AP PHYSICS 1 SUMMER WORK INTRODUCTION AND ANSWER SHEET

Greetings young scholars! Welcome to the summer preparation for the AP Physics 1 course. Because of the pace of the course it is important to make sure you are prepared mathematically before you come in on the first day of school. To that end I am expecting you to finish this packet by the end of the summer and be prepared to turn in all completed work on the first day of school. There will be a quiz over this material in the first week of school, so it is imperative you learn it now. Class time will not be spent “catching up” on summer work. In this packet you will hopefully find a lot of information you are already familiar with, such as the sine, the cosine, and the tangent. Your goal is to become familiar with the use of these functions to find the sides and angles in various triangles. If you are unfamiliar with these concepts, consider doing some research this summer, either through watching videos (Khan academy in particular has some very good tutorials on the information being presented here, as does Mathwarehouse on Youtube), online help websites, or even good old fashioned math textbooks, which you should be able to check out from a library or buy online or from a used bookstore.

This packet contains both the lessons I want you to read through as well as the answer sheet you will be turning in to me on that first day of school. You don’t need to print off this entire document, only pages 60 through 75- the pages that require you to write your answers down.

One final note- there is a lot of work to do here, so don’t wait until the last minute to start. Pace yourself through the summer so that you give yourself your best possible path to success.

Enjoy your summer and I’ll see you in August!

Mr. Rott

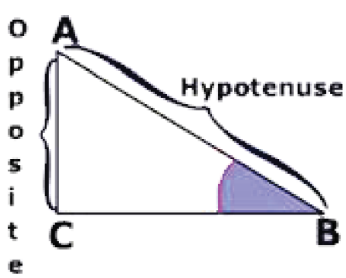
Part I- The Parts of a Right Triangle

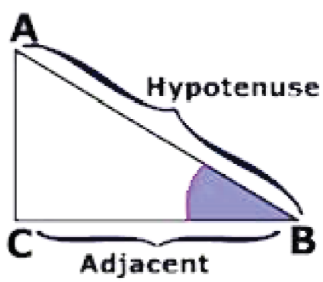
The foundation of all of trigonometry is understanding the relationships between the sides and angles in right triangles. In order to do that, we need to be able to identify those sides and angles. In the right triangle below, angle C is the 90° angle. The side across from angle C (side AB) is called the hypotenuse, while the other two sides (sides AC and BC) are called the legs. When referencing the other angles in the triangle, one of the legs is called the opposite side and the other is called the adjacent side. Which is which depends on which angle you are referencing. In the diagram below, with respect to angle B you can see that side AC is opposite the angle and side BC is adjacent to the angle.

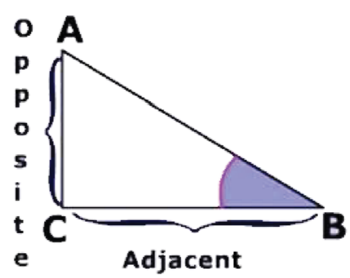
The ratios of the sides have names. They are called the sine, the cosine, and the tangent of the angle. You can see in the triangle below that these ratios are shown for angle B. They are read as "the sine of angle B is the ratio of the opposite side to the hypotenuse, the cosine of angle B is the ratio of the adjacent side to the hypotenuse, and the tangent of angle B is the ratio of the opposite side to the adjacent side." A useful mnemonic for keeping these definitions straight is "SOH-CAH-TOA."

Model Problems

$$\sin(B) = \frac{opposite}{hypotenuse} \qquad \cos(B) = \frac{adjacent}{hypotenuse} \qquad \tan(B) = \frac{opposite}{adjacent}$$





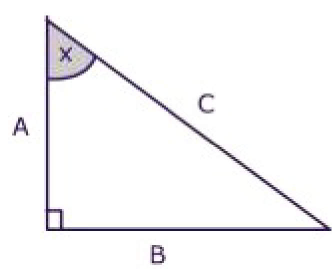


Model Problem 1: Identify the side adjacent, opposite to angle x and the hypotenuse

Adjacent to x : **A**

Opposite x : **B**

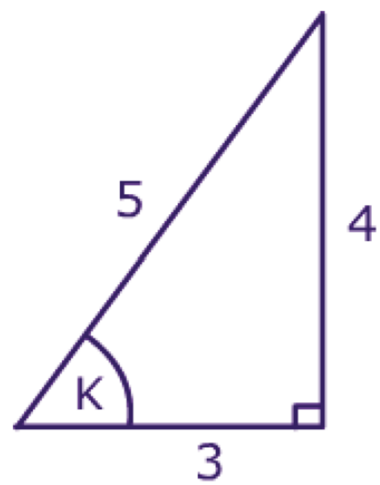
Hypotenuse : **C**



Model Problem 2: What are sin(*k*), cos(*k*), and tan(*k*)?

Use SOH-CAH-TOA

$$\sin(k) = \frac{opposite}{hypotenuse} = \frac{4}{5} = .8$$



$$\cos(k) = \frac{adjacent}{hypotenuse} = \frac{3}{5} = .6$$

$$\tan(k) = \frac{opposite}{adjacent} = \frac{4}{3} = 1.33$$

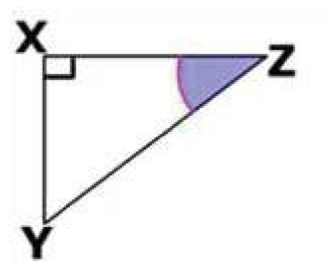
PROBLEM SET 1: *Identifying Opposite, Adjacent and Hypotenuse*

Identify

1) the hypotenuse

2) the side opposite of $\angle Z$

3) the side adjacent to $\angle Z$

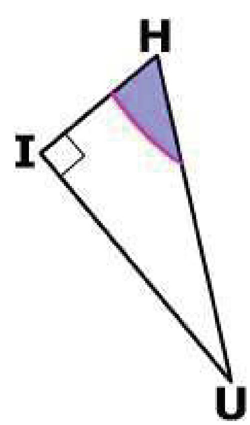


Identify

4) the hypotenuse

5) the side opposite of $\angle H$

6) the side adjacent to $\angle H$

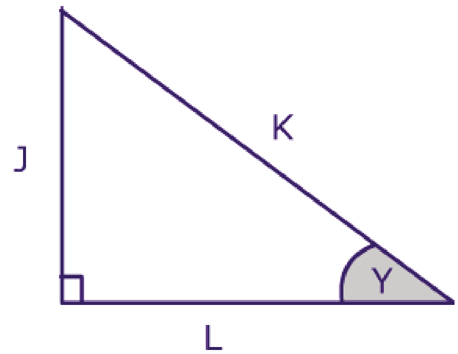


Identify

7) the hypotenuse

8) the side opposite of $\angle Y$

9) the side adjacent to $\angle Y$



PROBLEM SET 2: *Writing Sine, Cosine, Tangent Ratios*

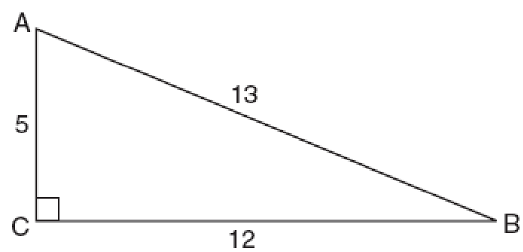
10) Which ratio represents cos(A) in the accompanying diagram of ΔABC?

(i) $\frac{5}{13}$

(ii) $\frac{12}{13}$

(iii) $\frac{12}{5}$

(iv) $\frac{13}{5}$



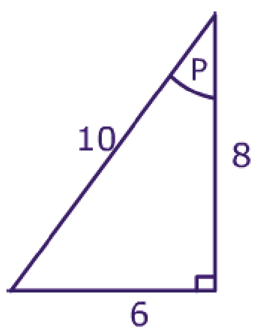
11) Which ratio represents sin(P) in the accompanying triangle?

(i) $\frac{6}{10}$

(ii) $\frac{8}{10}$

(iii) $\frac{6}{8}$

(iv) $\frac{10}{6}$



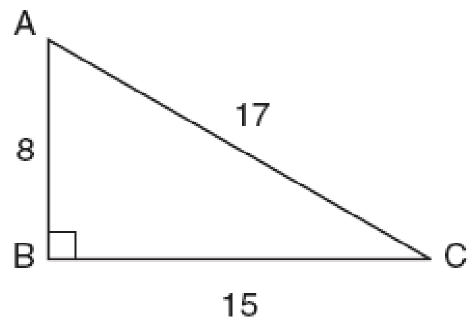
12) In the accompanying diagram of right triangle ABC, $AB = 8$, $BC = 15$, $AC = 17$, and $m\angle B = 90$.
What is $\tan(C)$?

(i) $\frac{8}{15}$

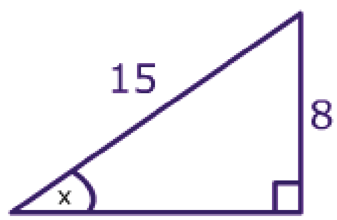
(ii) $\frac{7}{15}$

(iii) $\frac{8}{17}$

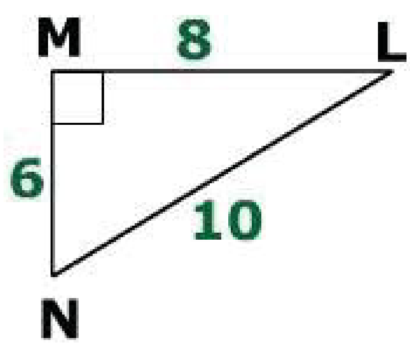
(iv) $\frac{15}{17}$



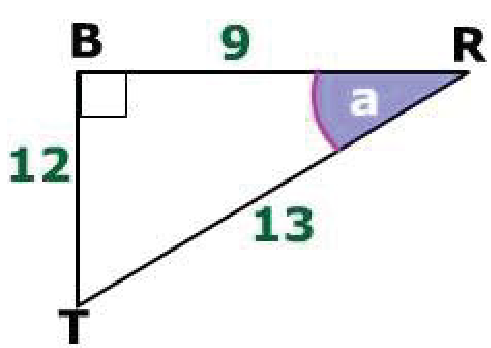
13) What is sin(x) for the triangle below?



14) What is $\sin(L)$, $\cos(L)$ and $\tan(L)$ for the triangle below?



15) What is $\sin(a)$, $\cos(a)$ and $\tan(a)$ for the triangle below?



16) In triangle XYZ, $\angle y = 90°$, XY = 7, Z = 24, and XZ = 25. Which ratio represents cosine of $\angle x$?
Hint: draw the triangle

(i) $\frac{7}{24}$ (ii) $\frac{24}{25}$ (iii) $\frac{7}{25}$ (iv) $\frac{24}{7}$

17) In triangle MCT, the measure of $\angle T = 90°$, $MC = 85$ cm, $CT = 84$ cm, and $TM = 13$ cm. Which ratio represents the sine of $\angle C$?

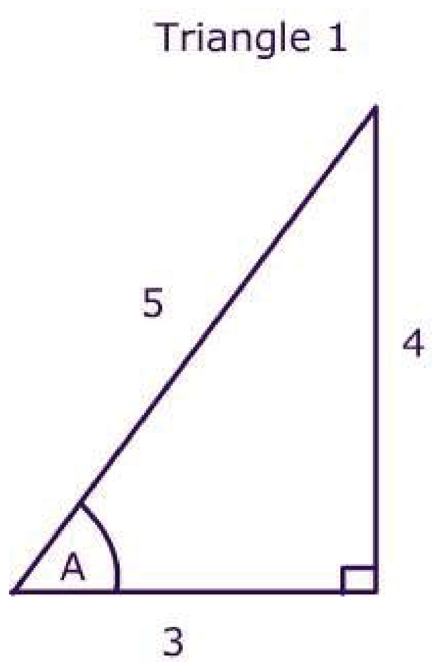
(i) $\frac{13}{85}$ (ii) $\frac{84}{85}$ (iii) $\frac{13}{84}$ (iv) $\frac{84}{13}$

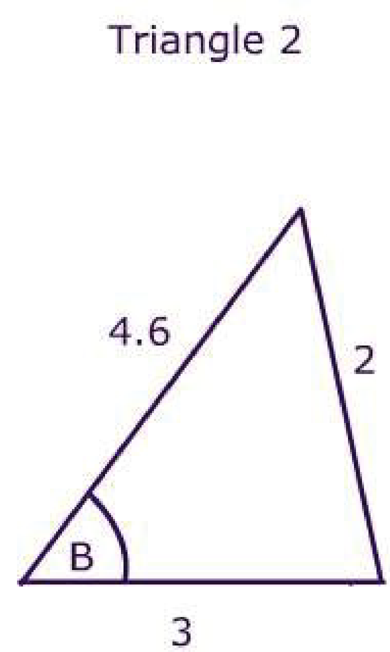
18) Error Analysis: A teacher asks the class if they can express sin(A) in Triangle 1 and sin(B) in Triangle 2.

Jose says sin(A) = $\frac{4}{5}$ and sin(B) does not exist.

Jenny says sin(A) = $\frac{4}{5}$ and sin(B) = $\frac{2}{4.6}$

Who, if either, is correct? (explain your reasoning)



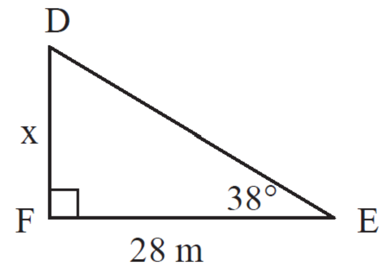


Part II- Right Triangles and SOH-CAH-TOA: Finding the Length of a Side Given One Side and One Angle

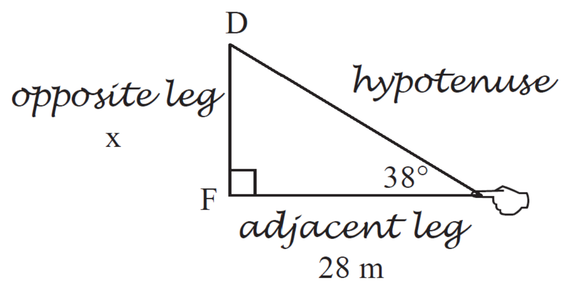
Preliminary Information: "SOH CAH TOA" is an acronym to represent the following three trigonometric ratios or formulas:

Model Problems

Example 1: Consider right ΔDEF pictured at right. We know one acute angle and one side, and our goal is to determine the length of the unknown side x.



Step 1: Place your finger on the 38° angle (the acute angle), and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg. (The word "adjacent" usually means "next to.")



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which side do I know?" In other words, which side has a length we already know? In this example, we know that one side is 28 m, so we know the adjacent leg. Underline both of the A's in SOH CAH TOA to indicate that we know the **A**djacent leg:

SOH C<u>A</u>H TO<u>A</u>

Step 4: Now ask yourself, "Which side do I want to find out?" In other words, which side length are we being asked to calculate? In this example, we are being asked to calculate the side marked x, so we want the opposite leg. Underline both of the O's in SOH CAH TOA to indicate that we want the **O**pposite leg:

<u>S</u>OH C<u>A</u>H T<u>OA</u>

Step 5: Consider which of the three ratios has the most information: we have one piece of information for the sine (one underline), only one piece of information for the cosine (one underline), yet we have two pieces of information for the tangent (two underlines). We are therefore going to use the tangent ratio formula:

$$\tan\theta = \frac{opposite\ leg}{adjacent\ leg}$$

Step 6: Substitute the known information into the formula:

$$\tan\theta = \frac{opposite\ leg}{adjacent\ leg} \Rightarrow \tan 38° = \frac{x}{28}$$

(Note that we dropped the units of "meters" for simplicity; the answer will be in meters.)

Step 7: Solve for x. In this example, it is probably simplest to multiply both sides by 28:

$$\tan 38° = \frac{x}{28}$$

$$28 \cdot \tan 38° = 28 \cdot \frac{x}{28}$$

$$x = 28 \cdot \tan 38°$$

Step 8: Simplify. You shoul use a handheld calculator (in degrees mode), on online calculator, or a table of values from a chart. In this case, an approximate value for the tangent of 38 degrees is 0.78129:

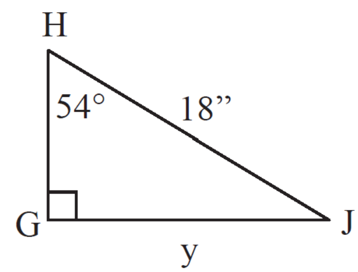
$$x = 28(0.78129)$$

$$x = 21.876m$$

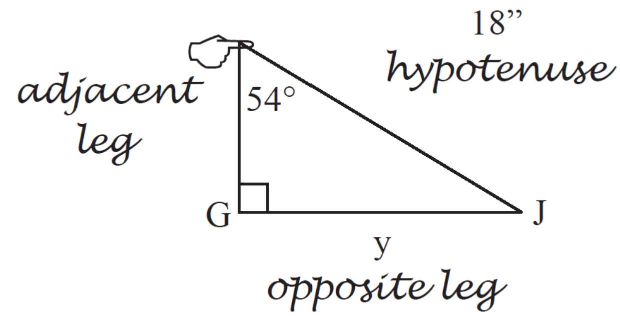
(Note that we have included units of meters, as the original side was specified in meters.)

Step 9: Check for reasonableness: In this case, the acute angle was 38°, which is less than 45°. (If it had been a 45° angle, both legs would be congruent.) It is reasonable that this leg should be less than 28m.

Example 2: Consider right ΔGHJ pictured at right. We know one acute angle and one side, and our goal is to determine the length of the unknown side y to the nearest inch.



Step 1: Place your finger on the 54° angle (the acute angle), and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which side do I know?" In this example, we know that the hypotenuse is 18 inches. Underline both of the H's in SOH CAH TOA:

SO<u>H</u> CA<u>H</u> TOA

Step 4: Now ask yourself, "Which side do I want to find out?" In this example, we are being asked to calculate the side marked y, so we want the opposite leg. Underline both of the O's in SOH CAH TOA:

S<u>O</u><u>H</u> CA<u>H</u> T<u>O</u>A

Step 5: Consider which of the three ratios has the most information: we have two pieces of information for the sine:

$$\sin\theta = \frac{opposite\ leg}{hypotenuse}$$

Step 6: Substitute the known information into the formula:

$$\sin\theta = \frac{opposite\ leg}{hypotenuse} \Rightarrow \sin 54° = \frac{y}{18}$$

(Note that we dropped the units of "inches" for simplicity.)

