3}{x+1}-4}{\frac{2x}{x+1}}$$

## Radicals & rational exponents

Simplify radicals whenever possible:

$$\text{ex1)} \quad \sqrt{x^5 y^7 z^6}$$

$$\sqrt{x^2 x^2 x \cdot y^2 y^2 y \cdot z^2 z^2 z^2}$$

$$x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \sqrt{x \cdot y}$$

$$x^2 y^3 z^3 \sqrt{xy}$$

$$\text{ex2)} \quad \sqrt[3]{-32}$$

$$\sqrt[3]{-2 \cdot -2 \cdot -2 \cdot 2 \cdot 2}$$

$$-2 \sqrt[3]{4}$$

$$\text{ex3)} \quad 3x\sqrt{2x} \cdot 4x^2\sqrt{5x}$$

$$3x \cdot 4x^2 \sqrt{2x \cdot 5x}$$

$$12x^3\sqrt{10x^2}$$

$$12x^4\sqrt{10}$$

$$\text{ex4)} \quad 5\sqrt{27} + 4\sqrt{2} + 7\sqrt{3}$$

$$15\sqrt{3} + 4\sqrt{2} + 7\sqrt{3}$$

$$22\sqrt{3} + 4\sqrt{2}$$

Rationalize fractions with radicals in the denominator:

$$\text{ex5)} \quad \frac{\sqrt[4]{24}}{\sqrt{15}}$$

$$\frac{\sqrt[4]{8}}{\sqrt{5}}$$

$$\frac{\sqrt[3]{2}}{\sqrt{5}}$$

$$\frac{\sqrt[3]{2}\sqrt[3]{5}}{\sqrt{5}\sqrt{5}}$$

$$\frac{2\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$\frac{2\sqrt{10}}{5}$$

$$\text{ex6)} \quad \frac{2}{\sqrt[4]{32}}$$

$$\frac{2}{\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}}$$

$$\frac{\cancel{2}}{\cancel{2} \cdot \cancel{\sqrt[4]{2}}}$$

$$\frac{1}{\sqrt[4]{2}}, \text{ now rationalize}$$

$$\frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2 \cdot 2 \cdot 2}}{\sqrt[4]{2 \cdot 2 \cdot 2}}$$

$$\frac{\sqrt[4]{8}}{\sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}}$$

$$\frac{\sqrt[4]{8}}{2}$$

$$\text{ex7)} \quad \frac{1}{2-\sqrt{3}}$$

$$\frac{1}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{2+\sqrt{3}}{2^2 - (\sqrt{3})^2}$$

$$\frac{2+\sqrt{3}}{4-3}$$

$$2+\sqrt{3}$$

**J. Simplify each:**

64)  $\sqrt[3]{24}$

65)  $\sqrt[3]{-40x^6y^7}$

66)  $\sqrt{75x^3} \cdot \sqrt{5x^3}$

67)  $2\sqrt{48} - 3\sqrt{27}$

**K. Simplify each:**

68) Write in radical form and simplify:  $64^{\frac{1}{2}}$

69) Simplify, then add like radicals:  
 $\sqrt{18} - \sqrt{75} + \sqrt{8} - 4\sqrt{2}$

70) Write in rational exponent form:

$10 \sqrt[5]{x^3}$

71) Write in rational exponent form:

$\sqrt[5]{10x^3}$

72)  $\sqrt[3]{27x^3y^6}$

73)  $\sqrt{16x^5}$

**L. Rationalize the following:**

74)  $\frac{5}{2\sqrt{3}}$

75)  $\frac{2}{\sqrt[3]{5}}$

76)  $\frac{2}{3+\sqrt{7}}$

Adapted from <https://www.cotterschools.org/>

**M. Evaluate using the order of operations, PERMDAS:**

77)  $\frac{3xy^2-1}{3x^3y+1}$  when  $x = -3, y = -2$

78)  $-x^2 - 4x$  when  $= -1$

**N. Solve equations:**

79)  $-3(3x + 2) = 6(-3 - 2x) - 2$

80)  $2(5x - 9) = 3 - 8(x + 2)$

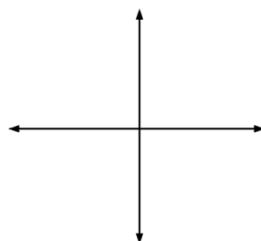
**O. Name the quadrant:**

81) In which quadrant is  $(5, -4)$ ?

