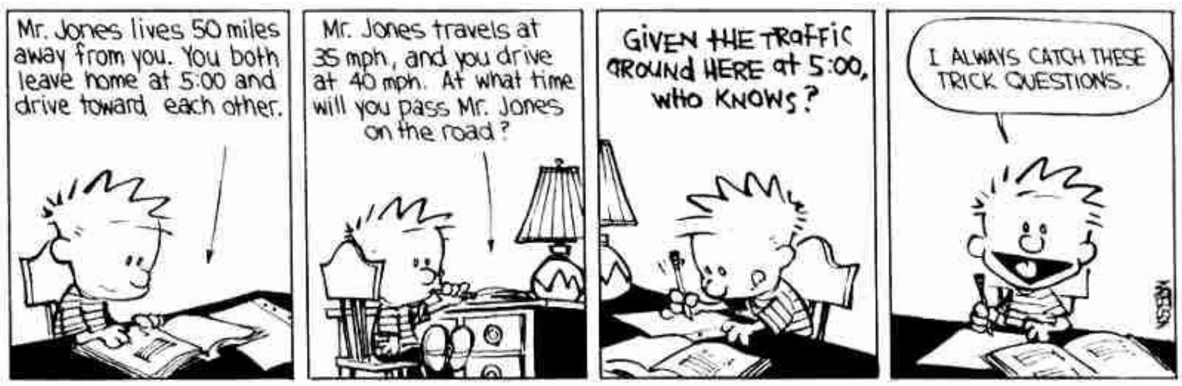
Name ______________________________

AP Calculus BC Summer Packet

If you prefer NOT to show work on the packet, show your numbered/labeled work neatly.



Welcome to Calc BC, the equivalent of college Calc I & II. Calculus is a great subject, and in BC you'll start to see its applications. As a chemistry major with a math minor, I took three semesters of calculus and also saw it used in chemistry and physics courses. Calculus is my favorite math class ☺ because it explores a wide variety of concepts and draws on skills from many other math classes. I hope you enjoy it, too, and become more curious about math as a result of taking Calc BC.

What will it take to succeed?

- A memorized **unit circle** (know how to find sin, cos, tan, csc, sec, and cot of any angle)
- Solid **algebra** and **trig skills**
- A thorough grasp of concepts and skills learned in the **first two chapters of Calculus**

To make sure you're ready, you'll need to do a few things over the summer.

- Complete the **first page of the packet soon** so you know what needs to be **memorized**.
- Complete the remaining pages now while your skills are fresh **OR** late in July so that your skills are fresh at the start of next year. If you worry that you'll forget, complete the rest of the packet now and print a second copy to complete right before school starts. **Your completed work is due on the first day of school**. Also **review the first page** for material that needs to be memorized.
- Begin **learning the flashcards** at the back of this packet. If given the front, you should be able to write the back. The third day of class, you'll have a flashcard quiz. You will receive a sturdier copy of flashcards on the first day.

We'll spend the first day of class addressing lingering questions from the review packet and clarifying misconceptions from the first two chapters. Then we will tackle the rest of calculus.

If you have any problems completing the assignment, message me on StudentSquare or by email (or ask a classmate, if you prefer). I look forward to working through Calc BC with you next school year!

Mrs. Mendenhall (emendenhall@brownsburg.k12.in.us)

AP Calculus BC Summer Homework

Complete each question, including work or explanations as directed. Except when specified, do NOT use your calculator.

Precalculus – do now

1. Sketch the graph of each common function below, using your calculator as needed. If not in your memory now, they should be when school starts.

$y = \sin x$	$y = \tan x$	$y = \sqrt{x}$
$y = \cos x$	$y = \sec x$	$y = \|x\|$
$y = e^x$ (or $y = a^x$)	$y = \tan^{-1} x$	$y = \frac{\|x\|}{x}$
$y = \ln x$	$y = \frac{1}{x}$	$y = \sqrt{3^2 - x^2}$ (or $y = \sqrt{a^2 - x^2}$)

2. Practice filling in the angles (in radians) and points on the unit circle, available separately. These should be also memorized by the first day of school.