Step 7: Solve for y. In this example, it is probably simplest to multiply both sides by 18:

$$\sin 54° = \frac{y}{18}$$

$$18 \cdot \sin 54° = 18 \cdot \frac{y}{18}$$

$$y = 18 \cdot \sin 54°$$

Step 8: Simplify. In this case, an approximate value for the sine of 54 degrees is 0.80902:

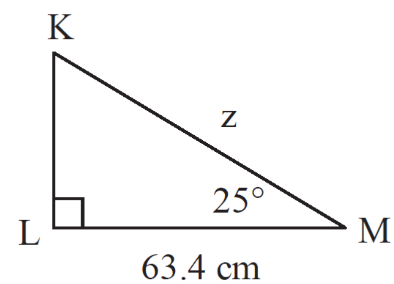
$y = 18(0.80902)$
$y = 4.5623"$

To the nearest inch, we get $y = 15"$
(Note that we have included the unit inches.)

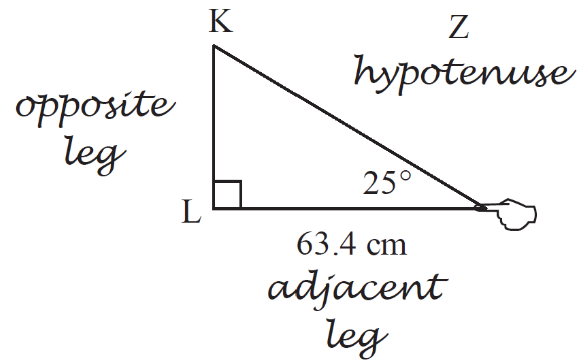
Step 9: Check for reasonableness: In this case, the hypotenuse must be longest at 18 inches, so a leg of 15" seems reasonable.

Example 3: (Note: This example is generally more difficult for students to complete correctly due to a significant change in the algebra required: we will end up with an equation in which the variable is in the denominator of a fraction, and the algebra steps required are different.)

Consider right ΔKLM pictured at right. We know one acute angle and one side, and our goal is to determine the length of the unknown side marked z to the nearest tenth of a centimeter.



Step 1: Place your finger on the acute angle, and then label the three sides: the hypotenuse is always the longest side; the side opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which side do I know?" In this example, we know that the adjacent leg is 63.4 cm. Underline both of the A's in SOH CAH TOA:

SOH C<u>A</u>H TO<u>A</u>

Step 4: Now ask yourself, "Which side do I want to find out?" In this example, we are being asked to calculate the side marked z, the hypotenuse. Underline both of the H's in SOH CAH TOA:

SO<u>H</u> CA<u>H</u> TO<u>A</u>

Step 5: Consider which of the three ratios has the most information: we have two pieces of information for the cosine:

$$\cos\theta = \frac{adjacent\ leg}{hypotenuse}$$

Step 6: Substitute the known information into the formula:

$$\cos\theta = \frac{adjacent\ leg}{hypotenuse} \Rightarrow \cos 25^\circ = \frac{63.4}{z}$$

(Note that we dropped the units of "centimeters" for simplicity.)

Step 7: Solve for the variable. In this example, note that the variable is in the denominator of the expression, so we cannot multiply both sides of the equation by 63.4: Instead, we need a different approach. Two of the most common techniques are shown below. Both are correct.

Method 1: Multiply both sides by the denominator	**Method 2: Cross-multiply**
$\cos 25° = \frac{63.4}{z}$ $z \cdot \cos 25° = z \cdot \frac{63.4}{z}$ $z \cdot \cos 25° = 63.4$	$\cos 25° = \frac{63.4}{z}$ *rewrite as a fraction* : $\frac{\cos 25°}{1} = \frac{63.4}{z}$ $z \cdot \cos 25° = 63.4$

We now can get z by itself by dividing both sides by cos 25°:

$$z \cdot \cos 25° = 63.4$$

$$\frac{z \cdot \cos 25°}{\cos 25°} = \frac{63.4}{\cos 25°}$$

$$z = \frac{63.4}{\cos 25°}$$

Step 8: Simplify. The approximate value for the cosine of 25 degrees is 0.90631:

$$z = \frac{63.4}{\cos 25°}$$

$$z = \frac{63.4}{0.90631}$$

$$y = 69.9542$$

To the nearest tenth of a centimeter, we get $z = 70.0cm$

(Note that we have included centimeters.)

Step 9: Check for reasonableness: In this case, the hypotenuse must be longest, and 70.0 cm is greater than 63.4 cm, so it seems reasonable.

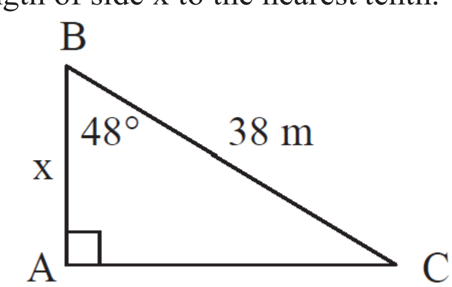
PROBLEM SET 3: *Using the Sine, Cosine, and Tangent*

19. Calculate the value of x to the nearest tenth: $\sin 38° = \frac{x}{80}$

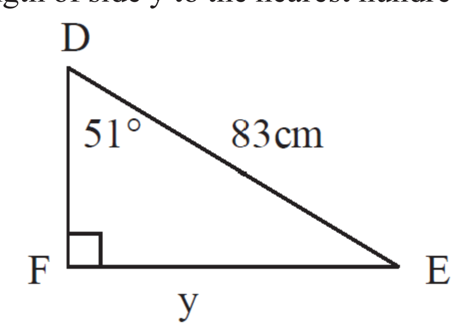
20. Calculate the value of y to the nearest tenth: $\cos 52° = \frac{x}{80}$

21. Calculate the value of z to the nearest hundredth: $\tan 24° = \frac{z}{34.267}$

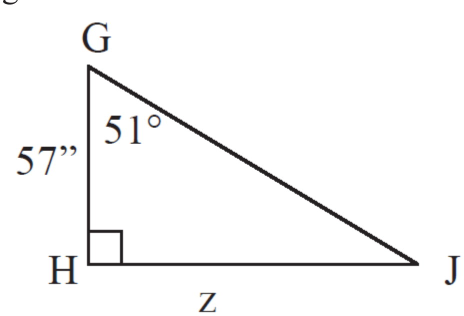
22. Determine the length of side x to the nearest tenth.



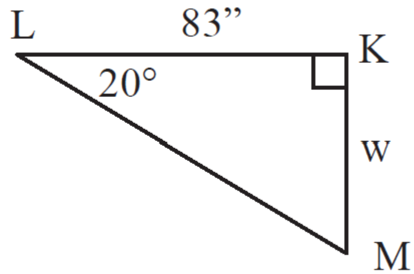
23. Determine the length of side y to the nearest hundredth.



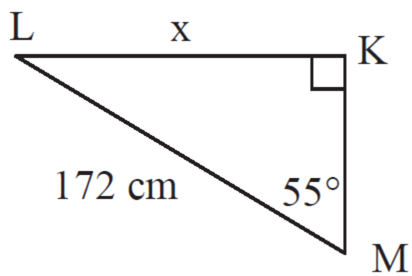
24. Determine the length of side z to the nearest inch.



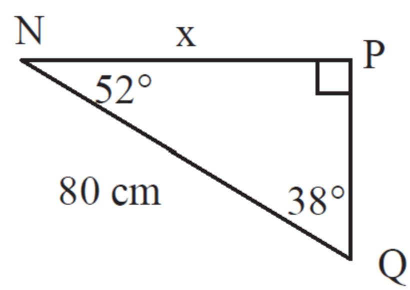
25. Determine the length of side w to the nearest inch.



26. Determine the length of side x to the nearest hundredth.



27. For the triangle pictured, Marcy placed her finger on the 38° angle and concluded that $\sin 38° = \frac{x}{80}$. Likewise, Darius placed his finger on the 52° angle and concluded that $\cos 52° = \frac{x}{80}$.



a) If you solve it Marcy's way, what answer will you get?
b) If you solve it Darius' way, what answer will you get?
c) Are these results sensible? Explain.

28. As we saw in problem 27, there is a connection between $\sin 38°$ and $\cos 52°$.

a) How are the angles 38° and 52° geometrically related? (Think back to what you know about angles from Geometry.)
b) Make a conjecture based on problems 27 and 28a: The sine of 20° must be equal to the cosine of ______° because the two angles are __________________.
c) State your conjecture by finishing this formula: $\sin \theta =$ __________
d) Verify that your formula works correctly for θ = 23°.

29. **Error Analysis**: Consider the following equation:

$$\tan 24° = \frac{34.627}{z}$$

Calculate the value of z to the nearest hundredth.

a) Substitute your answer for z into the expression and verify the two sides are in fact equivalent.
b) If your answers match, move on to the next problem. If your answers don't match, you probably multiplied both sides of the equation in part (a) by 34.627. Redo the problem by multiplying both sides by z or by using cross-multiplication. It may help to refer back to example 3.

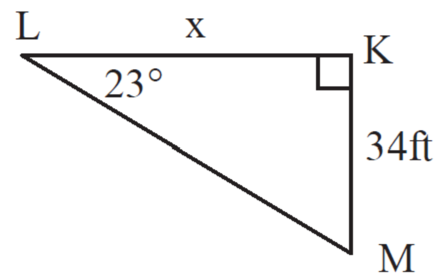
30. Consider the equation: $\tan 74° = \frac{x}{58cm}$

a) Sketch and label a right triangle that matches this equation.
b) Solve for x. Round to the nearest hundredth.
c) Determine the hypotenuse of your triangle. Round to the nearest hundredth.
d) Use the Pythagorean Theorem to confirm that this is, in fact, a right triangle.

31. Consider this information: In ΔABC with right $\angle C$, the measure of $\angle A = 31°$ and the length of side AB is 42cm.

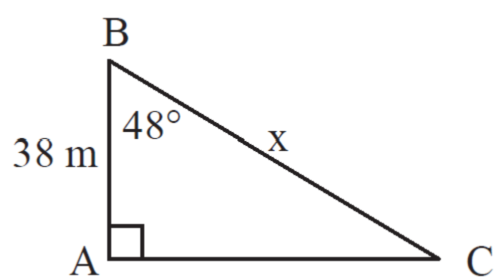
a) Sketch and label a right triangle that matches this description.
b) Determine the length of side BC.
c) Determine the length of the third side.

32. **Error Analysis:** Consider the right triangle pictured at right, which Camryn and Isabel are both trying to solve. They both set it up using the equation $\tan 23° = \frac{34}{x}$
The steps of their work are shown below. Analyze their work and determine who, if anyone, is doing it correctly.



Camryn's work	Isabel's work
$\tan 23° = \frac{34}{x}$ $34 \cdot \tan 23° = 34 \cdot \frac{34}{x}$ $34 \tan 23° = x$ $x = 14.43$	$\tan 23° = \frac{34}{x}$ *rewrite over* 1: $\frac{\tan 23°}{1} = \frac{34}{x}$ *cross – multiply* : $x \cdot \tan 23° = 34$ $\frac{x \cdot \tan 23°}{\tan 23°} = \frac{34}{\tan 23°}$ $x = \frac{34}{\tan 23°}$ $x = 80.10$

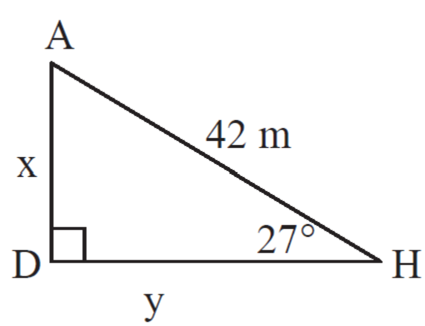
33. Consider the triangle at right:



a. Determine the length of side x to the nearest tenth.

b. Is side x, the hypotenuse, actually longer than 38 m? If not, find your error.

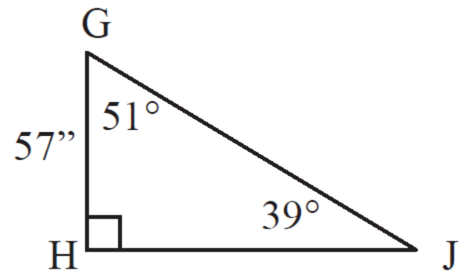
34. Answer the following questions about ΔDAH :



a. How long is side **x**? [Hint: Ignore side y. Just pretend
it's erased for a minute.]

b. How long is side **y**? [Hint: Ignore side x – just pretend it got erased for a minute.]

35. Determine the perimeter of the following triangle:



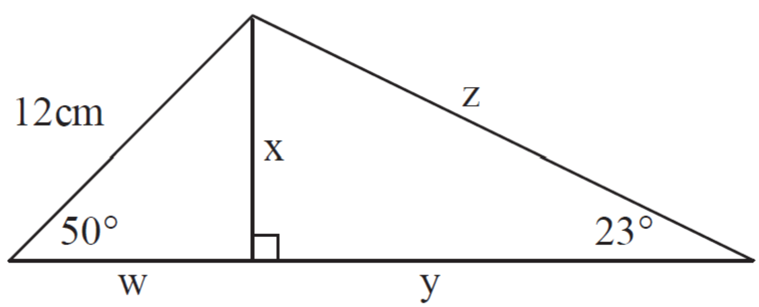
36. A 32-foot ladder is leaning against a tree. The ladder forms a 72° angle with the ground, not the tree. Assuming the tree is growing straight up:

   a. Make a labeled sketch of the situation.

   b. How high up the tree does the ladder reach?

   c. How far away from the tree is the base of the ladder?

37. Determine the lengths of sides w, x, y, and z in the figure. Round answers to the nearest hundredth:



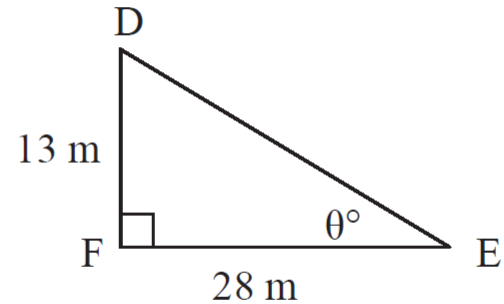
Part III: Right Triangles and SOH-CAH-TOA: Finding the Measure of an Angle Given any Two Sides

Preliminary Information: “SOH CAH TOA” is an acronym to represent the following three trigonometric ratios or formulas:

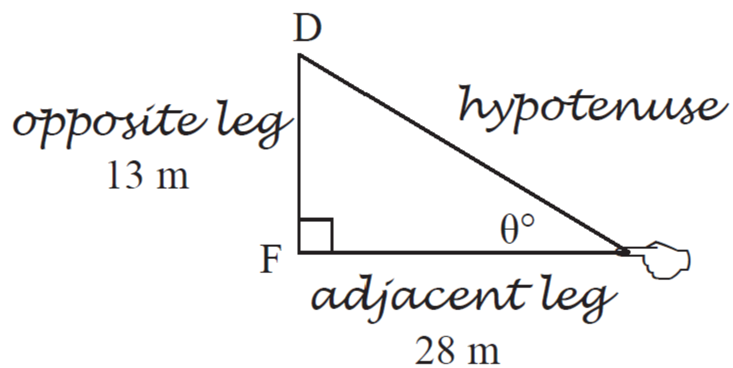
$$\sin\theta = \frac{\textit{opposite leg}}{\textit{hypotenuse}}\ ;\ \cos\theta = \frac{\textit{adjacent leg}}{\textit{hypotenuse}}\ ;\tan\theta = \frac{\textit{opposite leg}}{\textit{adjacent leg}}$$

Model Problems

Example 1: Consider right ΔDFE pictured at right. We know two sides, and our goal is to determine the measure of the unknown angle θ.



Step 1: Place your finger on the unknown angle, and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which sides do I know?" In this example, we know that the Adjacent leg is 28 m, and we know the Opposite leg is 13 m. To indicate that we know the Adjacent leg, underline both A's, and to indicate that we know the Opposite leg, underline both O's:

$S\underline{O}H\ C\underline{A}H\ T\underline{OA}$

Step 4: Consider which of the three ratios has the most information: we have one piece of information for the sine (one underline), only one piece of information for the cosine (one underline), yet we have two pieces of information for the tangent (two underlines). We are therefore going to use the tangent ratio formula:

Step 5: Substitute the known information into the formula:

$$\tan\theta = \frac{opposite\ leg}{adjacent\ leg} \Rightarrow \tan\theta = \frac{13m}{28m} \Rightarrow \tan\theta = \frac{13}{28}$$

Step 6: Determine the angle θ that satisfies this equation. There are generally two methods for finding this unknown angle:

Method 1: Table Lookup (approximate to the nearest degree)

We start with the equation

$$\tan\theta = \frac{13}{28}$$

First, we can approximate the fraction with a decimal:

$$\tan\theta = \frac{13}{28} = 0.4643$$

Next, we can examine a table of values from a chart and look for the closest "match" in the tangent column:

Angle	Sine	Cosine	Tangent
24°	0.40674	0.91355	0.44523
25°	0.42262	0.90631	0.46631 ☜
26°	0.43837	0.89879	0.48773

In which case, we pick an angle of 25°:

Angle	Sine	Cosine	Tangent
24°	0.40674	0.91355	0.44523
25° ☜	0.42262	0.90631	**0.46631**
26°	0.43837	0.89879	0.48773

So we conclude that θ = 25° to the nearest degree.

Method 2: Inverse Function on a Calculator (more accurate)

We rewrite the equation using the inverse tangent as

$$\theta = \tan^{-1}\left(\tfrac{13}{28}\right)$$

which is pronounced "theta is the inverse tangent of thirteen twenty-eighths."

What the inverse tangent function does is tell us what angle has a tangent of 13/28.