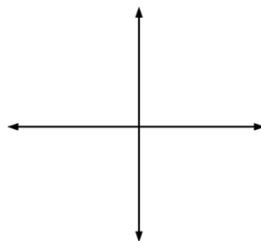
82) In which quadrant is the terminal side of  $\theta = -\frac{4\pi}{3}$ ?

**P. Sketch the graphs of the following linear equations:**

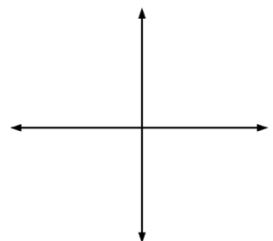
83)  $x = -3$



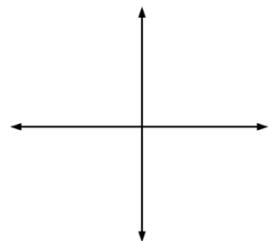
85)  $y = -3x + 5$



84)  $y = 5$



86)  $2x - 3y = 6$



**Q. Answer the following about linear functions:**

87) Find the slope of  $(-15, 11)$  and  $(8, -4)$

88) Find the intercepts of  $-4x - 3y = 16$

x-intercept \_\_\_\_\_

y-intercept \_\_\_\_\_

89) Rewrite in slope-intercept form:

$$-4x - 3y = 1$$

90) Write an equation of the line containing

$(1, 1)$  and  $(2, -2)$ .

slope: \_\_\_\_\_ y-intercept \_\_\_\_\_

91) Write an equation of the line passing through  $(7, 2)$  and having a slope of 0.

92) Write an equation of the line having an undefined slope (no slope) and passing through the point  $(-2, 5)$ .

**R. Rational expressions – Simplify COMPLETELY**

$$93) \quad \frac{x^2+6x+8}{x^2+3x} \div \frac{x^2+x-2}{4x^2+12x}$$

$$94) \quad \frac{\frac{2}{3x}}{\frac{1}{2} + \frac{5}{3x}}$$

$$95) \quad \frac{\frac{1}{x} + \frac{3}{x+1}}{\frac{3}{x-1} + \frac{1}{x}}$$

$$96) \quad \frac{\frac{x^2+2x+1}{x^2-4}}{\frac{x+1}{x^2-x-6}}$$

## Section 3 - Common Errors in Algebra

Many algebra errors come from not mastering the differences between the rules of addition and multiplication! Test yourself by doing the following problems. Check your answers by substituting numbers for the variables. There may be more than one correct form of the answer.

### I. ERRORS INVOLVING PARENTHESES

#### A. Distributing a negative sign

1.  $3 - (x - 2) =$       a)  $3 - x - 2$       b)  $1 - x$       c)  $5 - x$

2.  $\frac{3x}{x+2} - \frac{x+1}{x+2} =$       a)  $\frac{2x+1}{x+2}$       b)  $\frac{2x-1}{x+2}$

3.  $\frac{3x+2}{5x} - \frac{2(x+1)}{5x} =$       a)  $\frac{1}{5}$       b)  $\frac{x+1}{5x}$       c)  $\frac{x+4}{5x}$

#### B. Distributing Left and Right

1.  $4(x+2)(2) =$       a)  $8x+16$       b)  $4x+4$       c)  $4x+16$

#### C. DO NOT distribute exponents over addition or subtraction

Does  $(a+b)^2 = a^2 + b^2$  ?

Convince yourself: Does  $(3+4)^2 = 3^2 + 4^2$  ?