Calculus – do now

Derivative Summary*

3. Do you know these derivatives? You really should! (especially by the first day)

$\frac{d}{dx}[\sin x] =$

$\frac{d}{dx}[\cos x] =$

$\frac{d}{dx}[\tan x] =$

$\frac{d}{dx}[\sec x] =$

$\frac{d}{dx}[\csc x] =$

$\frac{d}{dx}[\cot x] =$

$\frac{d}{dx}[e^u] =$

$\frac{d}{dx}[\ln u] =$

$\frac{d}{dx}[\log_a u] =$

$\frac{d}{dx}[a^u] =$

$\frac{d}{dx}[\arcsin u] =$

$\frac{d}{dx}[\arccos u] =$

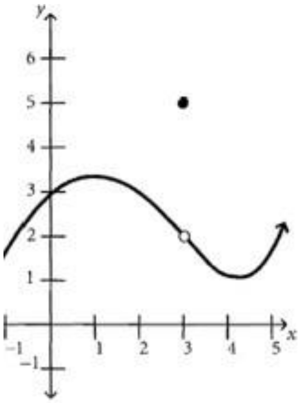
$\frac{d}{dx}[\arctan u] =$

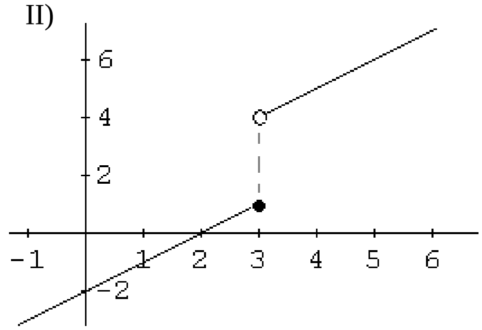
***Forgot any? Check the flashcards at the end of the packet.**

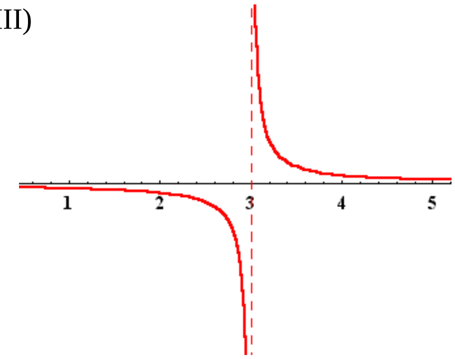
Calculus, Chapter 1 – do now or later

Limits – Graphically (1.2 & 1.4)

4. For which of the following does $\lim_{x\to 3} f(x)$ exist? **Explain why or why not.**

I) 

II) 

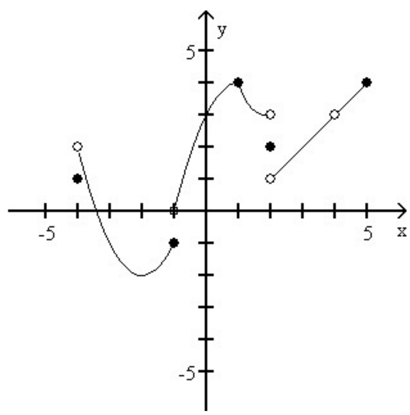
III) 

I)

II)

III)