To enter this on a calculator, you typically type the SHIFT , INV or 2^{nd} key, and then the tan or $\tan^{-1}$ key (make sure your calculator is in degrees mode):

$$\theta = \tan^{-1}\left(\tfrac{13}{28}\right) = 24.9048°$$

You could also use an online inverse tangent calculator

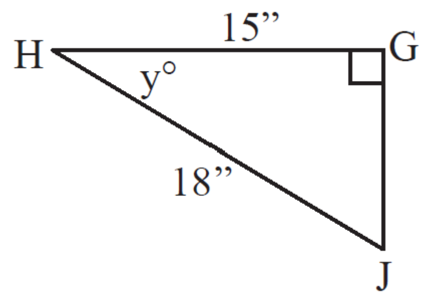
Step 7: Check for reasonableness: since the 13 m side is the shortest side, it makes sense that the opposite angle would be smallest. This answer seems reasonable.

Notice that both methods resulted in approximately the same angle.
Note: it is also possible to directly check your answer in the equation:

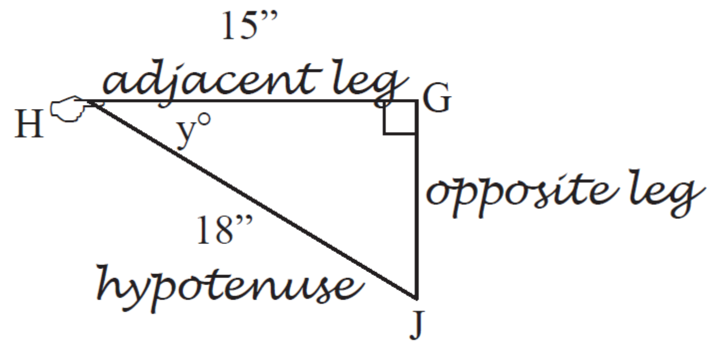
$$\tan 24.9048° = \frac{13}{28} \quad ☺$$

$$0.4643 = 0.4643$$

Example 2: Consider right ΔGHJ pictured at right. We know two sides, and our goal is to determine the degree measure of the unknown angle **y**.



Step 1: Place your finger on the unknown angle y, and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: Write “SOH CAH TOA” on your paper:

SOH CAH TOA

Step 3: Ask yourself, “Which sides do I know?” In this example, we know that the Adjacent leg is 15”, and we know the Hypotenuse is 18”. To indicate that we know the Adjacent leg, underline both A’s, and to indicate that we know the Hypotenuse, underline both H’s:

SOH CAH TOA

Step 4: Consider which of the three ratios has the most information: we have one piece of information for the sine and tangent (one underline each), yet we have two pieces of information for the cosine (two underlines). We are therefore going to use the cosine ratio formula:

$$\cos\theta = \frac{adjacent\ leg}{hypotenuse}$$

Step 5: Substitute the known information into the formula:

$$\cos y = \frac{adjacent\ leg}{hypotenuse} \Rightarrow \cos y = \frac{15''}{18''} \Rightarrow \cos y = \frac{15}{18} = \frac{5}{6} \quad (reduced)$$

Step 6: Determine the angle **y** that satisfies this equation.

Method 1: Table Lookup (approximate to the nearest degree)

We start with the equation

$$\cos y = \frac{15}{18} = \frac{5}{6}$$

First, we can approximate the fraction with a decimal:

$$\cos y = \frac{15}{18} = \frac{5}{6} = 0.8333$$

Next, we can examine a table of values from a chart and look for the closest "match" in the cosine column:

Angle	Sine	Cosine	Tangent
33°	0.54464	0.83867	0.64941
34°	0.55919	**0.82904** ☜	0.67451
35°	0.57358	0.81915	0.70021

In which case, we pick an angle of 34°:

Angle	Sine	Cosine	Tangent
33°	0.54464	0.83867	0.64941
34° ☜	0.55919	**0.82904**	0.67451
35°	0.57358	0.81915	0.70021

So we conclude that y = 34° to the nearest degree.

Method 2: Inverse Function on a Calculator (more accurate)

We rewrite the equation using the inverse cosine as

$$y = \cos^{-1}\left(\tfrac{15}{18}\right) = \cos^{-1}\left(\tfrac{5}{6}\right)$$

which is pronounced "y is the inverse cosine of five-sixths."

What the inverse cosine function does is tell us what angle has a cosine of 5/6.

To enter this on a calculator, you typically type the SHIFT , INV or 2nd key, and then the cos or $\cos^{-1}$ key (make sure your calculator is in degrees mode):

$$y = \cos^{-1}\left(\tfrac{5}{6}\right) = 33.5573°$$

You could also use an online inverse cosine calculator

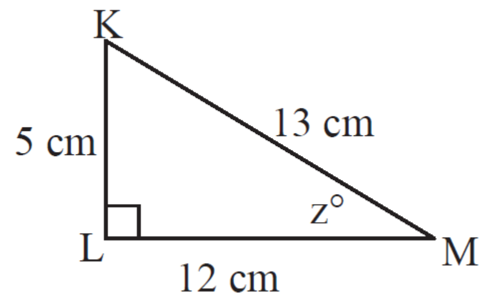
Step 7: Check for reasonableness: since the 15" side is almost as long as the hypotenuse, it makes sense that the angle would be less than 45°. This answer seems reasonable.

Note: it is also possible to directly check your answer in the equation:

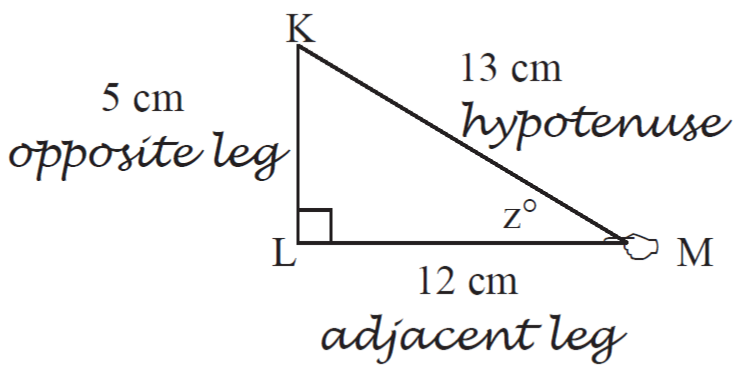
$$\cos 33.5573° = \frac{15}{18} \qquad ☺$$

$$0.8333 = 0.8333$$

Example 3: Consider right ΔKLM pictured at right. We know all three sides, and our goal is to determine the degree measure of the unknown angle **z**.



Step 1: Place your finger on the unknown angle z, and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: Write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which sides do I know?" In this example, we know all three sides: The Adjacent leg is 12 cm, the Hypotenuse is 13 cm, and we know the Opposite leg is 5 cm. To indicate that we know the Adjacent leg, underline both A's, to indicate that we know the Opposite leg, underline both O's, and to indicate that we know the Hypotenuse, underline both H's:

SOH CAH TOA

Step 4: Consider which of the three ratios has the most information: In this case, we could use any of them! For simplicity, let's favor the sine ratio in this example, but we'll carry the cosine and tangent ratios just to watch what happens:

$$\boxed{\sin z = \frac{\textit{opposite leg}}{\textit{hypotenuse}}};\ \cos z = \frac{\textit{adjacent leg}}{\textit{hypotenuse}};\ \tan z = \frac{\textit{opposite leg}}{\textit{adjacent leg}}$$

Step 5: Substitute the known information into the formula:

$$\boxed{\sin z = \frac{5}{13}};\ \cos z = \frac{12}{13};\ \tan z = \frac{5}{12}$$

Step 6: Determine the angle **z** that satisfies the equation.

Method 1: Table Lookup (approximate to the nearest degree)

We start with the equation

$$\sin z = \frac{5}{13};\ \cos z = \frac{12}{13};\ \tan z = \frac{5}{12}$$

Approximate the fractions with decimals:

$$\sin z = \frac{5}{13} = 0.3846;\ \cos z = \frac{12}{13} = 0.9231;$$

$$\tan z = \frac{5}{12} = 0.4167$$

Next, we can examine a

table of values from a chart

and look for the closest "match" in the sine column: (Notice how the cosine and tangent columns match as well!)

Angle	Sine	Cosine	Tangent
22°	0.37461	0.92718	0.40403
23°	0.39073 ☜	0.92050	0.42447
24°	0.40674	0.91355	0.44523

In which case, we pick an angle of 23°:

Angle	Sine	Cosine	Tangent
22°	0.37461	0.92718	0.40403
23° ☜	0.39073	0.92050	0.42447
24°	0.40674	0.91355	0.44523

So we conclude that z = 23° to the nearest degree.

Method 2: Inverse Function on a Calculator (more accurate)

We rewrite the equation using the inverse sine as

$$z = \sin^{-1}(\tfrac{5}{13})$$

which is pronounced "z is the inverse sine of five-thirteenths."

What the inverse sine function does is tell us what angle has a sine of 5/13.

To enter this on a calculator, you typically type the SHIFT , INV or 2nd key, and then the sin or $\sin^{-1}$ key (make sure your calculator is in degrees mode):

$$z = \sin^{-1}(\tfrac{5}{13}) = 22.6199°$$

You could also use an online inverse sine calculator

Note: Though this example favored the inverse sine, we could have used the inverse cosine or inverse tangent:

$$\cos^{-1}(\tfrac{12}{13}) = 22.6199°;\ \tan^{-1}(\tfrac{5}{12}) = 22.6199°,$$

both of which give identical answers.

Step 7: Check for reasonableness: since the 5 cm side is the shortest, it makes sense that the opposite angle is the smallest as well. Note: it is also possible to directly check your answer in the equations:

$$\sin 22.6199° = \frac{5}{13} \quad ☺$$
$$0.3846 = 0.3846$$

$$\cos 22.6199° = \frac{12}{13} \quad ☺$$
$$0.9231 = 0.9231$$

$$\tan 22.6199° = \frac{5}{12} \quad ☺$$
$$0.41671 = 0.4167$$

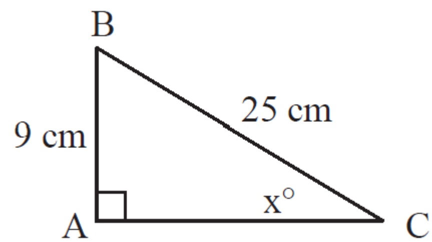
PROBLEM SET 4

38. Calculate the value of x to the nearest degree: $\sin x = 0.78801$

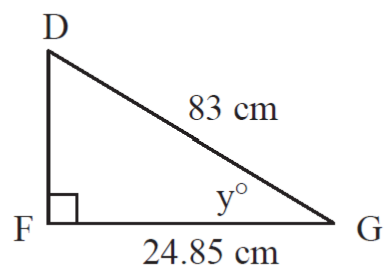
39. Calculate the value of y to the nearest tenth: $\cos y = \frac{24}{25}$

40. Calculate the value of z to the nearest hundredth: $\tan z = \frac{84.93}{34.627}$

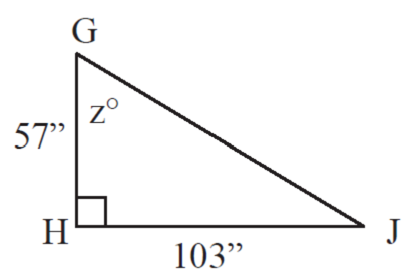
41. Determine the measure of angle x to the nearest tenth.



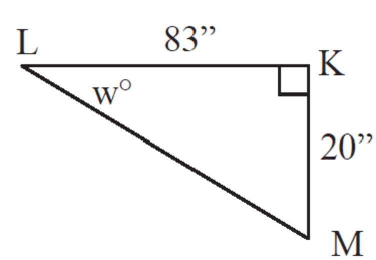
42. Determine the measure of angle y to the nearest hundredth.



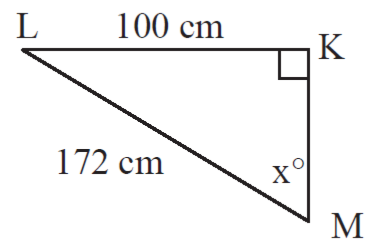
43. Determine the measure of angle z to the nearest degree.



44. Determine the measure of angle w to the nearest degree.



45. Error Analysis: Josh was asked to determine the measure of angle x to the nearest hundredth. His teacher marked it incorrect.



His work is shown below. Find his error, and then correct it.

$$\sin x = \tfrac{100}{172}$$

$$\textit{rewrite}:$$

$$x = \sin^{-1}(\tfrac{100}{172})$$

$$\textit{use a decimal approximation}:$$

$$x = \sin^{-1}(0.58140)$$

$$\textit{if you raise it to the } -1 \textit{ power, use a reciprocal}:$$

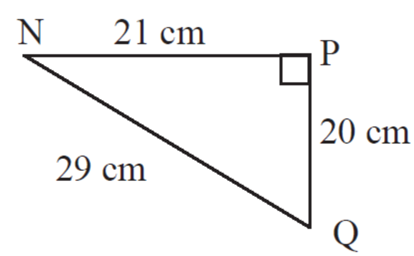
$$x = \frac{1}{\sin(0.58140)}$$

$$\textit{Simplify}:$$

$$x = \frac{1}{0.01015}$$

$$x = 98.52°$$

46. For the triangle pictured, Marcy placed her finger on the vertex of angle N and concluded that $\cos N = \frac{21}{29}$. Likewise, Timmy placed his finger on the vertex of angle N and concluded that $\sin N = \frac{20}{29}$.

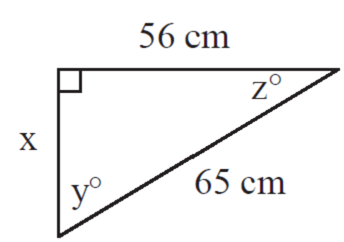


a. If you solve it beginning with Marcy's equation, what answer will she get?

b. If you solve it Timmy's way, what answer will he get?

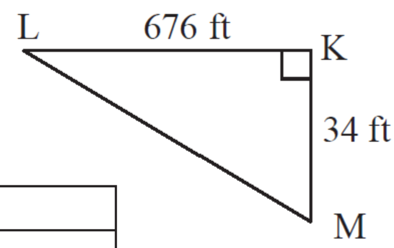
c. Are these results reasonable? Explain.

47. Use the Pythagorean Theorem, SOH-CAH-TOA, and the fact that the sum of the three interior angles of a triangle sum to 180° to determine all unknown sides and angles of the triangle pictured at right. Round all quantities, when necessary, to the nearest hundredth. (Note: you could also use a calculator.)



48. Error Analysis: Consider the right triangle pictured at right, which Annie and Lauren are both trying to solve. They both set it up using the equation .The steps of their work are shown below. Analyze their work to determine who, if anyone, is doing it correctl

(drawing not to scale)



Annie's work	Lauren's work
$\tan L = \frac{34}{676}$ $L = \tan^{-1}\left(\frac{34}{676}\right)$ $L = \mathbf{2.88}^{\circ}$	$\tan L = \frac{34}{676}$ $L = \tan\left(\frac{34}{676}\right)$ $L = \mathbf{0.00088}^{\circ}$

49. Consider the equation $\tan x = \frac{74\ cm}{58\ cm}$

   a. Sketch and label a right triangle that matches this equation.
   b. Solve for x. Round to the nearest hundredth.
   c. Determine the hypotenuse of your triangle. Round to the nearest hundredth.
   d. Use the Pythagorean Theorem to confirm that this is, in fact, a right triangle.

50. Consider the following information: In ΔABC with right $\angle C$, the length of side AB is 42 cm, and the length of side AC is 20 cm.

   a) Sketch and label a right triangle that matches this description.

   b) Determine the measure of angle B to the nearest hundredth.

   c) Determine the measure of the third angle.

51. For this problem, you will need to examine a chart of Sine, Cosine, and Tangent for each angle from 0° to 90°. You may use an online chart as well.

   For each question, determine, to the nearest degree, the angle between 0° and 90° that most closely matches each description below:

   a) The cosine of an angle is approximately twice as big as the sine of the same angle. (In other words, the number in the cosine column is very nearly twice as big as the number in the sine column.)
   b) The tangent of an angle is approximately five times as big as the cosine of the same angle.

52. The following equations specify a specific right triangle: $\tan C = \frac{38}{40}$; $\cos B = \frac{38}{x}$; $\sin A = 1$

a) Make a labeled sketch of this triangle.

b) Determine the measure of angle B to the nearest tenth.

c) Determine the measure of angle C to the nearest tenth.

d) Determine the length of side x to the nearest hundredth.

53. A right triangle has an area of 54 cm^2 and one of its legs is 9 cm. Determine the measures of its two acute angles to the nearest degree.

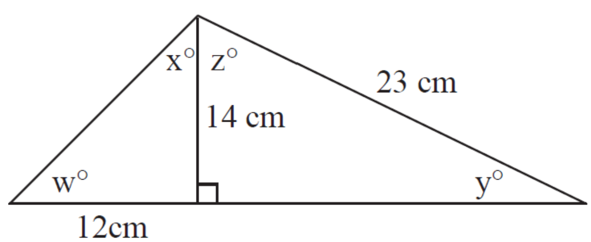
54. A 32-foot ladder is leaning against a tree. The base of the ladder rests 7 feet away from the foot of the tree. Assuming the tree is growing straight up:

a) Make a labeled sketch of the situation.

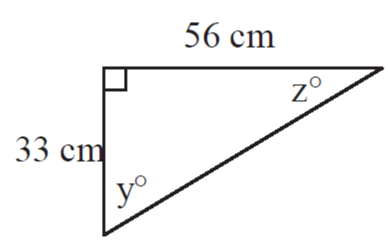
b) What acute angle does the ladder form with the ground?

c) How high up the tree does the ladder reach?

55. Determine the measures of angles w, x, y, and z. Round answers to the nearest hundredth:



56. Xavier, Yolanda and Zelda are examining the right triangle at right. Yolanda observes that $\tan y = \frac{56}{33}$, so $y = \tan^{-1}\left(\frac{56}{33}\right)$ and Zelda observes that $\tan z = \frac{33}{56}$, so $z = \tan^{-1}\left(\frac{33}{56}\right)$. Xavier comments that $\frac{56}{33}$ and $\frac{33}{56}$ are reciprocals of each other.



a) To the nearest degree, what angle will Yolanda get for angle y?

b) To the nearest degree, what angle will Zelda get for angle z?

c) What is the sum of Yolanda's and Zelda's answers?

d) Explain why the answer you got in part (c) is reasonable. Think back to the good ol' days of Geometry!

