Calculus Summer Work

About This Packet
(…and this class)

Welcome!

This packet includes a sampling of problems that students *entering* Calculus should be able to answer. The questions are organized by topic:

A Super-Basic Algebra Skills
T Trigonometry
F HigherLevel Factoring
L Logarithms and Exponential Functions
R Rational Expressions and Equations
G Graphing

In Calculus, it's rarely the calculus that'll get you; it's the algebra. Students entering Calculus absolutely *must* have a strong foundation in algebra. Most questions in this packet were included because they concern skills and concepts that will be used extensively in Calculus. Others have been included not so much because they are skills that are used frequently, but because being able to answer them indicates a strong grasp of important mathematical concepts and—more importantly—the ability to problem-solve.

An answer key to this packet has been provided at the end of this file. It is extremely important for all students to review the concepts contained in this packet and to be prepared for an <u>assessment of prerequisite skills to take place within the first 3-5 days of school.</u>

The curriculum (and your teacher) will expect you to approach problems with the mathematical *toolkit* needed to do the calculations and the mathematical *understanding* needed to make sense of unusual problems. This is <u>not</u> a class where every problem you see on tests and quizzes is identical to problems you've done dozens of times in class. This is because the AP test itself (and, truly, all "real" mathematics) requires you to take what you know and apply it, rather than to simply regurgitate a rote process.

Now that I've said all that, I encourage you to <u>take a deep breath</u> and start working. If you have the basics down and you put in the work needed, you'll see how amazing Calculus is! Calculus is challenging, demanding, rewarding, and—to put it simply—totally awesome.

A: SuperBasic Algebra Skills

A1. **True or false.** If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = \underline{x^{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = \underline{x^{\frac{3}{2}}}$ T F

c. $(x + 3)^2 = \underline{x^2 + 9}$ T F

d. $\dfrac{x^2 - 1}{x - 1} = \underline{x}$ T F

e. $(4x + 12)^2 = \underline{16}(x + 3)^2$ T F

f. $\underline{3} + 2\sqrt{x - 3} = 5\sqrt{x - 3}$ T F

g. If $(x + 3)(x - 10) = \underline{\ 2\ }$, then $x + 3 = \underline{\ 2\ }$ or $x - 10 = \underline{\ 2\ }$. T F

A2. More basic algebra.

a. If 6 is a zero of f, then _____ is a solution of $f(2x) = 0$.

b. Lucy has the equation $2(4x + 6)^2 - 8 = 16$. She multiplies both sides by ½. If she does this correctly, what is the resulting equation?

c. Simplify $\dfrac{2 \pm 4\sqrt{10}}{2}$

d. Rationalize the denominator of $\dfrac{12}{3 + \sqrt{x - 1}}$

e. If $f(x) = 3x^2 + x + 5$, then $f(x + h) - f(x) =$ (Give your answer in simplest form.)

f. A cone's volume is given by $V = \frac{1}{3}\pi r^2 h$. If $r = 3h$, write V in terms of h.

g. Write an expression for the area of an equilateral triangle with side length s.

h. Suppose an isosceles <u>right</u> triangle has hypotenuse h. Write an expression for its perimeter in terms of h.

T: Trigonometry

You should be able to answer these quickly, *without* referring to (or drawing) a unit circle.

T1. Find the value of each expression, in exact form.

a. $\sin \frac{2\pi}{3}$

b. $\cos \frac{11\pi}{6}$

c. $\tan \frac{3\pi}{4}$

d. $\sec \frac{5\pi}{3}$

e. $\csc \frac{7\pi}{4}$

f. $\cot \frac{5\pi}{6}$

T2. Find the value(s) of x in $[0, 2\pi)$ which solve each equation.

a. $\sin x = \frac{\sqrt{3}}{2}$

b. $\cos x = -1$

c. $\tan x = \sqrt{3}$

d. $\sec x = -2$

e. $\csc x$ is undefined

f. $\cot x = 1$

T3. Solve the equation. Give *all* real solutions, if any.

a. $\sin 3x = 1$

b. $2\sqrt{3}\cos(\pi x) = 3$

c. $\tan 2x = 0$

d. $4\sec x + 1 = 9$

e. $\csc(4x + 3) = 0$

f. $3\cot 6x + \sqrt{3} = 0$

T4. Solve by factoring. Give *all* real solutions, if any.