5. Use the graph of $y = g(x)$ to find the following values.



a) $\lim_{x\to 1} g(x) =$

e) $\lim_{x\to 2} g(x) =$

b) $\lim_{x\to 2^+} g(x) =$

f) $\lim_{x\to -1^-} g(x) =$

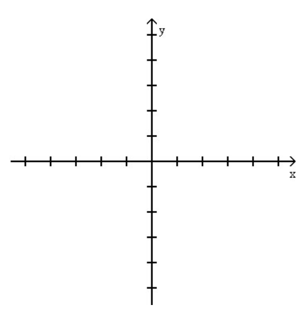
c) $\lim_{x\to 2^-} g(x) =$

g) $\lim_{x\to 4} g(x) =$

d) $g(2) =$

h) $\lim_{x\to -1^+} g(x) =$

6. Draw the graph of a function for which $\lim_{x\to 1} f(x)$ does not exist.



Limits – Numerically from Tables (1.2)

7. The table below gives the values of three functions, f, g, and h near $x = 0$. Based on the values given, for which of the functions does it appear that the limit as x approaches zero is 3? List all that apply.

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	3	3	3	3	2	2	2
g(x)	2.971	2.987	2.997	undefined	2.997	2.987	2.971
h(x)	3.018	3.008	3.002	3	3.002	3.008	3.018

Limits – Analytically from Expressions (1.3) and/or L'Hopital's Rule

8. If $a \neq 0$, then what is $\lim\limits_{x\to a} \dfrac{x^2 - a^2}{x^4 - a^4}$? **Show your work.**

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

9. If $f(x) = \begin{cases} \ln x^4 & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$ then find $\lim\limits_{x\to 2} f(x)$. **Show your work.**

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

10. What is $\lim\limits_{x\to 0} \dfrac{5x^5 + 12x^3}{3x^5 - 6x^3}$? **Show your work.**

(A) $-\dfrac{17}{3}$ (B) 0 (C) $\dfrac{5}{3}$ (D) -2 (E) nonexistent

Continuity (1.4)

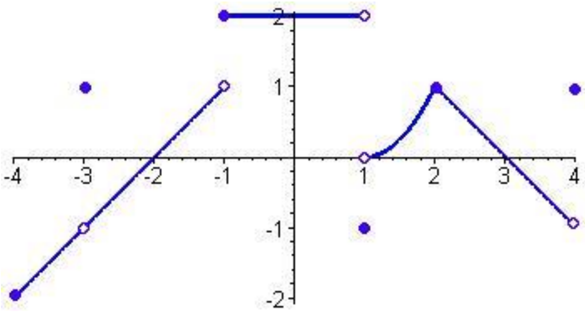
11. What makes a function, *f*, continuous at *x* = *a* ? (forgot? check the flashcards at the end of the packet)

12. Which of the following functions is/are continuous for all real numbers x? **Sketch each or explain**.

I. $y = \ln x$
II. $y = 3^x$
III. $y = \cos x$

(A) None (B) I only (C) II only (D) I and II (E) II and III

13. The graph of the function f is shown below. Identify whether each statement about f is true or false. **Explain**.



(A) f is continuous at $x = -1$. True or false?

(B) $x = -1$ is in the domain of f. True or false?

(C) $\lim_{x\to -1^+} f(x)$ is equal to $\lim_{x\to -1^-} f(x)$. True or false?

(D) $\lim_{x\to -1} f(x)$ exists. True or false?

14. Determine the values of *b* and *c* such that the function is continuous on the entire real line. **Show your work.**

$$h(x) = \begin{cases} x+1, & 1 < x < 3 \\ x^2 + bx + c, & x \le 1, x \ge 3 \end{cases}$$

Intermediate Value Theorem (1.4)

15. What does IVT state?

16. Suppose that f is a continuous function defined for all real numbers x and $f(-4) = 3$ and $f(-1) = -4$. If $f(x) = 0$ for one and only one value of x, then which of the following could be x? **Explain your answer. Sketching the graph may be helpful.**

(A) -7 (B) -3 (C) 0 (D) 1 (E) -4

17. Let f be a function that is continuous on the closed interval [2, 4] with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem? **Sketch or explain.**

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

Asymptotes and Limits at Infinity (1.5-1.6)