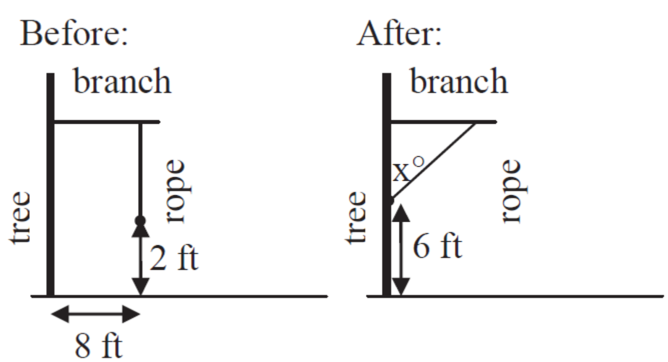
e) Xavier attempts to write a formula expressing this idea, but he gets stuck. Can you help him complete his formula?

$$\tan^{-1}\left(\tfrac{a}{b}\right) + \underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$

f) Verify that your formula works when a = 20 and b = 7

57. This problem will require you to assign variables to the unknown quantities and write equations to represent the given information. The Pythagorean Theorem will come in handy!

A rope swing (a rope with a large knot tied at the bottom, to sit on) is hanging straight down from a horizontal tree branch. In that position, the swing seat is two feet off the ground, and it hangs 8 feet from the tree. Ryan, who is 6 feet tall, pushes the swing seat toward the tree so that its edge is touching the tree, and Ryan realizes that he can fit directly underneath (it is as if he were wearing the seat as a hat!). Assume that the rope is still straight. A sketch of the situation is shown below:



a. How long is the rope?

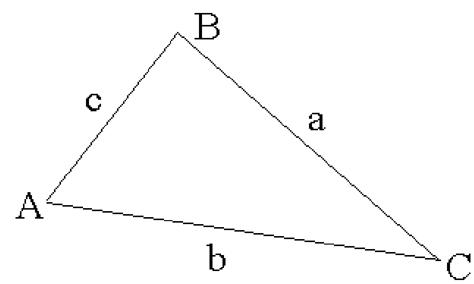
b. How high up from the ground is the branch?

c. What angle x does the rope form with the tree branch?

PART IV: Law of Sines and Law of Cosines

The previous sections have all dealt with the sides and angles in right triangles. But what if a triangle doesn't have a right angle in it? There are still relationships between the sides and angles, but they quite as simple. These relationships are the called the Law of Sines and the Law of Cosines. They hold true for <u>any</u> triangle, not just right triangles.

Take a look at the generic triangle at right. If the sides are lengths a, b, and c, and the angles opposite those sides are A, B, and C respectively, then the Law of Sines states that the ratio of the length of a side to the sine of the angle opposite that side is constant. In other words,



LAW OF SINES:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Law of Cosines is a generalized version of Pythagoras' Theorem. Recall that in a right triangle, if c is the hypotenuse and a and b are the two legs, then $c^2 = a^2 + b^2$. The Law of Cosines states that for any triangle, if side c is opposite angle C and sides a and b are adjacent to angle C (as in the drawing,) then:

LAW OF COSINES:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

The choice of whether to use the Law of Sines or the Law of Cosines to solve a triangle depends on what information you've been given about the triangle:

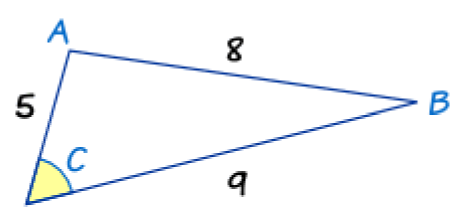
Law of Cosines is the best choice if:

Case1: The length of all three sides of a triangle are known and you are trying to find an angle:

Start with: $c^2 = a^2 + b^2 - 2ab \cos(C)$

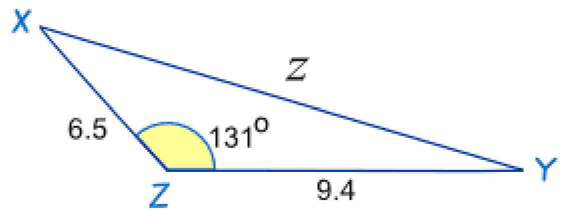
Put in a, b and c: $8^2 = 9^2 + 5^2 - 2 \cdot 9 \cdot 5 \cdot \cos(C)$

Calculate: $64 = 81 + 25 - 90 \cdot \cos(C)$



Subtract 25 from both sides: $39 = 81 - 90 \times \cos(C)$
Subtract 81 from both sides: $-42 = -90 \times \cos(C)$
Swap sides: $-90 \times \cos(C) = -42$
Divide both sides by −90: $\cos(C) = 42/90$
Inverse cosine: $C = \cos^{-1}(42/90)$
Calculator: C = **62.2°** (to 1 decimal place)

Case 2: Two sides and an enclosed angle are known and you are trying to find the side opposite the angle:



Start with: $z^2 = x^2 + y^2 - 2xy\cos(Z)$

Put in the values we know:

$$z^2 = 9.4^2 + 6.5^2 - 2 \cdot 9.4 \cdot 6.5 \cdot \cos(131^\circ)$$

Calculate: $z^2 = 88.36 + 42.25 - 122.2 \cdot (-0.656...)$

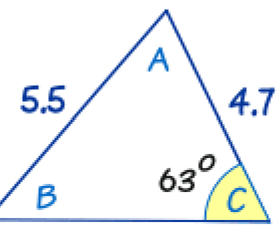
$$z^2 = 130.61 + 80.17...$$

$$z^2 = 210.78...$$

$z = \sqrt{210.78\,...} =$ **14.5** to 1 decimal place.

Law of Sines is the best choice if:

Case 3: Two sides and an angle opposite one of those sides are known and you are trying to find the other angle(s). For example, to find angle B in the triangle shown:



Start with:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Put in the values we know:

$$\frac{a}{\sin A} = \frac{4.7}{\sin B} = \frac{5.5}{\sin 63^\circ}$$

Ignore "$\frac{a}{\sin A}$" and invert both sides of the remaining equation:

$$\frac{\sin B}{4.7} = \frac{\sin 63^\circ}{5.5}$$

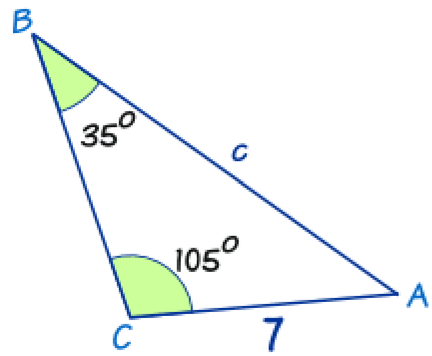
Multiply both sides by 4.7: $\sin B = \left(\frac{\sin 63^\circ}{5.5}\right) \times 4.7$

Calculate: $\sin B = 0.7614...$

Inverse Sine: $B = \sin^{-1}(0.7614...)$

$B =$ **49.6°**

Case 4: Two angles and one side are known and you are trying to find a missing side:



Law of Sines:	a/sin A = b/sin B = c/sin C
Put in the values we know:	a/sin A = 7/sin(35°) = c/sin(105°)
Ignore a/sin A (not useful to us):	7/sin(35°) = c/sin(105°)

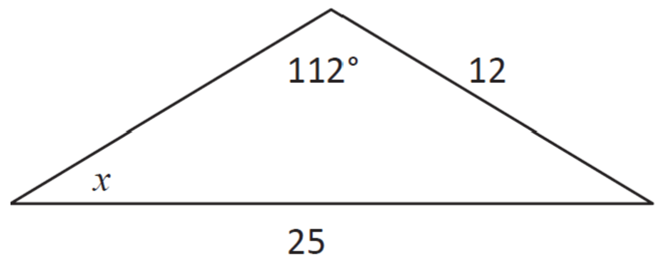
Now we use our algebra skills to rearrange and solve:

Swap sides:	c/sin(105°) = 7/sin(35°)
Multiply both sides by sin(105°):	c = (7 / sin(35°)) × sin(105°)
Calculate:	c = (7 / 0.574...) × 0.966...
	c = **11.8** (to 1 decimal place)

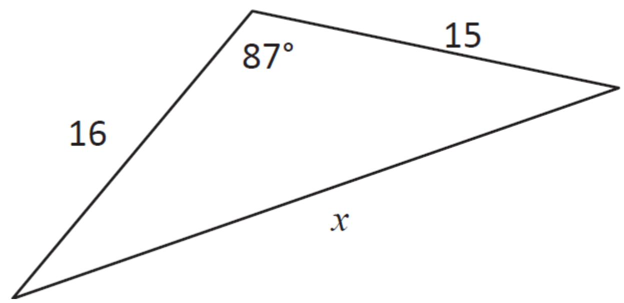
In general, the Law of Sines is easier to use so always check to see if you can use it first.

PROBLEM SET 5: Determine whether the Law of Cosines or the Law of Sines is the best choice.

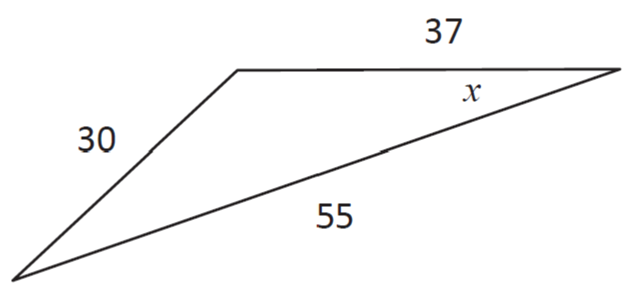
58. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



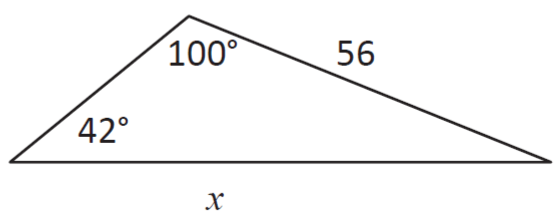
59. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



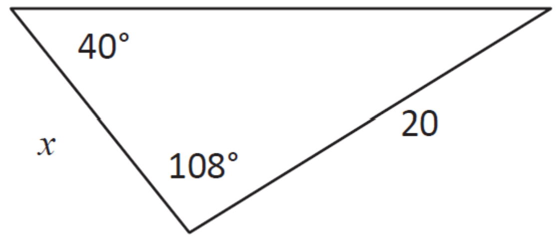
60. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).state whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



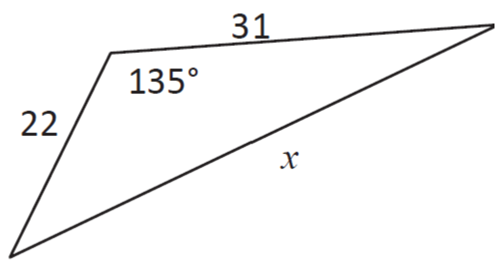
61. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



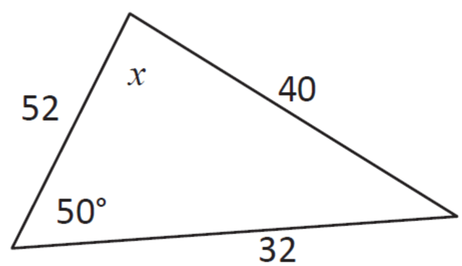
62. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



63. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



64. State whether the Law of Sines or Law of Cosines is the best choice to solve for x for the given figure. Substitute the values into the appropriate formula (do not solve).



65. State whether the Law of Sines or Law of Cosines is the best choice to solve for the requested value from the information given. Substitute the values into the appropriate formula (do not solve).

In ΔABC, $a = 17$, $c = 34$, and $m\angle B = 94°$. Find b.

66. State whether the Law of Sines or Law of Cosines is the best choice to solve for the requested value from the information given. Substitute the values into the appropriate formula (do not solve).

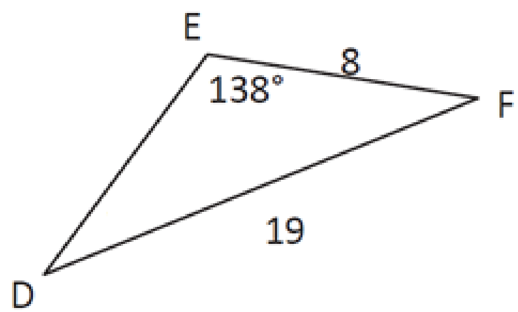
In ΔHJK, $j = 31$, $m\angle H = 132°$, $m\angle J = 21°$, and $m\angle K = 27°$. Find h.

67. State whether the Law of Sines or Law of Cosines is the best choice to solve for the requested value from the information given. Substitute the values into the appropriate formula (do not solve).

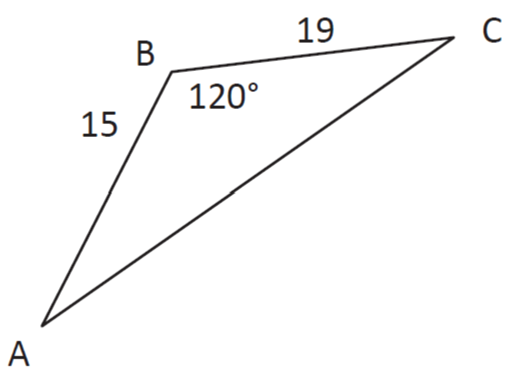
In ΔXYZ, $x = 6$, $y = 9$, and $z = 12$. Find $m\angle Y$.

PROBLEM SET 6: Use the Law of Sines and Law of Cosines to find missing dimensions.

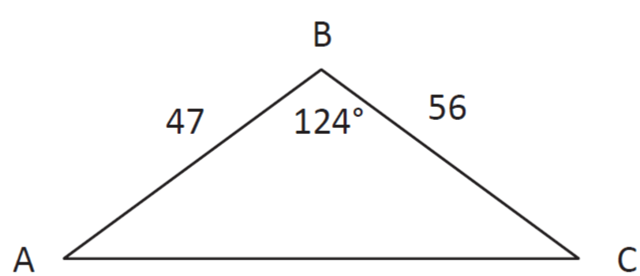
68. Find the length of side DE and the measure of angle F in the triangle below. Round your answers to the nearest whole number.



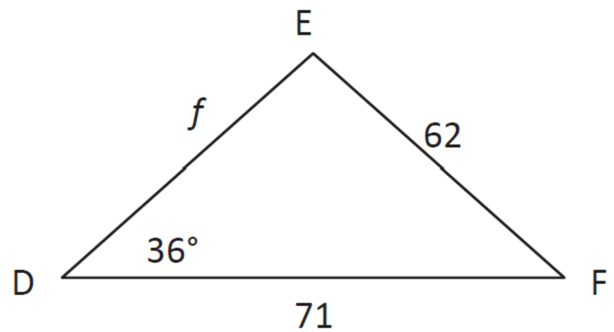
69. Find the $m\angle C$ to the nearest whole degree.



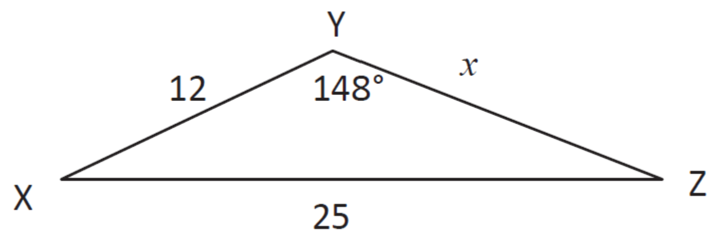
70. Find the length of side AC and the measures of angles A and C in the triangle below. Round your answers to the nearest whole number.



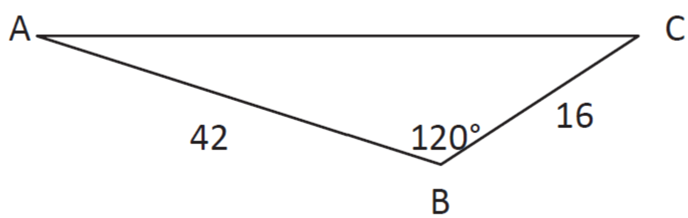
71. Find f to the nearest whole number in the triangle below.



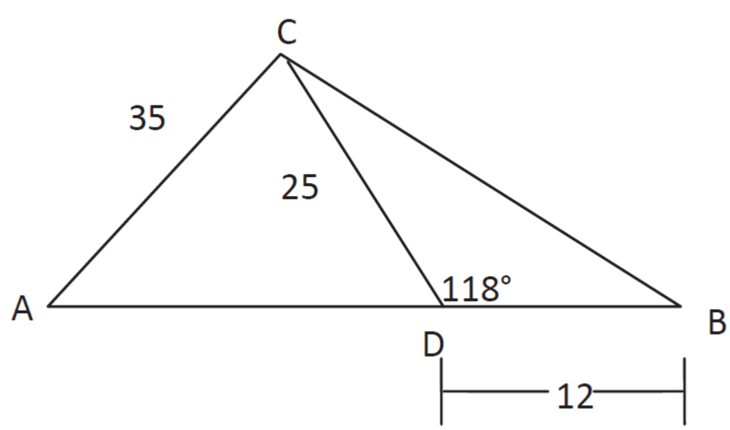
72. Find x to the nearest whole number in the triangle below.



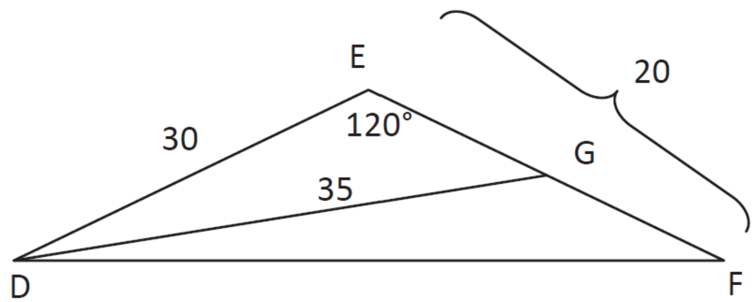
73. Find the measure of angle A to the nearest whole degree.