a. $4\sin^2 x + 4\sin x + 1 = 0$

b. $\cos^2 x - \cos x = 0$

c. $\sin x \cos x - \sin^2 x = 0$

d. $x\tan x + 3\tan x = x + 3$

T5. Graph each function, identifying x and *y-intercepts*, if any, and asymptotes, if any.

a. $y = -\sin(2x)$

b. $y = 4 + \cos x$

c. $y = \tan x - 1$

d. $y = \sec x + 1$

e. $y = \csc(\pi x)$

f. $y = 2\cot x$

F: HigherLevel Factoring

F1. **Solve by factoring.**

a. $x^3 + 5x^2 - x - 5 = 0$

b. $4x^4 + 36 = 40x^2$

c. $(x^3 - 6)^2 + 3(x^3 - 6) - 10 = 0$

d. $x^5 + 8 = x^3 + 8x^2$

F2. **Solve by factoring.** You should be able to solve each of these *without* multiplying the whole thing out. (In fact, for goodness' sake, please *don't* multiply it all out!)

a. $(x + 2)^2 (x + 6)^3 + (x + 2)(x + 6)^4 = 0$

b. $(2x - 3)^3 (x^2 - 9)^2 + (2x - 3)^5 (x^2 - 9) = 0$

c. $(3x + 11)^5 (x + 5)^2 (2x - 1)^3 + (3x + 11)^4 (x + 5)^4 (2x - 1)^3 = 0$

d. $6x^2 - 5x - 4 = (2x + 1)^2 (3x - 4)^2$

F3. **Solve.** Each question *can* be solved by factoring, but there are other methods, too.

a. $a(3a + 2)^{1/2} + 2(3a + 2)^{3/2} = 0$

b. $\sqrt{2x^2 + x - 6} + \sqrt{2x - 3} = 0$

c. $2\sqrt{x + 3} = x + 3$

d. $\dfrac{6}{(2x + 1)^2} + \dfrac{3}{2x + 1} = 1 + \dfrac{2}{2x + 1}$

L: Logarithms and Exponential Functions

L1. Expand as much as possible.

a. $\ln x^2y^3$

b. $\ln \dfrac{x+3}{4y}$

c. $\ln 3\sqrt{x}$

d. $\ln 4xy$

L2. Condense into the logarithm of a single expression.

a. $4\ln x + 5\ln y$

b. $\frac{2}{3}\ln a + 5\ln 2$

c. $\ln x - \ln 2$

d. $\dfrac{\ln x}{\ln 2}$

(contrast with part **c**)

L3. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $\ln(x+3) = 2$

b. $\ln x + \ln 4 = 1$

c. $\ln x + \ln(x+2) = \ln 3$

d. $\ln(x+1) - \ln(2x-3) = \ln 2$

L4. Solve. Give your answer in exact form *and* rounded to three decimal places.

a. $e^{4x+5} = 1$

b. $2^x = 8^{4x-1}$

c. $100e^{x\ln 4} = 50$

d. $2^x = 3^{x-1}$

(need rounded answer only on **d**)

L5. Round final answers to 3 decimal places.

a. At $t = 0$ there were 140 million bacteria cells in a petri dish. After 6 hours, there were 320 million cells. If the population grew exponentially for $t \geq 0$…

…how many cells were in the dish 11 hours after the experiment began?

…after how many hours will there be 1 billion cells?

b. The *halflife* of a substance is the time it takes for half of the substance to decay. The *halflife* of Carbon14 is 5568 years. If the decay is exponential…

…what percentage of a Carbon14 specimen decays in 100 years?

…how many years does it take for 90% of a Carbon14 specimen to decay?

R: Rational Expressions and Equations

R1.