18. How do you find a vertical asymptote? (1.5)

19. How do you find a horizontal asymptote? (1.6)

20. The line $y = 5$ is a horizontal asymptote to the graph of a function. Which one? **Show work or explain.**

(A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2-x}{1+4x^2}$

21. The vertical line $x = 2$ is an asymptote for the graph of the function f. Which of the following statements must be **false**? **Explain and/or sketch a graph.**

(A) $\lim_{x\to 2} f(x) = 0$ (B) $\lim_{x\to 2} f(x) = -\infty$ (C) $\lim_{x\to 2} f(x) = \infty$
(D) $\lim_{x\to\infty} f(x) = 2$ (E) $\lim_{x\to\infty} f(x) = \infty$

22. Find $\lim_{x\to\infty} \frac{\sqrt{9x^4+1}}{4x^2+3}$. **Show work or explain**.

(A) 1/3 (B) 3/4 (C) 3/2 (D) 9/4 (E) infinite

Calculus, Chapter 2 – do now or later

Power Rule, Differentiability, and Tangent Lines (2.1 & 2.2)

23. In general, how can you find the average rate of change of f on $[a, b]$?

24. In general, how can you find the instantaneous rate of change of f at $x = a$?

25. What two things do you need to find in order to write the equation of a tangent line?

26. What makes a function, f, differentiable at $x = a$?

27. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$? **Show work**.

(A) $y = 7x - 3$ (B) $y = 7x + 7$ (C) $y = 7x + 11$ (D) $y = -5x - 1$ (E) $y = -5x - 5$

28. Let f be the function $f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & if\ x\neq 2\\ 1 & if\ x=2\end{cases}$. Is each statement about f true or false? **Explain and/or show work**.

I. f has a limit at $x=2$. T/F?

II. f is continuous at $x=2$. T/F?

III. f is differentiable at $x=2$. T/F?

29. Let f be the function defined by $f(x)=\sqrt{|x-2|}$ for all x. Which of the following statements is true? **Explain. (could you sketch this graph without a calculator?)**

(A) f is continuous but not differentiable at $x=2$.

(B) f is differentiable at $x=2$.

(C) f is not continuous at $x=2$.

(D) $\lim_{x\to 2} f(x)\neq 0$

(E) $x=2$ is a vertical asymptote of the graph of f.

30. If the line tangent to the graph of the function f at the point (1, 7) passes through the point (-2, -2), then what is $f'(1)$?

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

31. The graph of $y=e^{\tan x}-2$ crosses the x-axis at one point in the interval [0, 1]. What is the slope of the graph at this point? **(calculator allowed)**

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

32. The function $P(t)$ models the population of the world, in billions of people, where t is the number of years since January 1, 2010. Which of the following is the best interpretation of the statement $P'(1)=0.076$?

(A) On February 1, 2010, the population of the world was increasing at a rate of 0.076 billion people per year.

(B) On January 1, 2011, the population of the world was increasing at a rate of 0.076 billion people per year.

(C) On January 1, 2011, the population of the world was 0.076 billion people.

(D) From January 1, 2010 to January 1, 2011, the population of the world was increasing at an average rate of 0.076 billion people per year.

(E) When the population of the world was 1 billion people, the population of the world was increasing at a rate of 0.076 billion people per year.

Product Rule, Quotient Rule, and Trig (2.3)

33. If $y = \frac{2x+3}{3x+2}$, find $\frac{dy}{dx}$. **Show work.**

(A) $\frac{12x+13}{(3x+2)^2}$ (B) $\frac{12x-13}{(3x+2)^2}$ (C) $\frac{5}{(3x+2)^2}$ (D) $\frac{-5}{(3x+2)^2}$ (E) $\frac{2}{3}$

34. Given that $f'(x) = 3\sec^2 x + 2$, which of the following could be $f(x)$?

(A) $3\tan x$ (B) $3\tan x + 2x$ (C) $3\sec x + 2x$ (D) $\sec^3 x + 2x$ (E) $6\sec^2 x \tan x$