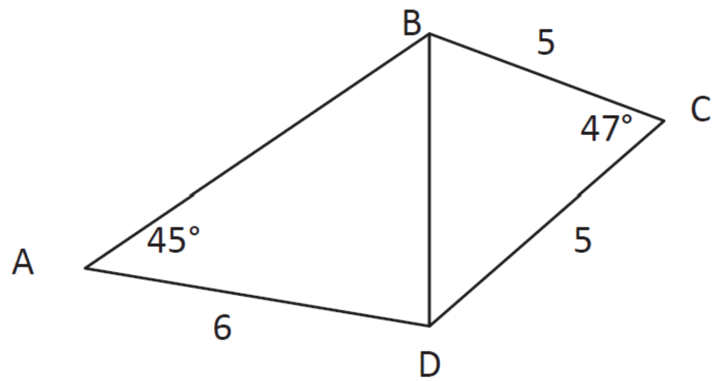
74. Find measure of angle A to the nearest whole degree.



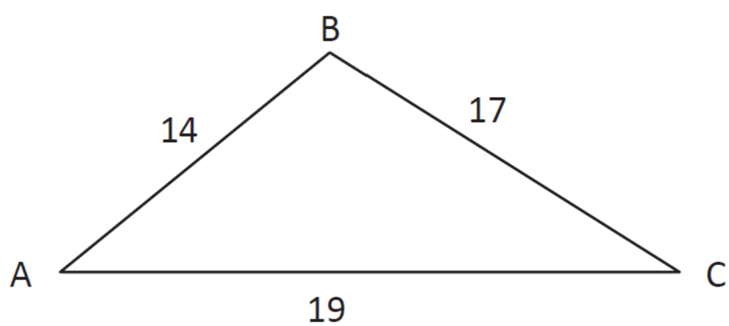
75. Find $m\angle DGF$ to the nearest whole degree:



76. Find $m\angle ABD$ to the nearest whole degree:



77. Find the measures of all three angles in the triangle below:



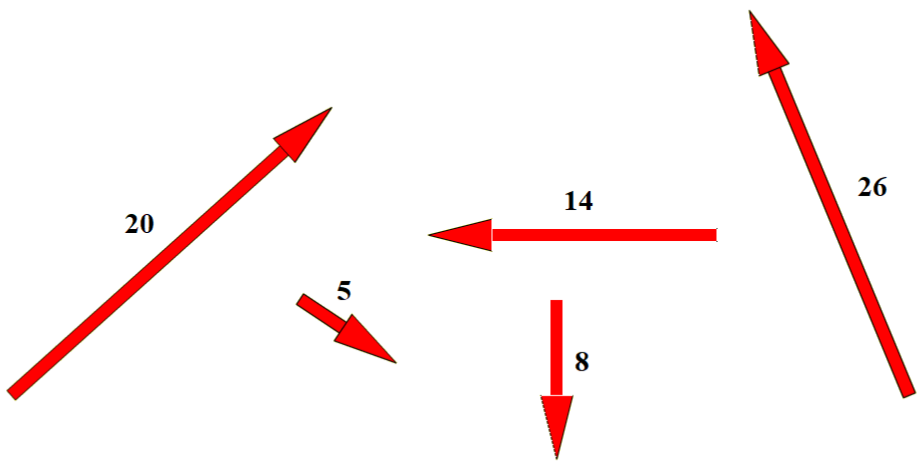
PART V: VECTORS

Let's learn a little bit about vectors…
In science, we deal with measureable quantities. Measureable quantities are attributes of an object that can be quantitatively determined and compared to an accepted standard. For example, a person is not a measureable quantity, but things about a person (height, weight, age, heart rate, blood sugar level, etc.) are. When talking about a measureable quantity, the accepted standard is called the unit. So, height can be given in meters, weight in newtons, age in years, heart rate in beats per minute, and so on. In this course, our units will use SI.

In physics, some quantities are distinguished not only by their size but also by the direction in which they act. For example, we distinguish between moving at a certain speed heading north and moving at that same speed but heading south. The reason is that these lead to two different outcomes- you can't get to your friend's house by driving the wrong way! When a quantity requires a direction to fully describe it, it is called a vector quantity. Quantities such as time, temperature, or mass do not have direction. These types of quantities are called scalars.

VECTOR QUANTITY: any quantity that has both magnitude and direction.
SCALAR QUANTITY: any quantity that has only magnitude.

Vector quantities are represented by arrows. The length of the arrow indicates the magnitude of the quantity and the orientation of the arrow indicates the direction.



In the above diagram, we see five different vectors. The magnitude of the vector pointing directly to the left is given as 14 units. What those units might be depends on the type of vector quantity the arrow is supposed to represent. Examples of vector quantities in physics and their units are listed below:

Quantity	**SI unit**
Position	Meters
Displacement	Meters
Velocity	Meters/second
Acceleration	Meters/second^2
Force	Newtons

Designating Vectors

In a given question, you are often asked to find a solution that involves a vector quantity. How do you give your answer? In order to provide a full answer, we need to know what the symbol for a vector looks like, then we need to understand how to provide the direction a vector points in.

If a quantity is a vector, then the symbol for it has a little arrow drawn over it. If we're only interested in the magnitude of the vector, there won't be an arrow over the symbol. $\vec{\boldsymbol{A}}$ means "the vector A," but $\boldsymbol{A}$ means "the *magnitude* of vector A." To fully describe the vector, you need to provide the direction. In physics, the best way to provide the direction is to give it as an angle relative to an axis. In introductory physics, we will focus almost exclusively on phenomena that occur either along a line or in a plane, i.e., one- or two-dimensional situations. Therefore, we will only at most have to give the angle a vector makes to either the x- or y-axis.

In 1-D situations it is even more simplistic: the direction will be either in one direction or the other. We can label these two directions the same way we do on a number line: as either positive or negative. For vectors, it is then NOT true that the negative of a number is less than the positive. For example, a temperature of -10 °C is definitely less than a temperature of 10 °C because temperature is a scalar. However, if car A is moving at -10 m/s and car B is moving at +10 m/s, car A is not slower than car B. In both cases the car is moving at 10 m/s; the only difference is that the cars are heading in opposite directions. We would state:

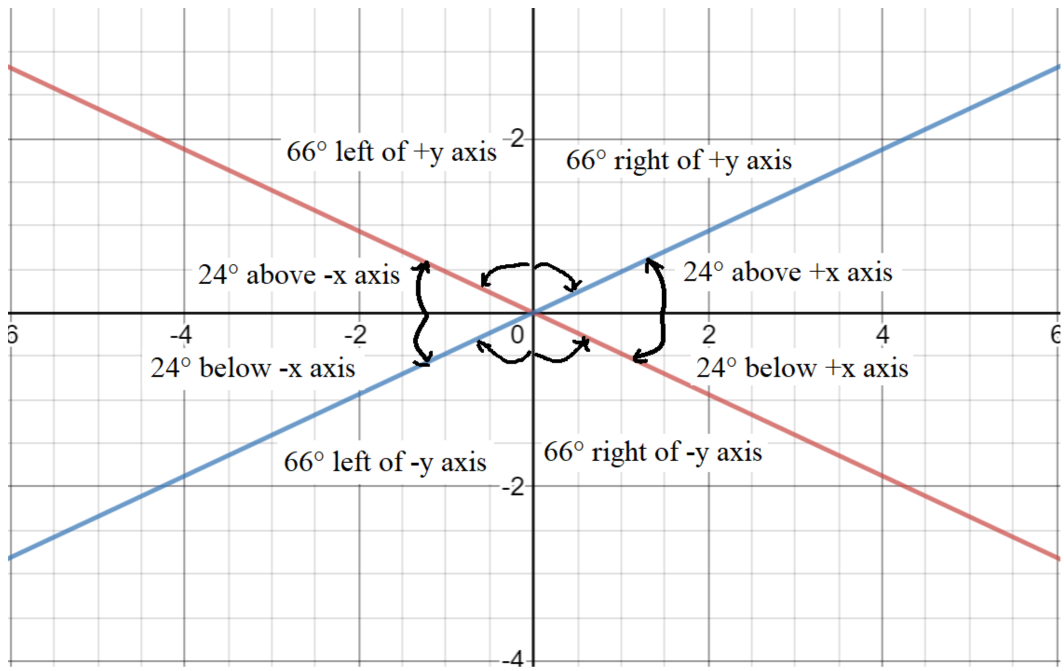
$$\vec{v}_A = -10\ \frac{m}{s}$$

$$\vec{v}_B = +10\frac{m}{s}$$

And

$$v_A = v_B = 10\frac{m}{s}$$

If a situation involves two dimensions, the direction of the vector is given as an angle measured from one of the axes in a plane. If the angle is measured from the x-axis, the angle is named as being either *above or below* either the positive or negative x-axis. If the angle is measured from the y-axis, the angle is named as being either *to the left or to the right* of either the positive or negative y-axis. The diagram shows this for some angles in the quadrants:

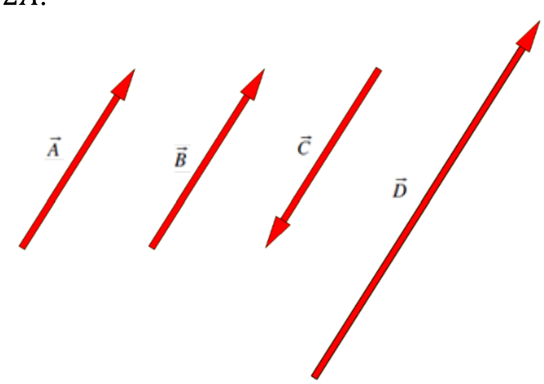


Basic Properties of Vectors:

1. Two vectors are equal if they have the same magnitude as well as the same direction. In the diagram below, $\vec{A} = \vec{B}$.

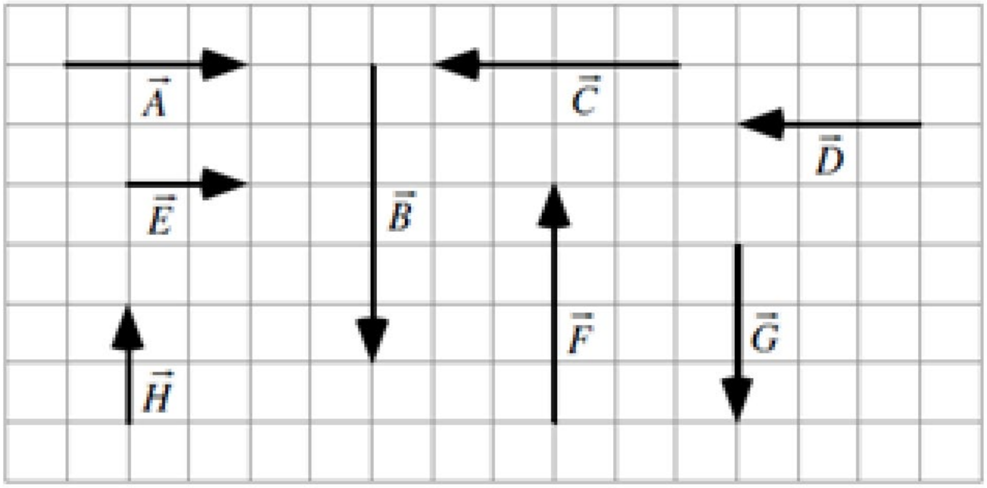
2. The opposite of a vector is a vector of the same magnitude but pointing in the opposite direction. In the diagram below, $\vec{C} = -\vec{A}$.

3. To multiply a vector by a scalar, multiply the magnitude of the vector by the scalar. In the diagram below, $\vec{D} = 2\vec{A}$.



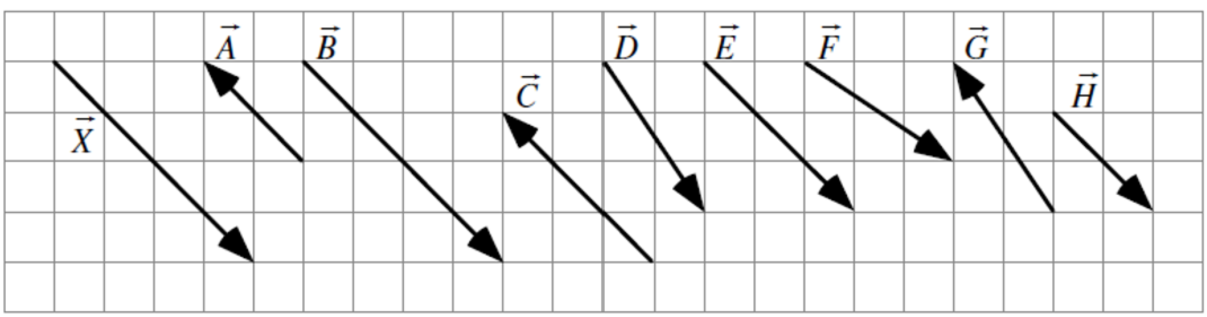
PROBLEM SET 7

Here are eight vectors imposed on a grid:



78. List all the vectors that have the same magnitude as vector $\vec{A}$.
79. List all the vectors that have the same magnitude as vector $\vec{B}$.
80. List all the vectors that have the same magnitude as vector $\vec{C}$.
81. List all the vectors that are equal to $-\vec{A}$.

Nine vectors are superimposed on a grid:



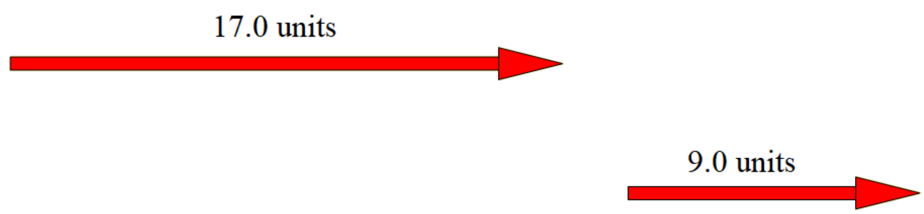
82. List all the vectors that have the same direction as vector $\vec{X}$.
83. List all the vectors that are equal to vector $\vec{X}$.
84. List all the vectors that are equal to $2\vec{H}$.
85. List all the vectors that are equal to $-1.5\vec{H}$.

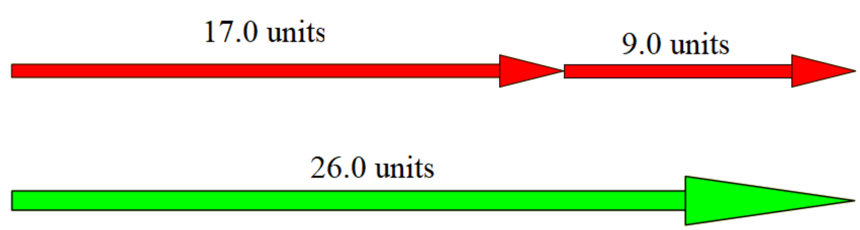
Vector Math

Like scalar quantities, vectors can be added or subtracted. However, the process isn't as simple as it is with scalars. For example, mass is a scalar. If 3.0 kg of apples are loaded onto a 4.0 kg wagon, there is a total mass of 7.0 kg. Eat 1.0 kg worth of apples, and there will be 6.0 kg of mass left. However, if a plane heading due east with a velocity of 4.0 m/s encounters a north-blowing wind of 3.0 m/s, the plane does not end up going 7.0 m/s. When adding vectors such as velocity, both the magnitude and direction of the vectors must be considered.
Since vectors are represented by arrows, the addition of vectors is accomplished by adding the arrows together. This is called the *tip-to-tail* method. The arrow representing the vector being added is moved until it is placed on the tip of the arrow being added to. The sum of the vectors, called the **resultant**, is then found by drawing an arrow from the tail of the first to the tip of the second. In the diagrams below, the red vectors represent the vectors being added together and the green arrow represents the resultant.

Example 1: Collinear vectors in the same direction

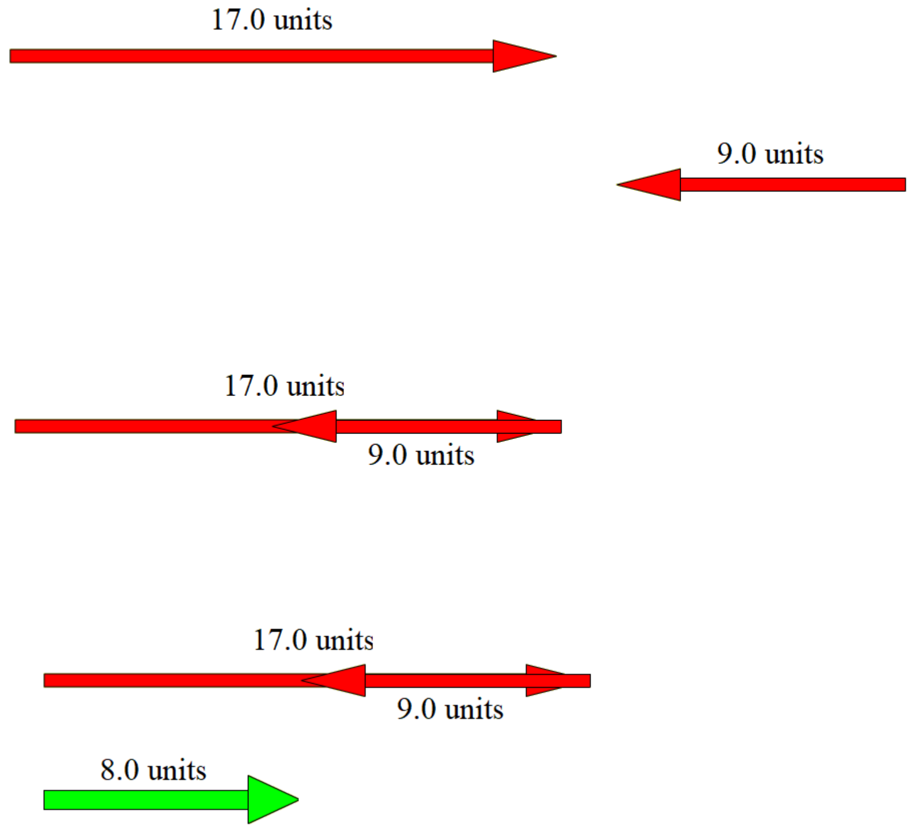
Two vectors that point along the same line are said to be collinear. In this example, the 9.0 unit long vector is moved to the tip of the 17.0 unit long vector. The resultant vector is 26.0 units long and points in the same direction.





Example 2: Collinear vectors in opposite directions

Again, the 9.0 unit long vector is moved so that its tail rest s on the tip of the 17.0 unit long vector, and the resultant is drawn from the tail of the first to the tip of the last. The resultant is an 8.0 unit long vector that points in the direction of the longer vector.