	Function	Domain	Hole(s): (x, y) if any	Horiz. Asym., if any	Vert. Asym.(s), if any
a.	$f(x)=\dfrac{4x^2+7x-15}{8x^2-14x+5}$				
b.	$f(x)=\dfrac{3(4+x)^2-48}{x}$				
c.	$f(x)=\dfrac{6x+4}{\sqrt{3x^2-10x-8}}$		skip	skip	

R2. Write the equation of a function that has…

a. asymptotes $y = 4$ and $x = 1$, and a hole at (3, 5)

b. holes at (-2, 1) and (2, -1), an asymptote $x = 0$, and no horizontal asymptote

R3. Find the *x*coordinates where the function's output is zero and where it is undefined.

a. For what real value(s) of *x*, if any, is the output of the function $f(x)=\dfrac{x^2+4}{e^{6x}-1}$

…equal to zero? …undefined?

b. For what real value(s) of *x*, if any, is the output of $g(x)=\dfrac{\cos^2(\pi x)}{\sin x+2}$ …

…equal to zero? …undefined?

R4. Simplify completely.

a. $\dfrac{2}{\sqrt{x^2+4}}-\dfrac{x^2+4}{3}$ (Don't worry about rationalizing)

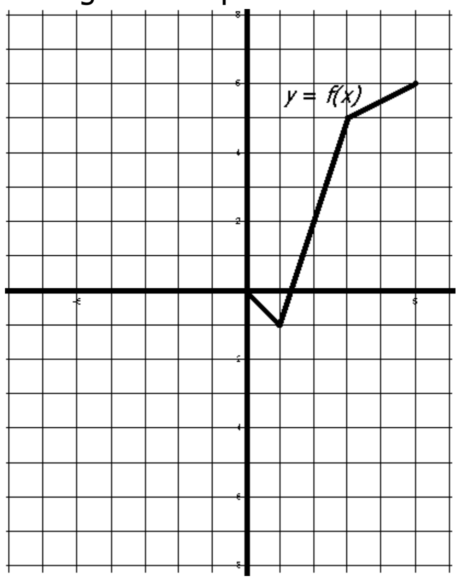
b. $\dfrac{3}{\left(\frac{4}{x}\right)^2+1}$ (Your final answer should have just one numerator and one denominator)

c. $\dfrac{5}{x^2+3x+2}-\dfrac{2x}{x^2+2x+1}$

d. $\dfrac{3}{(x+2)^{1/2}}+\dfrac{x}{(x+2)^{5/2}}$ (Don't worry about rationalizing)

G: Graphing

G1. <u>PART</u> of the graph of f is given. Each gridline represents 1 unit.



a. Complete the graph to make f an EVEN function.

b. What are the domain and range of f_{even}?

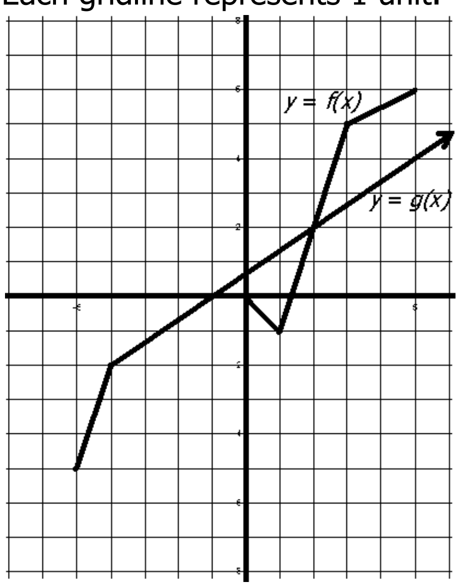
c. What is $f_{even}(-3)$?

d. Complete the graph to make f an ODD function.

e. What are the domain and range of f_{odd}?

f. What is $f_{odd}(-3)$?

G2. The graphs of f and g are given. Answer each question, if possible. If impossible, explain why. Each gridline represents 1 unit.



a. $f^{-1}(5) =$

b. $f(g(5)) =$

c. $(g \circ f)(3) =$

d. Solve for x: $f(g(x)) = 5$

e. Solve for x: $f(x) = g(x)$

For parts **f** – **i**, respond in interval notation.

f. For what values of x is $f(x)$ increasing?

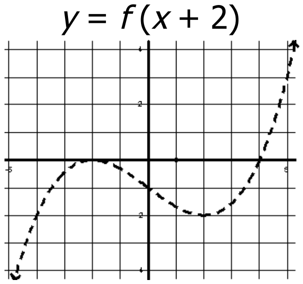
g. For what values of x is $g(x)$ positive?

h. Solve for x: $f(x) < 4$

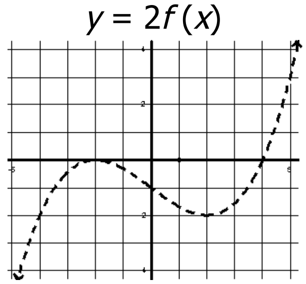
i. Solve for x: $f(x) \geq g(x)$

G3. Given the graph of $y = f(x)$ (dashed graph), sketch each transformed graph.