35. Let $f(x)$ be a differentiable function. Let $g(x) = \frac{1}{f(x)}$. Use the table to find the value of $g'(2)$. **Show work.**

x	1	2	3	4
$f(x)$	–3	–8	–9	0
$f'(x)$	–5	–4	3	16

(A) $-\frac{1}{8}$ (B) 0 (C) $\frac{1}{16}$ (D) $\frac{1}{64}$ (E) 16

36. Let f be a differentiable function with $f(2) = 3$ and $f'(2) = -5$, and let g be the function defined by $g(x) = x\,f(x)$. What is the equation of the line tangent to the graph of g at the point where $x = 2$? **Show work.**

Chain Rule and Combinations of Rules (2.4)

37. If $y = (x^3+1)^2$, then $\frac{dy}{dx} =$

(A) $(3x^2)^2$ (B) $2(x^3+1)$ (C) $2(3x^2+1)$ (D) $3x^2(x^3+1)$ (E) $6x^2(x^3+1)$

38. If $y = x^2 \sin(2x)$, then $\frac{dy}{dx} =$? **Show work.**

(A) $2x\cos 2x$ (B) $4x\cos 2x$ (C) $2x(\sin 2x + \cos 2x)$ (D) $2x(\sin 2x - x\cos 2x)$ (E) $2x(\sin 2x + x\cos 2x)$

39. Use the following table to find $h'(1)$, given that $h(x) = f(g(x))$. **Show work.**

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
–1	6	5	3	–2
1	3	–3	–1	2
3	1	–2	2	3

40. If $f(x) = \sqrt{4\sin x + 2}$, then $f'(0) = ?$ **Show work.**

(A) -2 (B) 0 (C) $\sqrt{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) 1

41. What is the average rate of change of $y = \cos(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$? **Show work.**

(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

Implicit Differentiation (2.5)

42. What is the slope of the tangent line to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$? **Show work.**

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

43. If $(x + 2y)\frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$? **Show work.**

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

Derivatives of Exponentials and Logarithms (2.6)

44. What do you know about the slope of each?

a. parallel lines b. horizontal lines c. vertical lines

45. $\frac{d}{dx}(2^x) =$

(A) 2^x (B) $(\ln 2)2^x$ (C) $\frac{2^x}{\ln 2} + C$ (D) $(x)2^{x-1}$

46. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a horizontal tangent line at $x = \frac{2}{3}$? **Show work.**

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

Derivatives of Inverse Functions (2.7)

47. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$? **Show work.**

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

Motion (ch. 2)

48. A particle moves along the *x*-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time(s) t is the particle at rest? **Show work**.

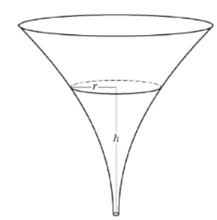
49. A particle moves along the *x*-axis so that its velocity at time t is given by $v(t) = -(t+2)\sin\frac{t^2}{3}$. **You may use a graphing calculator.**

a) Find the acceleration of the particle at time $t = 5$. **Show work or concept.**

b) Find all times t in the open interval $0 < t < 4$ when the particle changes direction. **Justify your answer**.

Related Rates (2.8)

50. The inside of a funnel of height 10 inches has circular cross sections. At height h, the radius of the funnel is given by $r = \frac{3}{20} + \frac{h^2}{20}$, where $0 \leq h \leq 10$. The units of r and h are inches.