You probably notice in the first example the resultant is the sum of the two magnitudes, while in the second the resultant is the difference. This should suggest that like scalars, subtracting a vector is the same as adding its opposite. *The opposite of a vector is a vector that is the same magnitude but points in the opposite direction*.

If $\vec{A}$ is the 17.0 unit vector, $\vec{B}$ is the 9.0 unit vector that points to the right, and $\vec{C}$ is the 9.0 unit long vector, then example 1 shows $\vec{A}+\vec{B}$ and example 2 shows $\vec{A}+\vec{C}$ *or* $\vec{A}-\vec{B}$, since $\vec{C}=-\vec{B}$.
You might also notice that if instead you move the 17.0 unit vector instead of the 9.0 unit long vector, you will get the same resultants. In other words, $\vec{A}+\vec{B}=\vec{B}+\vec{A}$. This means that vector addition obeys the Commutative Property. Vector addition also obeys the Associative Property.

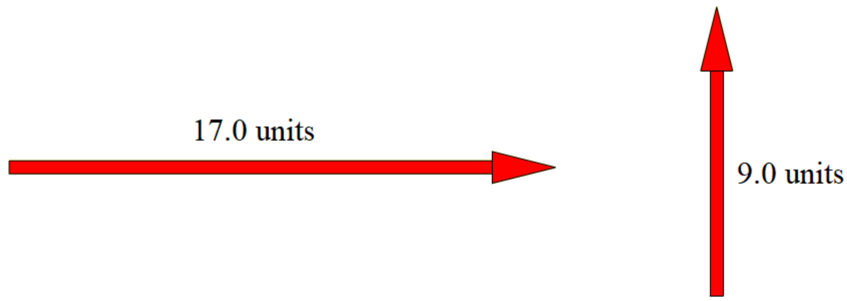
Example 3: Perpendicular Vectors

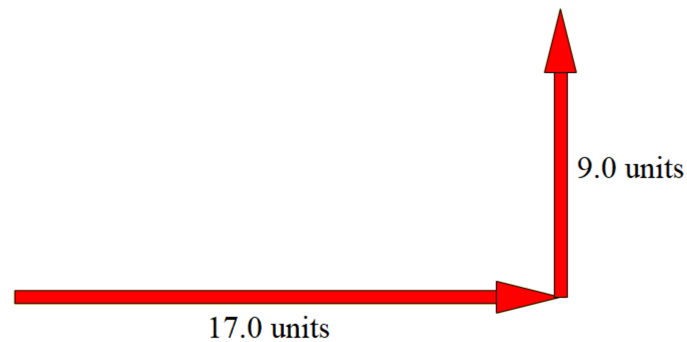
When the vectors are perpendicular, the vectors are still added tip-to-tail, but the resultant will now be the hypotenuse of a right triangle.
The magnitude of the resultant is found from the Pythagorean Theorem, and the direction in this example is given as the angle the resultant makes with the horizontal. This angle can be fund by using the inverse tangent function:

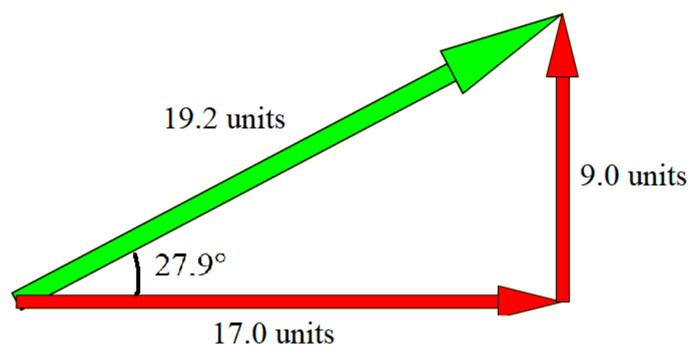
$$R = \sqrt{17.0^2 + 9.0^2}$$
$$= 19.2 \text{ units}$$
$$\theta = \tan^{-1}\left(\frac{9.0}{17.0}\right)$$
$$= 27.9°$$

$\vec{R} = 19.2\ units$ @ $27.9°$ above the + x-axis.

You should be able to verify that once again, the sum of the two red vectors is the same if the order in which the two vectors is switched.

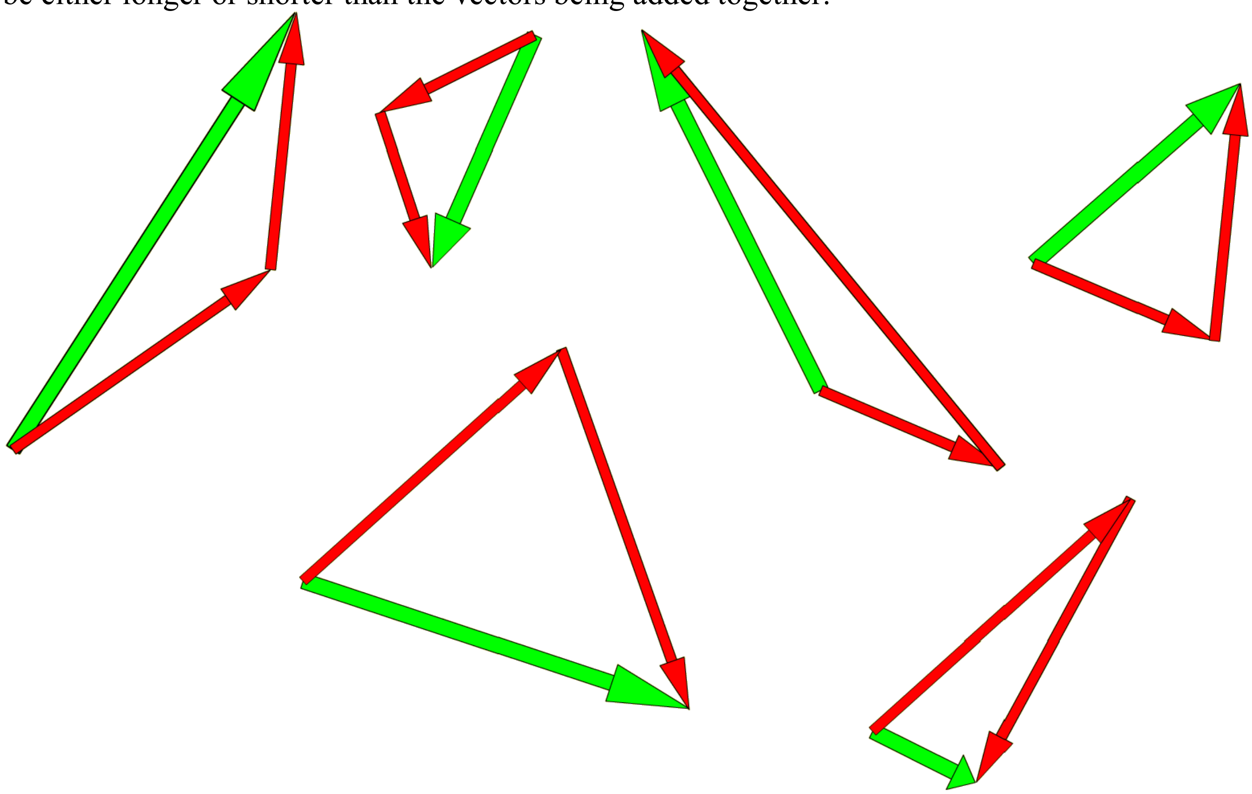




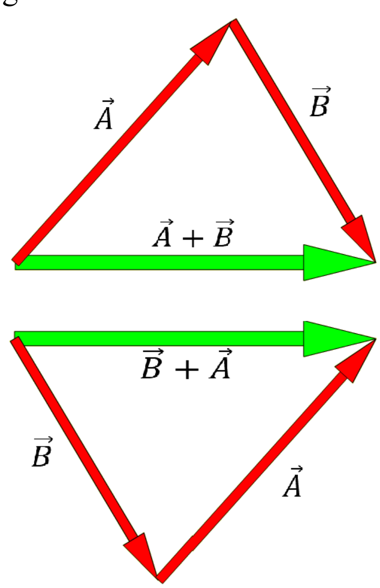


Example 4: Non-perpendicular, non-colinear vectors

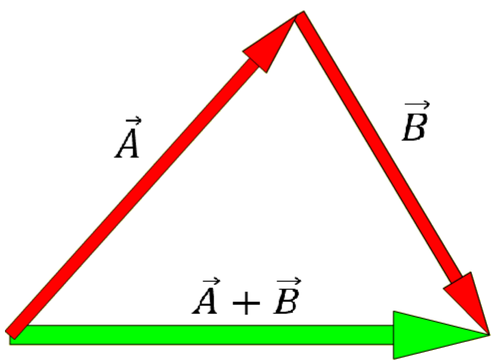
The tip-to-tail method of adding vectors together gives the resultant in the general case of any two vectors. The diagrams below show two vectors (red) and their resultant (green.) Note, the resultant can be either longer or shorter than the vectors being added together:

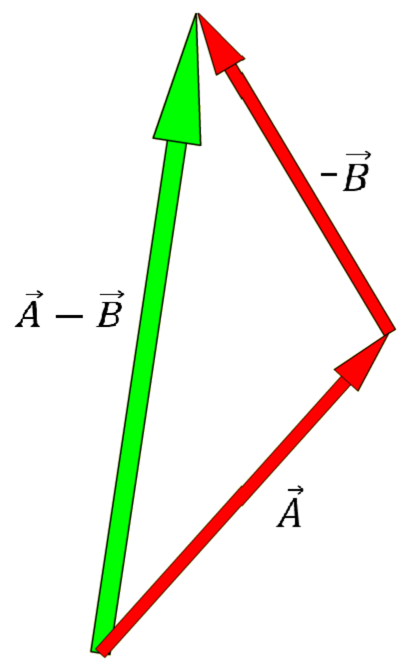


The Commutative Property still holds in the general case. Note that the vector sum $\vec{A} + \vec{B}$ in the diagram below has the same magnitude and same direction as the vector sum $\vec{B} + \vec{A}$.



Since the opposite of a vector is a vector of the same length but pointing in the opposite direction, when performing vector subtraction, add the opposite of the vector you are subtracting and follow the same rules. In the figures below, note that in the second figure the vector $-\vec{B}$ has the same magnitude but points in the opposite direction as the vector $\vec{B}$ in the first figure. Adding $\vec{A} + (-\vec{B})$ is the same as $\vec{A} - \vec{B}$.

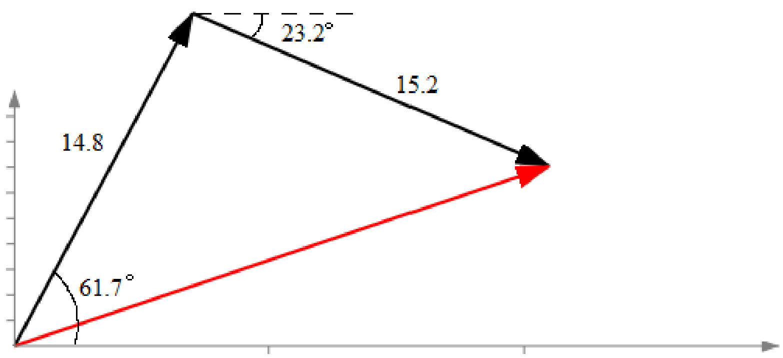




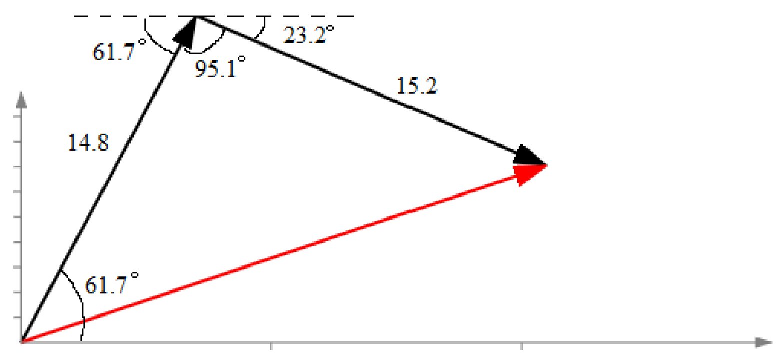
There are two methods by which the sum or difference of two vectors is found. One is by using the Law of Cosines and the Law of Sines to determine the magntiude and direction of the resultant.

Example 5:

Vector $\vec{A}$ is 14.8 units long directed at an angle of 61.7° above the +x-axis. Vector $\vec{B}$ is 15.2 units long directed at an angle of 23.2° below the +x-axis. Find $\vec{A} + \vec{B}$.
Draw the two vectors tip-to-tail and draw in the resultant:



Next, find the angle made by the tip of $\vec{A}$ and the tail of $\vec{B}$:



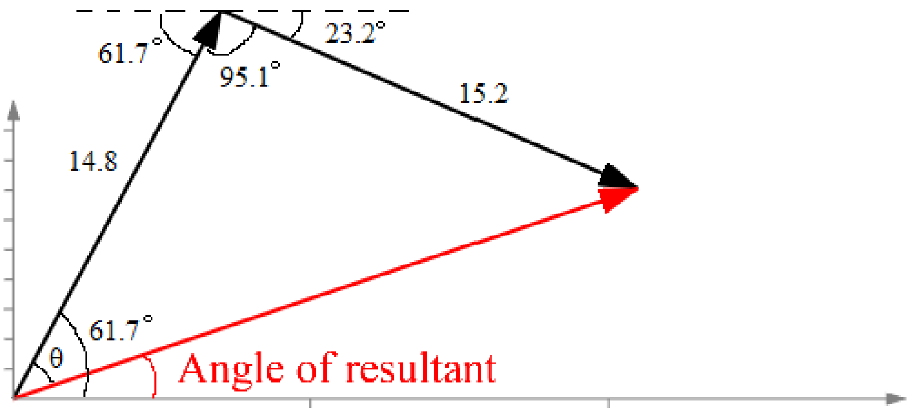
Note that the angle is found through the application of geometry. The dotted line is parallel to the x-axis, so the angle at the tip of vector $\vec{A}$ is congruent to the angle made at its tail, and the three angles together must add to 180°. So the included angle (which is the angle we need for the Law of Cosines) must be $180° - 61.7° - 23.2° = 95.1°$.

Now that we have two sides and the included angle, we apply the Law of Cosines to find the magnitude of the resultant:

$$R^2 = A^2 + B^2 - 2AB\cos\theta$$
$$R^2 = 14.8^2 + 15.2^2 - 2(14.8)(15.2)\cos 95.1°$$
$$R^2 = 219.04 + 231.04 - (-39.9953220462)$$
$$R^2 = 490.075322046$$
$$R = 22.1376449074$$

So the magnitude of the resultant is 22.1 units.

To find the angle the resultant makes with the horizontal, we will use the Law of Sines to find the angle the resultant makes with vector $\vec{A}$ and then subtract from $\vec{A}$'s angle to find $\vec{R}$'s angle:



$$\frac{\sin\theta}{15.2} = \frac{\sin 95.1°}{22.1376449074}$$

$$\frac{\sin\theta}{15.2} = 0.044993090709$$

$$\sin\theta = 0.683894978783$$

$$\theta = \sin^{-1} 0.683894978783 = 43.1487653438$$

$$\text{Angle of resultant} = 61.7° - 43.1487653438° = 18.5512346562°$$

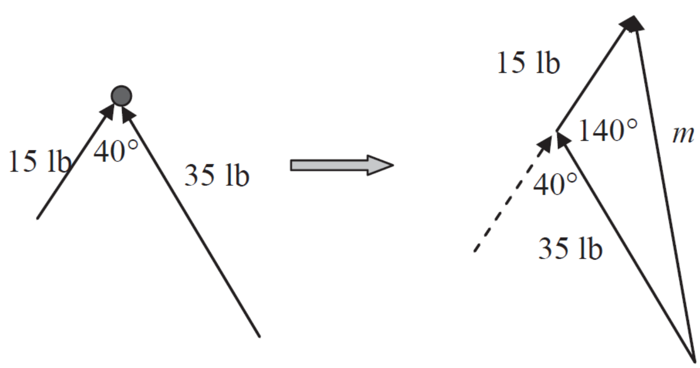
So the angle the resultant makes with the horizontal is 18.6°.

Answer: $\vec{R} = 22.1 \; units \; @ \; 18.6° \; above \; the + x \; axis$

Example 6:

Problem: Two forces with magnitudes of 15 pounds and 35 pounds and an angle of 40° between them are applied to an object. Find the magnitude of the resultant vector.

Sketch the problem. Remember this is a sketch. The actual angles may look very different.

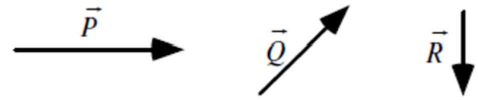


Law of Cosines
Substitute.
Simplify.