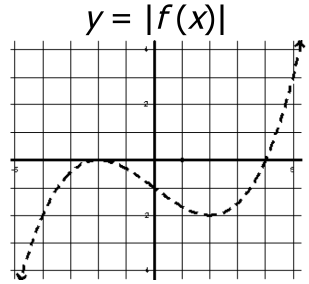
a. $y = f(x + 2)$



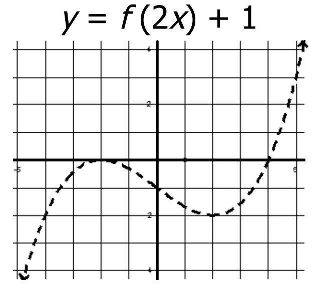
b. $y = 2f(x)$



c. $y = |f(x)|$



d. $y = f(2x) + 1$



Answer Key

A1.

a. true

b. false; 7/2

c. false; $x^2 + 6x + 9$

d. false; $x + 1$

e. true

f. false; $3\sqrt{x-3}$

g. false; 0, 0, 0

A2.

a. $x = 3$

b. $(4x + 6)^2 - 4 = 8$

c. $1 \pm 2\sqrt{10}$

d. $\dfrac{12(3-\sqrt{x-1})}{10-x}$ or $\dfrac{36-12\sqrt{x-1}}{10-x}$

e. $6xh + 3h^2 + h$

f. $V = 3\pi h^3$

g. $\frac{\sqrt{3}}{4}s^2$

h. $h(1+\sqrt{2})$ or anything equivalent

T1.

a. $\frac{\sqrt{3}}{2}$ **b.** $\frac{\sqrt{3}}{2}$

c. -1 **d.** 2

e. $-\sqrt{2}$ **f.** $-\sqrt{3}$

T2.

a. $\frac{\pi}{3}$, $\frac{2\pi}{3}$ **b.** π

c. $\frac{\pi}{3}$, $\frac{4\pi}{3}$ **d.** $\frac{2\pi}{3}$, $\frac{4\pi}{3}$

e. 0, π **f.** $\frac{\pi}{4}$, $\frac{3\pi}{4}$

T3.

a. $x = \frac{\pi}{6} + \frac{2}{3}\pi n$

b. $x = \frac{1}{6} + 2n$, $x = -\frac{1}{6} + 2n$

c. $x = \frac{\pi}{2}n$

d. $x = \frac{\pi}{3} + 2\pi n$, $x = -\frac{\pi}{3} + 2\pi n$

e. no solution

f. $x = \frac{\pi}{9} + \frac{\pi}{6}n$

T4.

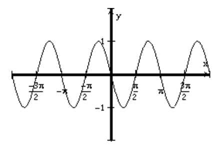
a. $x = -\frac{\pi}{6} + 2\pi n$, $x = \frac{7\pi}{6} + 2\pi n$

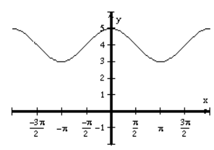
b. $x = \frac{\pi}{2} + \pi n$, $x = 2\pi n$

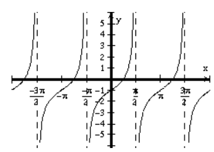
c. $x = \frac{\pi}{4} + \pi n$, $x = \pi n$

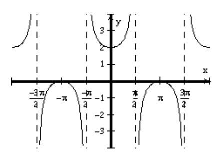
d. $x = \frac{\pi}{4} + \pi n$, $x = -3$

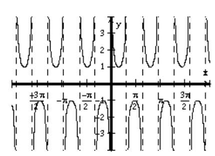
T5.

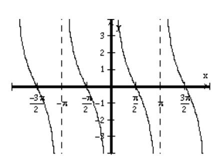
a. 

b. 

c. 

d. 

e. 

f. 