The funnel contains liquid that is draining from the bottom. At the instant when the height of liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time? **Show work and include units.**

(A) -2/3 (B) 2/3 (C) 3/10 (D) -3/10

Units:

Other Limits (multiple ways to solve)

51. Find $\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h}$. **Show work or explain.**

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

Flashcards – Chapters 1-2A (you'll receive sturdier ones in the fall)

f is continuous at x = a 1	*** IVT (v.1) (Intermediate Value Theorem) *** 1	*** IVT (v.2) (Intermediate Value Theorem) *** 1
$\lim_{x\to\infty} \frac{ax+b}{cx^2+d}$ 1	$\lim_{x\to\infty} \frac{ax^2+b}{cx^2+d}$ 1	$\lim_{x\to\infty} \frac{ax^2+b}{cx+d}$ 1
horizontal asymptote 1	vertical asymptote 1	$\lim_{x\to a} f(x) = \frac{0}{0} \ or \ \frac{\infty}{\infty}$ 1
f is differentiable at $x = a$ 2A	average velocity 2A	sign of velocity 2A
average rate of change of f(x) on [a, b] 2A	average acceleration 2A	sign of acceleration 2A
$\frac{d}{dx}[\sin x]$ 2A	$\frac{d}{dx}[\cos x]$ 2A	$\frac{d}{dx}[\tan x]$ 2A

Flashcards – Chapters 1-2A

Since f is continuous on $[a, b]$, by IVT, f takes on every value between $f(a)$ and $f(b)$. $f(a) <$ __ $< f(b)$	Since f is continuous on $[a, b]$ and $f(a) = A$ and $f(b) = B$, then IVT guarantees that $f(x) = C$ in $[a, b]$ because $f(a) < C < f(b)$.	$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$ (left limit = right limit = $f(a)$)
∞ or $-\infty$ (DNE)	$\frac{a}{c}$	0
Indeterminate form; simplify limit or use l'Hopital	Denominator of f equals zero; factor and cancel first	Limit of f at ∞ AND $-\infty$
+ is moving up or right 0 is stopped - is moving down or left	Slope of position	(continuous first) $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$ *and* $\lim_{x\to a^-} f'(x) = \lim_{x\to a^+} f'(x) = f'(a)$
+ is accel. up or right 0 is constant velocity (no acceleration) - is accel. down or left	Slope of velocity	$\frac{f(b) - f(a)}{b - a}$ (slope of f)
$\sec^2 x$	$-\sin x$	$\cos x$

Flashcards – Chapter 2A-2B (sturdier ones coming in class)

$\frac{d}{dx}[\csc x]$ 2A	$\frac{d}{dx}[\sec x]$ 2A	$\frac{d}{dx}[\cot x]$ 2A
$\frac{d}{dx}[e^u]$ 2A	$\frac{d}{dx}[\ln u]$ 2A	
$\frac{d}{dx}[a^u]$ 2B	$\frac{d}{dx}[\log_a u]$ 2B	$\frac{d}{dx}[\arcsin u]$ AKA $\frac{d}{dx}(\sin^{-1} u)$ 2B
$\frac{d}{dx}[\arccos u]$ AKA $\frac{d}{dx}(\cos^{-1} u)$ 2B	$\frac{d}{dx}[\arctan u]$ AKA $\frac{d}{dx}(\tan^{-1} u)$ 2B	$\frac{d}{dx}[\operatorname{arcsec} u]$ AKA $\frac{d}{dx}(\sec^{-1} u)$ 2B
l'Hopital's Rule applies to limits that are ___ 2B	l'Hopital's Rule 2B	If f and g are inverses, then $f'(x)$ = 2B

$-\csc^2 x$	$\sec x \tan x$	$-\csc x \cot x$
	$\frac{u'}{u}$ or $\frac{1}{u}(u')$	$e^u\,(u')$
$\frac{u'}{\sqrt{1-u^2}}$	$\frac{u'}{u}\left(\frac{1}{\ln a}\right)$ or $\frac{1}{u}\left(\frac{u'}{\ln a}\right)$	$a^u\,(\ln a)\,(u')$
$\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	$\frac{u'}{1+u^2}$	$\frac{-u'}{\sqrt{1-u^2}}$
$\frac{1}{g'(y)}$	$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ if f and g are continuous	indeterminate: $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$