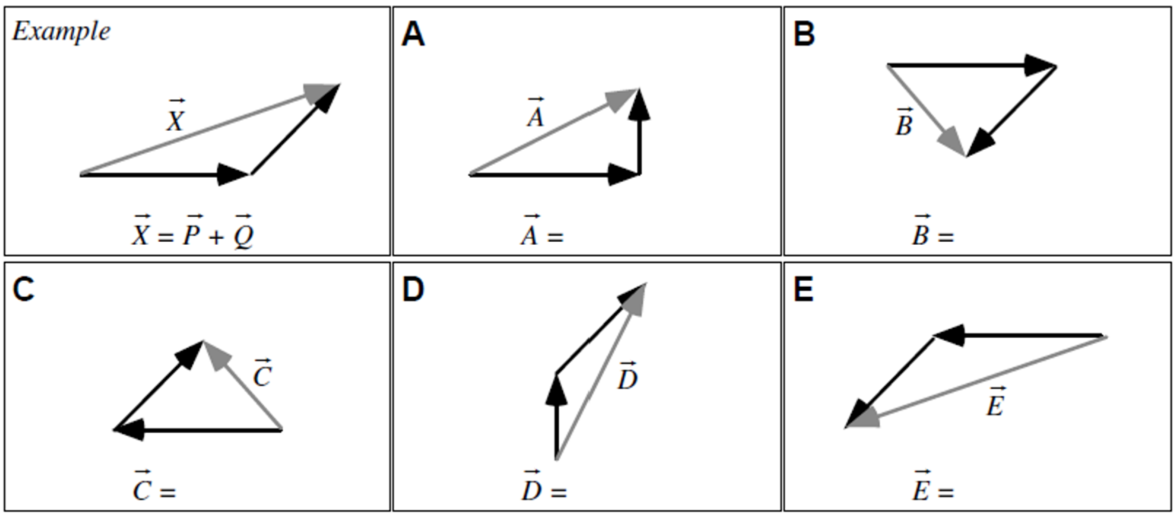
$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos\theta \\ m^2 &= (15)^2 + (35)^2 - 2(15)(35)\cos 140° \\ m^2 &= 225 + 1225 - 1050\cos 140° \\ \sqrt{m^2} &= \sqrt{225 + 1225 - 1050\cos 140°} \\ m &\approx 47 \text{ lb} \end{aligned}$$

PROBLEM SET 8

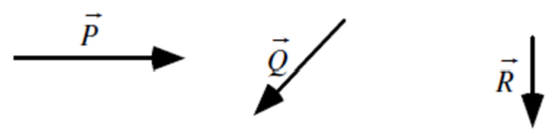
86. Three vectors are shown here:



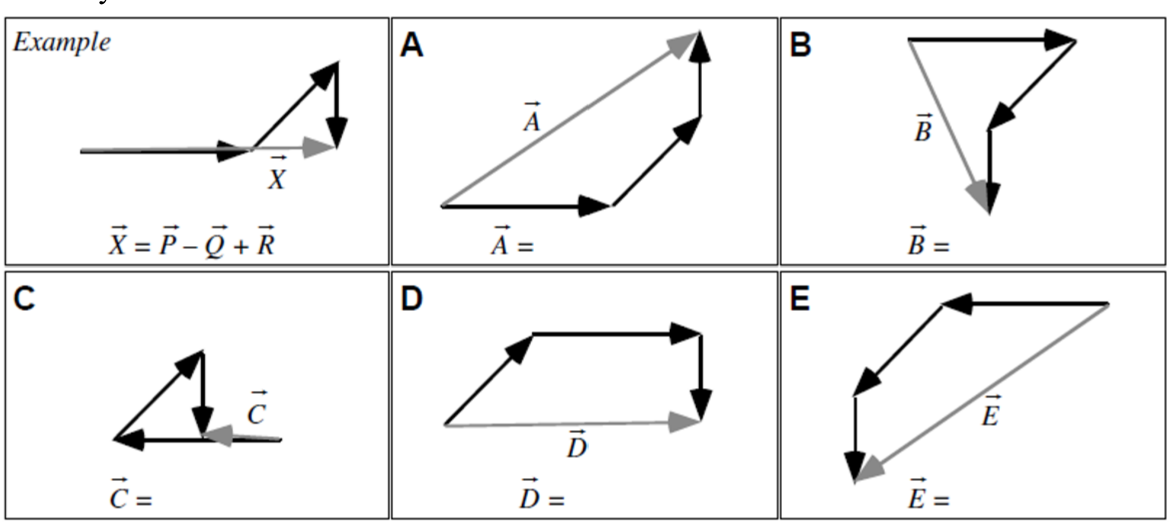
Write an expression for the following resultant vectors using the three vectors above. An example has been done for you.



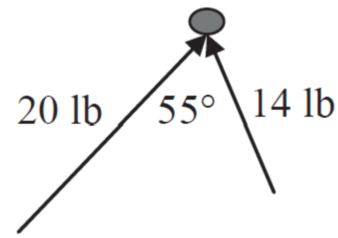
87. Three vectors are shown here:



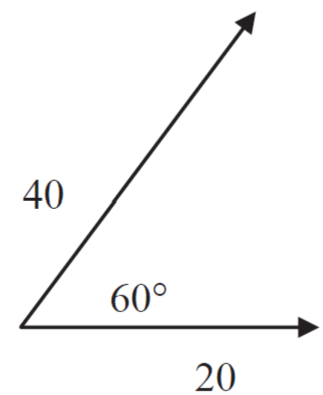
Write an expression for the following resultant vectors using the three vectors above. An example has been done for you.



88. In the diagram at right two forces are pushing on a small marble. The magntiudes of the two forces are given and the angle between the two forces has also been given. Determine the total force (magntiude and direction) acting on the marble by adding the vectors together. Remember, vectors are added tip-to-tail.

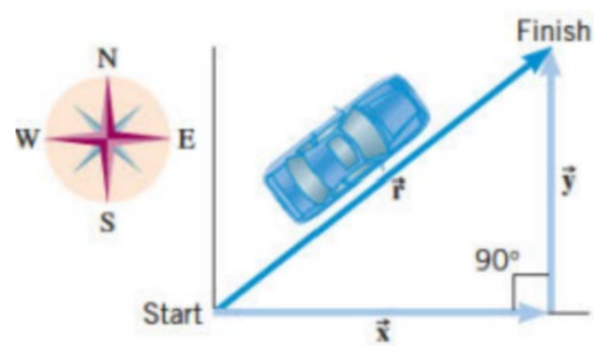


89. Find the magnitude and direction of the resultant when the two vectors shown are added together. Remember, vectors are added tip-to-tail.



The other method for finding the sum or difference of two vectors is by use of **components**.

Components of a Vector

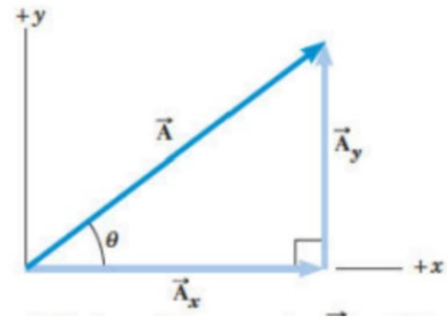


Suppose a car moves along a straight line from start to finish, as in the figure, the corresponding displacement vector being $\vec{r}$. The magnitude and direction of the vector $\vec{r}$ give the distance and direction traveled along the straight line. However, the car could also arrive at the finish point by first moving due east, turning through 90°, and then moving due north. This alternative path is shown in the drawing and is associated with the two displacement vectors $\vec{x}$ and $\vec{y}$. The vectors $\vec{x}$ and $\vec{y}$ are called the x-vector component and the y-vector component of $\vec{r}$. Vector components are very important in physics and have two basic features that are apparent in the figure. One is that the components add together to equal the original vector:

$$\vec{r} = \vec{x} + \vec{y}$$

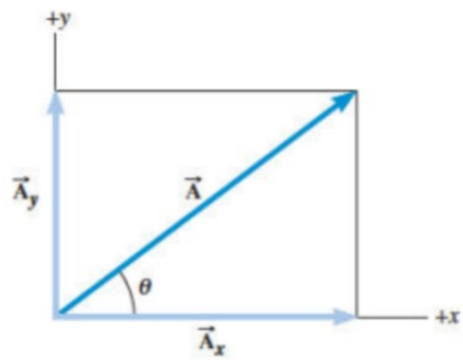
The components $\vec{x}$ and $\vec{y}$, when added vectorially, convey exactly the same meaning as does the original vector $\vec{r}$: they indicate how the finish point is displaced relative to the starting point. The other feature of vector components that is apparent in the figure is that $\vec{x}$ and $\vec{y}$ are not just any two vectors that add together to give the original vector $\vec{r}$: they are perpendicular vectors. This perpendicularity is a valuable characteristic, as we will soon see.

Any type of vector may be expressed in terms of its components, in a way similar to that illustrated for the displacement vector in the figure. This figure shows an arbitrary vector $\vec{A}$ and its vector components $\vec{A}_x$ and $\vec{A}_y$. The components are drawn parallel to convenient x and y axes and are perpendicular. They add vectorially to equal the original vector $\vec{A}$:



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

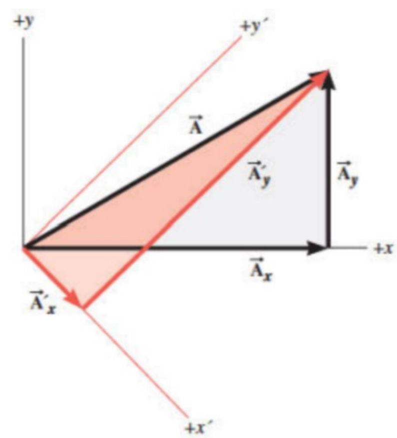
There are times when a drawing such as the previous figure is not the most convenient way to represent vector components, and this figure presents an alternative method. The disadvantage of this alternative is that the tail-to-head arrangement of $\vec{A}_x$ and $\vec{A}_y$ is missing, an arrangement that is a nice reminder that $\vec{A}_x$ and $\vec{A}_y$ add together to equal $\vec{A}$.



The definition that follows summarizes the meaning of vector components:

> **VECTOR COMPONENTS:** In two dimensions, the vector components of a vector $\vec{A}$ are two perpendicular vectors $\vec{A}_x$ and $\vec{A}_y$ that are parallel to the x and y axes, respectively, and add together vectorially according to $\vec{A} = \vec{A}_x + \vec{A}_y$.

In general, the components of any vector can be used in place of the vector itself in any calculation where it is convenient to do so. The values calculated for vector components depend on the orientation of the vector relative to the axes used as a reference. This figure illustrates this fact for a vector $\vec{A}$ by showing two sets of axes, one set being rotated clock wise relative to the other. With respect to the black axes, vector $\vec{A}$ has perpendicular vector components $\vec{A}_x$ and $\vec{A}_y$; with respect to the colored rotated axes, vector $\vec{A}$ has different vector components $\vec{A}'_x$ and $\vec{A}'_y$. The choice of which set of components to use is purely a matter of convenience.



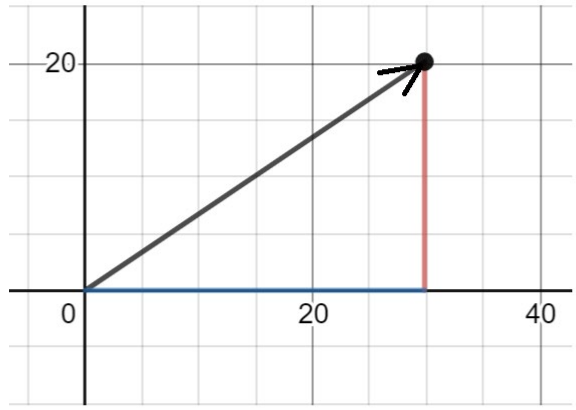
It is often easier to work with the scalar components, A_x and A_y rather than the vector components $\vec{A}_x$ and $\vec{A}_y$. Scalar components are positive or negative numbers (with units) that are defined as follows: The scalar component A_x has a magnitude equal to that of $\vec{A}_x$ and is given a positive sign if $\vec{A}_x$ points along the +x axis and a negative sign if $\vec{A}_x$

points along the -x axis. The scalar component A_y is defined in a similar manner. As an example, if the x-component of a vector is 8.0 units directed along the –x axis, the scalar component would be given as A_x = -8.0 units.

The scalar components correspond to the coordinates in the plane the vector points to when you place the tail of the vector at the origin. As examples:

The scalar components of the vector at right are

$$A_x = +30, A_y = +20$$

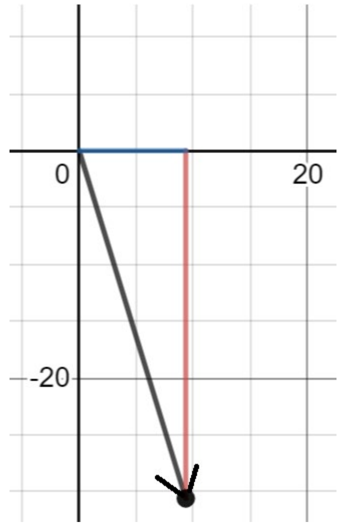


The scalar components of the vector at left are

$$B_x = -20, B_y = +30$$

The scalar components of the vector at right are

$$C_x = +10, C_y = -30$$



Resolving a Vector into Its Components

If the magnitude and direction of a vector are known, it is possible to find the components of the vector. The process of finding the components is called "resolving the vector into its components." As the next example illustrates, this process can be carried out with the aid of trigonometry, because the two perpendicular vector components and the original vector form a right triangle.

Example 6

The vector $\vec{r}$ at right is 175 units long and points at 50° above the +x axis. The components of the vector, $\vec{x}$ and $\vec{y}$, are shown. To determine their scalar values, note that the components form the legs of a right triangle with the vector's magnitude as the hypotenuse. Knowing the direction angle allows us to find the lengths of the sides:

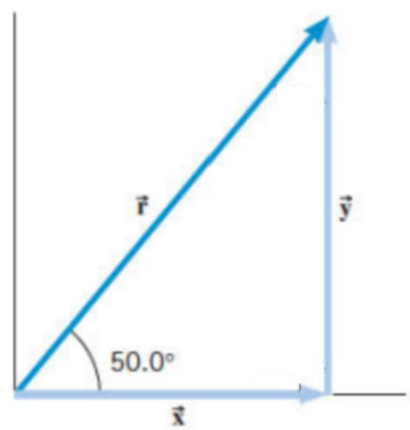
$$\sin 50.0° = \frac{y}{175}$$

$$y = 175 \sin 50.0° = 134.057777546$$

$$x = 175 \cos 50.0° = 112.487831695$$

Since the vector is in the 1st quadrant, both components are positive and we would write

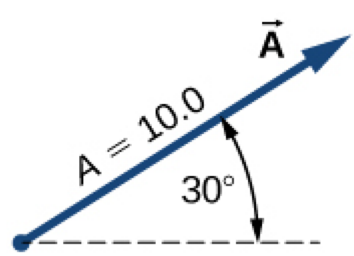
$$x = +112\ units, y = +134\ units$$



Example 7

Find the components of the vector.

The vector shown points into the 1st quadrant, so both components of the vector will be positive. As before, the vector's magnitude is the hypotenuse of a right triangle. Since the angle given is to the horizontal, the x-component is the adjacent side of the triangle and the y-component is the side opposite. We find the scalar components as before:

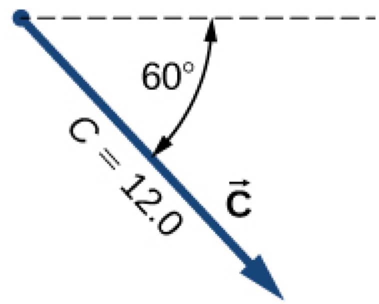


$$A_x = A\cos\theta = 10.0\cos 30° = 8.66025403784$$

$$A_y = A\sin\theta = 10.0\sin 30° = 5.00$$

So the components are $A_x = 8.66$ units and $A_y = 5.00$ units.

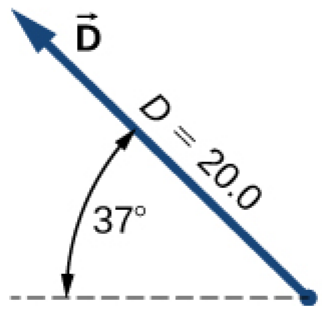
Examples 8-11
Find the components of each of the following vectors.



Vector $\vec{C}$ points into the 4th quadrant, so it will have a positive x-component and a negative y-component. Since it makes a 60° angle with the horizontal, the x-compnent once again is the adjacent side and the y-component is the opposite side:

$$C_x = C\cos\theta = 12.0\cos 60° = 6.00$$
$$C_y = -C\sin\theta = -12\sin 60° = -10.3923048$$

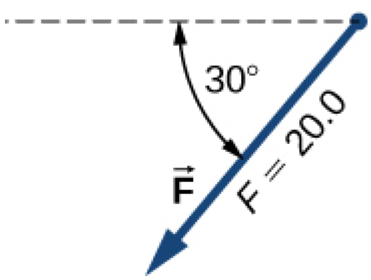
The components are C_x = 6.00 units, C_y = -10.4 units.



Vector $\vec{D}$ points into the 2nd quadrant, so it will have a negative x-component and a positive y-component. Note the angle is still given to the horizontal:

$$D_x = -20.0\cos 37° = -15.972710201$$
$$D_y = 20.0\sin 37° = 12.036300463$$

The components are D_x = -16.0 units, D_y = 12.0 units.



Vector $\vec{F}$ points into the 3rd quadrant, so both components will be negative. Yet again, the angle is to the horizontal:

$$F_x = -20.0\cos 30° = -17.32050807$$
$$F_y = -20.0\sin 30° = -10.0$$

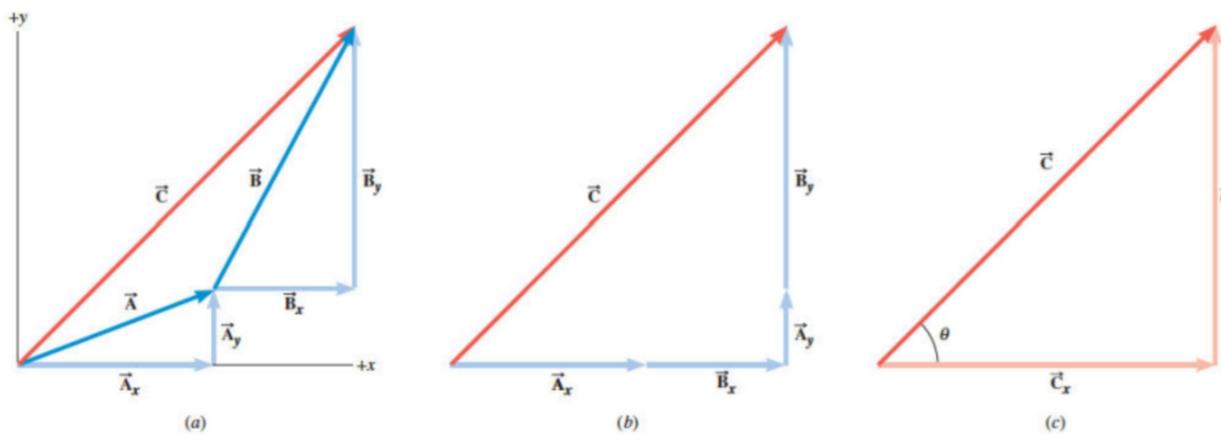
The components are F_x = -17.3 units, F_y = -10.0 units.