F1. **a.** -5, -1, 1
b. -3, -1, 1, 3
c. 1, 2
d. -1, 1, 2

F2. **a.** -6, -4, -2
b. -3, 0, $\frac{3}{2}$, $\frac{12}{5}$, 3
c. -9, -5, -4, $-\frac{11}{3}$, $\frac{1}{2}$
d. $-\frac{1}{2}$, $\frac{4}{3}$, $\frac{5\pm\sqrt{145}}{12}$

F3. **a.** $-\frac{2}{3}$, $-\frac{4}{7}$
b. $\frac{3}{2}$
c. -3, 1
d. $-\frac{3}{2}$, 1

L1. **a.** $2\ln x + 3\ln y$ **b.** $\ln(x+3) - \ln 4 - \ln y$
c. $\ln 3 + \frac{1}{2}\ln x$ **d.** $\ln 4 + \ln x + \ln y$

L2. **a.** $\ln x^4y^5$ **b.** $\ln 32a^{2/3}$
c. $\ln \frac{x}{2}$ **d.** $\log_2 x$ (change of base)

L3. **a.** $x = e^2 - 3 \approx 4.389$ **b.** $\frac{e}{4} \approx 0.680$
c. $x = 1$ (-3 is extraneous) **d.** $x = \frac{7}{3}$

L4. **a.** $x = -\frac{5}{4}$ **b.** $x = \frac{3}{11}$
c. $x = -\frac{1}{2}$ **d.** $x \approx 2.710$

L5. **a.** 637.287 million cells
14.270 hours
b. 1.237%
18496.496 years

R1.

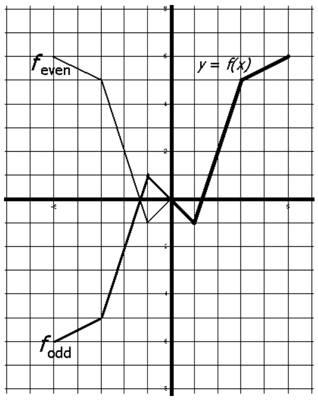
a.	$x \neq \frac{1}{2}, \frac{5}{4}$	$\left(\frac{5}{4}, \frac{17}{6}\right)$	$y = \frac{1}{2}$	$x = \frac{1}{2}$
b.	$x \neq 0$	(0, 24)	none	none
c.	$(-\infty, -\frac{2}{3}) \cup (4, \infty)$	skip	skip	$x = 4$

R2. **a.** Answers vary. One possibility: $\frac{(4x-2)(x-3)}{(x-1)(x-3)}$
b. Answers vary. One possibility: $\frac{-2(x^2-3)(x+2)(x-2)}{x(x+2)(x-2)}$

R3. **a.** = 0: never undefined: at $x = 0$
b. = 0: at $x = 0.5 + n$ undefined: never

R4. **a.** $\frac{6-(x^2+4)^{3/2}}{3(x^2+4)^{1/2}}$ **b.** $\frac{3x^2}{x^2+16}$ **c.** $\frac{-2x^2+x+5}{(x+1)^2(x+2)}$ **d.** $\frac{3x^2+13x+12}{(x+2)^{5/2}}$

G1.



a. see graph

b. D: [-5, 5]   R: [-1, 6]

c. 5

d. see graph

e. D: [-5, 5]   R: [-6, 6]

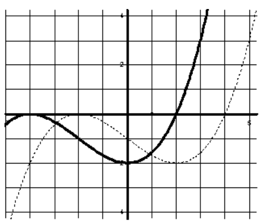
f. -5

G2.

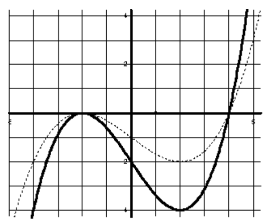
a. 3
b. 5.5
c. 4—that notation means the same thing as $g(f(3))$
d. $x = 3.5$
e. $x = 2$
f. (1, 6)
g. (-1, ∞)
h. $[0, 2\frac{2}{3})$
i. [2, 5]

G3.

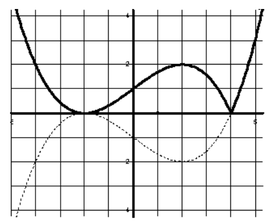
a.



b.



c.



d.