#### D. DO NOT distribute when there is only multiplication

1.  $\left(\frac{1}{2}\right)(a)(b) =$       a)  $\left(\frac{1}{2}a\right)\left(\frac{1}{2}b\right)$       b)  $\left(\frac{1}{2}\right)ab$       c)  $\frac{1}{2}(ab)$       d)  $\frac{ab}{2}$

### II. ERRORS INVOLVING FRACTIONS

1. Does  $\frac{a+b}{x} = \frac{a}{x} + \frac{b}{x}$  ?      Does  $\frac{x}{a+b} = \frac{x}{a} + \frac{x}{b}$  ?

2.  $\frac{1}{a} + \frac{1}{b} =$       a)  $\frac{1}{a+b}$       b)  $\frac{b+a}{ab}$

3.  $\frac{x/a}{b} =$       a)  $\frac{bx}{a}$       b)  $\frac{x}{ab}$

4.  $\left(\frac{1}{3}\right)x =$       a)  $\frac{1}{3x}$       b)  $\frac{x}{3}$

5.  $\frac{1}{x} + 2 =$       a)  $\frac{1}{x+2}$       b)  $\frac{1+2x}{x}$

6.  $\frac{x+1}{x+1} =$       a) 1      b) 0

7.  $\frac{(x+1)}{(x+1)(x+3)} =$       a)  $\frac{1}{x+3}$       b)  $x+3$

8. Which are possible steps for doing on a calculator  $\frac{50}{(5)(2)}$ ?  
 a)  $50 \div 5 \times 2$       b)  $50 \div 5 \div 2$       c)  $50 \div (5 \times 2)$

### III. ERRORS INVOLVING EXPONENTS

1.  $(x^2)^3 =$       a)  $x^5$       b)  $x^6$

2.  $x^3 \cdot x^5 =$       a)  $x^8$       b)  $x^{15}$

3. Does  $4x^2 - 2x^2 = 4x^2 - (2x)^2$ ?

4.  $\frac{9}{5x^3} =$       a)  $\frac{9}{5}(x^{-3})$       b)  $\left(\frac{9}{5}\right)x^3$

5.  $\frac{7}{\sqrt{2x-3}} =$       a)  $7(2x-3)^{\frac{1}{2}}$       b)  $7(2x-3)^{-\frac{1}{2}}$

6. Which are correct steps for doing on a calculator  $2^{2+3}$ ?

a)  $2^2 \times 3$       b)  $2^{\wedge}(2 \times 3)$

### IV. ERRORS INVOLVING RADICALS

1.  $\sqrt{5x} =$       a)  $5\sqrt{x}$       b)  $\sqrt{5x}$       c)  $\sqrt{5} \cdot \sqrt{x}$

2. Does  $\sqrt{a^2 + b^2} = a + b$ ?      Check: Does  $\sqrt{3^2 + 4^2} = 3 + 4$ ?

3. Does  $\sqrt{(a+b)^2} = a + b$ ?      Check: Does  $\sqrt{(3+4)^2} = 3 + 4$ ?

4.  $\frac{\sqrt{64}}{2} =$       a)  $\sqrt{32}$       b)  $\frac{8}{2}$  or 4

5.  $\frac{\sqrt{32}}{2} =$       a)  $\sqrt{16}$       b)  $\frac{4\sqrt{2}}{2}$  or  $2\sqrt{2}$

6. Which are correct calculator steps for  $\sqrt{3^2 + 4^2}$ ?

a)  $\sqrt{3^2} + \sqrt{4^2}$       b)  $\sqrt{(3^2 + 4^2)}$   
 c)  $(3^2 + 4^2)^{1 \div 2}$       d)  $(3^2 + 4^2)^{(1 \div 2)}$

## Answers to *Common Errors in Algebra*

I.      A.    1. c    2. b    3. a

B.    1. a

C.    NO!!!

D.    1. b or c or d

II.    1. Yes, No

2. b

3. b

4. b

5. b

6. a

7. a

8. b or c

III.    1. b

2. a

3. NO!!

4. a

5. b

6. b

IV.    1. c

2. NO!!

3. Yes

4. b

5. b

6. b or d

V. A.    1. b or c

2. b

3. b

4. a

5. c

6. a