It is possible for one of the components of a vector to be zero. This does not mean that the vector itself is zero, however. For a vector to be zero, *every* vector component must individually be zero. Thus, in two dimensions, saying that $\vec{A} = 0$ would be the equivalent of saying that $\vec{A}_x = 0$ and $\vec{A}_y = 0$. Or, stated in terms of scalar components, if $A = 0$, then $A_x = 0$ and $A_y = 0$.

Two vectors are equal if, and only if, they have the same magnitude and direction. Thus, if one displacement vector points east and another points north, they are not equal, even if each has the same magnitude of 480 units. In terms of vector components, two vectors $\vec{A}$ and $\vec{B}$ are equal if, and only if, each vector component of one is equal to the corresponding vector component of the other. In two dimensions, if $\vec{A} = \vec{B}$, then $\vec{A}_x = \vec{B}_x$ and $\vec{A}_y = \vec{B}_y$. Alternatively, using scalar components, we write that $A_x = B_x$ and $A_y = B_y$.

Addition of Vectors Using Components

The components of a vector provide the most convenient and accurate way of adding (or subtracting) any number of vectors. For example, suppose that vector $\vec{A}$ is added to vector $\vec{B}$. The resultant vector is $\vec{C}$. The figure below illustrates this vector addition. Part (a) shows the sum $\vec{C} = \vec{A} + \vec{B}$ along with the x- and y- vector components of $\vec{A}$ and $\vec{B}$.



In part (b) of the drawing, the vectors $\vec{A}$ and $\vec{B}$ have been removed, because we can use the vector components of these vectors in place of them. The vector component $\vec{B}_x$ has been shifted downward and arranged tail to head with vector component $\vec{A}_x$. Similarly, the vector component $\vec{A}_y$ has been shifted to the right and arranged tail to head with the vector component $\vec{B}_y$. The x components are colinear and add together to give the x component of the resultant vector $\vec{C}$. In like fashion, the y components are colinear and add together to give the y component of $\vec{C}$. In terms of scalar components, we can write $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

The vector components $\vec{C}_x$ and $\vec{C}_y$ of the resultant vector form the sides of the right triangle shown in (c) in the above figure. Thus, we can find the magnitude of $\vec{C}$ by using the Pythagorean theorem:

$$C = \sqrt{C_x^2 + C_y^2}$$

The angle θ that C makes with the x axis is given by $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$.

Let's revisit Example 5.

Example 5 (Revisited):
Vector $\vec{A}$ is 14.8 units long directed at an angle of 61.7° above the +x-axis. Vector $\vec{B}$ is 15.2 units long directed at an angle of 23.2° below the +x-axis. Find $\vec{A} + \vec{B}$.
The easiest way to add vectors together by components is to set up a chart where each vector is a row and each component is a column:

	x-component	y-component
$\vec{A}$		
$\vec{B}$		
$\vec{A} + \vec{B}$		

Since the angles for the two vectors are referenced from the x-axis, the x-components are found using the cosine of the angles and the y-components from the sines of the angles. Vector $\vec{A}$ points into the 1st quadrant, so both of its components are positive. Vector $\vec{B}$ points into the 4th quadrant, so it has a positive x-component and a negative y-component:

	x-component	y-component
$\vec{A}$	$14.8\cos(61.7) = 7.0165054$	$14.8\sin(61.7) = 13.03106$
$\vec{B}$	$15.2\cos(23.2) = 13.970857$	$-15.2\cos(23.2) = -5.987917$
$\vec{R} = \vec{A} + \vec{B}$	**20.98736265**	**7.043147806**

Note that the components of the resultant are found by adding the x-components of the two vectors together and the y-components of the two vectors together. The components of the resultant are both positive, so the sum points into the 1st quadrant. To find its magnitude we use the Pythagorean Theorem:

$$R = \sqrt{20.98736265^2 + 7.043147806^2} = 22.1376449074$$

And the angle is found by using the inverse tangent function:

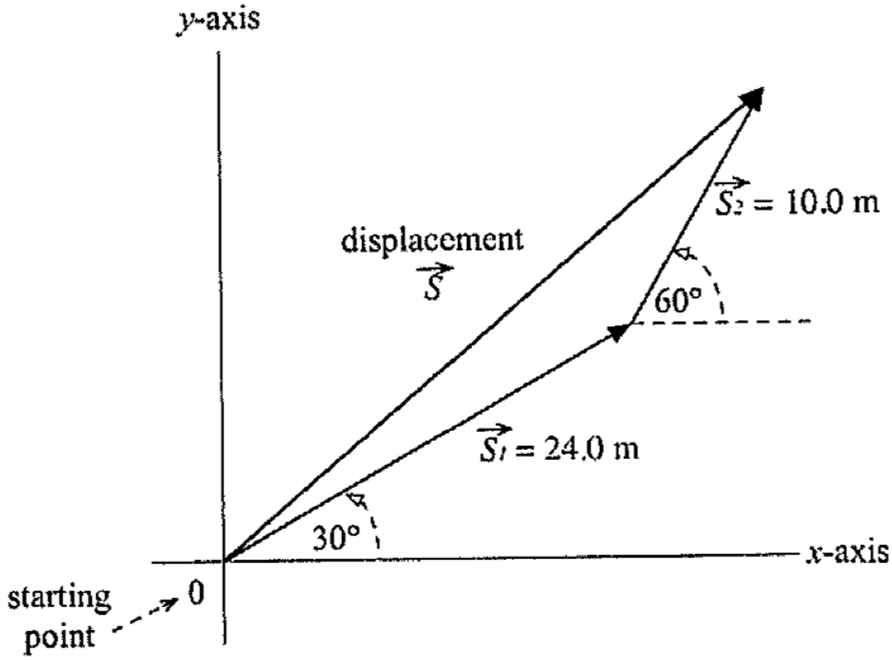
$$\theta = \tan^{-1}\left(\frac{7.043147806}{20.98736265}\right) = 18.55123465$$

So the resultant is

$$\vec{R} = 22.1\ units\ @\ 18.6°\ above\ the + x\ axis$$

Just as before.

Example 7:
An object moves from its starting point a straight line distance of 24.0 m in a direction of 30° above the positive x-axis. It then turns through an angle of 60° above the +x –axis and moves another 10.0 m in a straight line. Determine the total displacement (magnitude and direction) of the object from its starting point.



In the diagram above the two movements of the object are shown, along with the vector that represents its total displacement. Note that the displacment is the resultant of the two movements.

	x-component	y-component
$\vec{S}_1$	$24.0\cos 30 = 20.7846096$	$24.0\sin 30 = 12.0$
$\vec{S}_2$	$10.0\cos 60 = 5.0$	$10.0\sin 60 = 8.66025404$
$\vec{S}_1 + \vec{S}_2$	25.7846096	20.66025404

$$Displacement = \sqrt{25.7846096^2 + 20.66025404^2} = 33.04076564$$

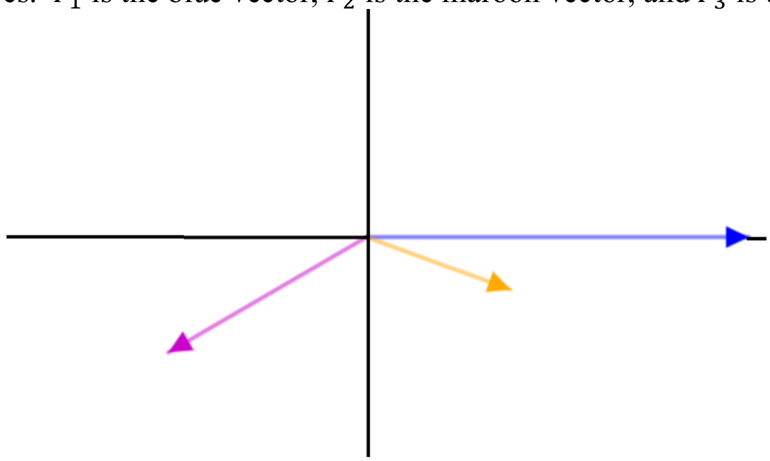
$$\theta = \tan^{-1}\left(\frac{20.66025404}{25.7846096}\right) = 38.70390625$$

The displacement is 33.0 m @ 38.7° above the +x axis.

Example 8:

Three concurrent forces act on a small box. The three forces are $\vec{F}_1 = 200\ N\ along + x\ axis$, $\vec{F}_2 = 120\ N\ @\ 30°\ below - x\ axis$, and $\vec{F}_3 = 80\ N\ @\ 70°\ right\ of - y\ axis$. Find the resultant of the three forces.

Here are the three forces. $\vec{F}_1$ is the blue vector, $\vec{F}_2$ is the maroon vector, and $\vec{F}_3$ is the gold vector:



Note that $\vec{F}_2$ points into quadrant III and $\vec{F}_3$ points into quadrant IV. Note also that the angle given for $\vec{F}_3$ is given to the vertical axis, not the horizontal. This means that the component opposite the angle lies along the x-direction and the component adjacent to the angle lies along the y-axis. When you determine the components of these vectors, pay attention to the signs on the components:

	x-component	y-component
$\vec{F}_1$	$200 \cos 0 = 200$	$200 \sin 0 = 0$
$\vec{F}_2$	$-120 \cos 30 = -103.92304$	$-120 \sin 30 = -60$
$\vec{F}_3$	$80 \sin 70 = 75.1754097$	$-80 \cos 70 = -27.3616114$
Resultant	171.252361209	-87.361611466

The x component of the resultant turns out to be positive and the y-component turns out to be negative. This means the resultant is in quadrant IV. When we find the angle the vector makes with the horizontal we will ignore the signs on the components since we know which quadrant the vector will be in:

$$Resultant = \sqrt{171.252361209^2 + (-87.361611466)^2} = 192.248335175$$

$$\theta = \tan^{-1}\left(\frac{87.361611466}{171.252361209}\right) = 27.0276633681$$

The answer is 192 N @ 27.0° below the +x-axis.

PROBLEM SET 9

Find the x- and y-components of the following vectors.

90) $\vec{A}$ = 145 N @ 75° above the +x axis

91) $\vec{B}$ = 0.988 m @ 15.4° above the –x axis

92) $\vec{C}$ = 38.44 m/s @ 76.87° below the –x axis

93) $\vec{D}$ = 1.087 N @ 81.21° left of the –y axis

94) $\vec{E}$ = 2000. m along the +y axis

95) $\vec{F}$ = 6.77 m/s^2 @ 25° below the +x axis

96) $\vec{G}$ = 50.114 N along the –x axis

97) $\vec{H}$ = 414.56 m/s @ 27.86° right of the +y axis

98) $\vec{I}$ = 0.36221 cm @ 14.58° left of the +y axis

99) $\vec{J}$ = 948 m/s^2 @ 20.7° right of the –y axis

You are given the x- and y-components of a vector. Find the magnitude (size) of the original vector and its direction measured from the x-axis.
<u>DRAW A SKETCH OF THE VECTOR TO HELP YOU DETERMINE THE CORRECT ANGLE.</u>

100) K_x = –3.567 m, K_y = 11.883 m

101) L_x = 250 N, L_y = –250 N

102) M_x = 0.000 m/s^2, M_y = –15.89 m/s^2

103) N_x = –900.0 N, N_y = –1345 N

104) P_x = 88.88 m, P_y = 0.000 m

105) Q_x = –12.5 N, Q_y = –7.02 N

106) R_x = 4.48 m/s, R_y = 13.12 m/s

107) S_x = –108.1 m/s, S_y = 21.9 m/s

108) T_x = 9.224 N, T_y = –15.249 N

109) U_x = 477 m, U_y = 37.9 m

Here are ten vectors:

$\vec{A}$ = 1.45 N @ 75º above the +x axis

$\vec{B}$ = 0.988 m @ 15.4º above the –x axis

$\vec{C}$ = 38.44 m/s^2 @ 76.87º below the –x axis

$\vec{D}$ = 1.087 N @ 81.21º left of the –y axis

$\vec{E}$ = 2.00 m along the +y axis

$\vec{F}$ = 6.77 m/s^2 @ 25º below the +x axis

$\vec{G}$ = 5.14 N along the –x axis

$\vec{H}$ = 414.56 m/s @ 27.86º right of the +y axis

$\vec{I}$ = 362.21 m/s @ 14.58º left of the +y axis

$\vec{J}$ = 94.8 m/s^2 @ 20.7º right of the –y axis

Use the component method to find the following:

110. $\vec{A} + \vec{D}$
111. $\vec{B} - \vec{E}$
112. $\vec{C} + \vec{F}$
113. $\vec{F} - \vec{J}$
114. $\vec{A} - \vec{D} + \vec{G}$
115. $2\vec{H} + 3\vec{I}$
116. $\vec{E} - \vec{B}$

Place your answers to the Problem Sets in the blanks provided below:

PROBLEM SET 1

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

PROBLEM SET 2

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. $\sin L =$ ____________ $\cos L =$ ____________ $\tan L =$ ____________

15. $\sin a =$ ____________ $\cos a =$ ____________ $\tan a =$ ____________

16. ______________________

17. ______________________

18. Give your explanation in the box below:

PROBLEM SET 3

19. ______________________

20. ______________________

21. ______________________

22. ______________________

23. ______________________

24. ______________________

25. ______________________

26. ______________________

27.

   a. ______________________

   b. ______________________

c. Explain in the text box below:

28.

a. ___________________________

b. The sine of 20° must be equal to the cosine of ______° because the two angles are___________________.

c. $\sin\theta =$ __________

d. Show your work in the text box below:

29. Show your work in the text box below:

30.

a. Show your sketch in the box below.

b. ________________________

c. ________________________

d. Show your work in the box below.

31.

a. Show your sketch in the box below.

b. ______________________

c. ______________________

32. Show your work in the text box below:

33.

a. ______________________

b. ______________________

34.

a. ______________________

b. ______________________

35. ______________________

36.

a. Show your sketch in the box below.

b. ______________________

c. ______________________

37. $w =$ ______________, $x =$ ______________, $y =$ ______________, $z =$ ____________

PROBLEM SET 4

38. ______________________

39. ______________________

40. ______________________

41. ______________________

42. ______________________

43. ______________________

44. ______________________

45. Show the correct work in the text box below:

46.

a. ___________________

b. ___________________

c. Give your explanation in the text box below:

47. $x =$ ______________, $y =$ ______________, $z =$ ______________

48. Who, if anyone, did it correctly? ____________________

49.

a. Show your sketch in the box below.

b. ________________________

c. ________________________

d. Show your work in the text box below:

50.

a. Show your sketch in the box below.

b. ____________________________

c. ____________________________

51.

a. ____________________________

b. ____________________________

52.

a. Show your sketch in the box below.

b. ____________________________

c. ____________________________

d. ____________________________

53. Angle 1: ______________________ Angle 2: _________________________

54.

a. Show your sketch in the box below.

b. ________________________

c. ________________________

55. $w =$ ______________, $x =$ ______________, $y =$ ______________, $z =$ ____________

56.

a. ________________________

b. ________________________

c. ________________________

d. Give your explanation in the text box below:

e. $\tan^{-1}(\frac{a}{b}) +$ ______________ $=$ ________________

f. Show your work in the text box below:

57.

a. ________________

b. ________________

c. ________________

PROBLEM SET 5

58. State and show your equation with substituted values in the text box below:

59. State and show your equation with substituted values in the text box below:

60. State and show your equation with substituted values in the text box below:

61. State and show your equation with substituted values in the text box below:

62. State and show your equation with substituted values in the text box below:

63. State and show your equation with substituted values in the text box below:

64. State and show your equation with substituted values in the text box below:

65. State and show your equation with substituted values in the text box below:

66. State and show your equation with substituted values in the text box below:

67. State and show your equation with substituted values in the text box below:

PROBLEM SET 6

68. $DE =$ ______________________, $m\angle F =$ ______________________

69. $m\angle C =$ ____________________

70. $AC =$ ______________________, $m\angle A$____________________, $m\angle C =$ ____________________

71. $f =$ ____________________

72. $x =$ ____________________

73. $A =$ ____________________

74. $A =$ ____________________

75. $m\angle DGF =$ ________________________

76. $m\angle ABD =$ ________________________

77. $A =$ ___________________, $B =$ ________________________, $C =$ ___________________

PROBLEM SET 7

78. ___________________________________

79. ___________________________________

80. ___________________________________

81. ___________________________________

82. ___________________________________

83. ___________________________________

84. ___________________________________

85. ___________________________________

PROBLEM SET 8

86. $\vec{A} =$ ___________________ $\vec{B} =$ ___________________ $\vec{C} =$ ___________________

$\vec{D} =$ ___________________ $\vec{E} =$ ___________________

87. $\vec{A} =$ ___________________ $\vec{B} =$ ___________________ $\vec{C} =$ ___________________

$\vec{D} =$ ___________________ $\vec{E} =$ ___________________

88. Magnitude= ___________________ Direction = _________________________

89. Magnitude= ___________________ Direction = _________________________

PROBLEM SET 9

90. x-component= _________________________, y-component = ___________________________

91. x-component= _________________________, y-component = ___________________________

92. x-component= _________________________, y-component = ___________________________

93. x-component= _________________________, y-component = ___________________________

94. x-component= _________________________, y-component = ___________________________

95. x-component= _________________________, y-component = ___________________________

96. x-component= _________________________, y-component = ___________________________

97. x-component= _________________________, y-component = ___________________________

98. x-component= _________________________, y-component = ___________________________

99. x-component= _________________________, y-component = ___________________________

100. Magnitude= ___________________ Direction = _________________________

101. Magnitude= ___________________ Direction = _________________________

102. Magnitude= ___________________ Direction = _________________________

103. Magnitude= ___________________ Direction = _________________________

104. Magnitude= ___________________ Direction = _________________________

105. Magnitude= ___________________ Direction = _________________________

106. Magnitude= ___________________ Direction = _________________________

107. Magnitude= ___________________ Direction = _________________________

108. Magnitude= ___________________ Direction = _________________________

109. Magnitude= ___________________ Direction = _________________________

110. Magnitude= ___________________ Direction = _________________________

111. Magnitude= ___________________ Direction = _________________________

112. Magnitude= ___________________ Direction = _________________________

113. Magnitude= ___________________ Direction = _________________________

114. Magnitude= ___________________ Direction = _________________________

115. Magnitude= ___________________ Direction = _________________________

116. Magnitude= ___________________ Direction = _________________________