Summer Work AP Pre-Calculus

These are skills that you need prior to the beginning of school. We will have a quiz over these the first week of school.

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$	2. $f(x+2) =$	3. $f(x+h) =$

Write the equation of the line meets the following conditions. Use point-slope form.
$\boldsymbol{y - y_1 = m(x - x_1)}$

5. slope $= 3$ and $(4, -2)$	6. $m = -\frac{3}{2}$ and $f(-5) = 7$	7. $f(4) = -8$ and $f(-3) = 12$

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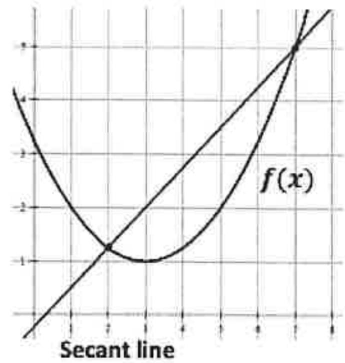
MULTIPLE CHOICE! Remember slope = $\frac{y_2-y_1}{x_2-x_1}$

10. Which choice represents the slope of the secant line shown?

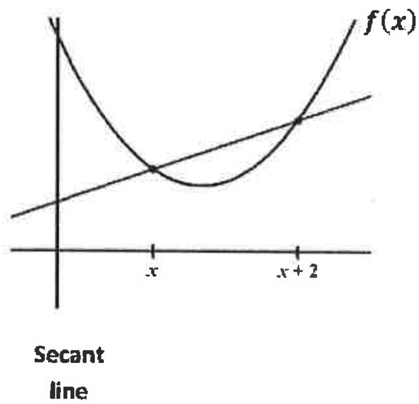
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

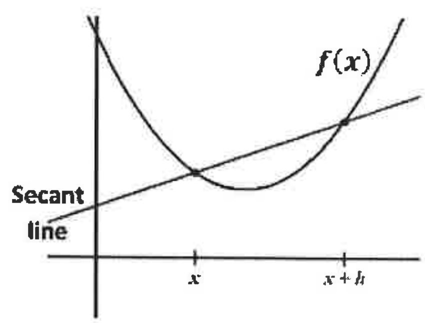
D) $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

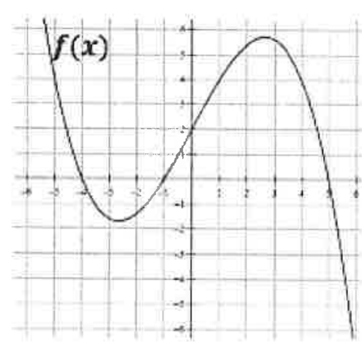
D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?

I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



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Using the equation identify the following: $f(x) = \dfrac{x^2 + x - 12}{x^2 - 16}$

1. Vertical asymptote(s)
2. Horizontal asymptote(s)
3. Hole(s)
4. x-intercept(s)
5. y-intercept(s)

Using the equation identify the following: $f(x) = \dfrac{x^2 - 1}{-4x - 12}$

1. Vertical asymptote
2. Horizontal asymptote
3. Hole(s)
4. x-intercept(s)
5. y-intercept(s)

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$		
19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

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Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$		
25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.		
31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Summer Work AP Pre-Calculus

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.		
46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

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Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$
72. $\ln 1$	73. $\ln e^7$	74. $\log_3 \frac{1}{3}$
75. $\log_{1/2} 8$	76. $\ln \frac{1}{2}$	77. $27^{\frac{2}{3}}$
78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$	79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$	80. $\left(4a^{5/3}\right)^{3/2}$

AP Pre-Calculus Review of Skills

Part 1 – Lines and Coordinate Geometry

Algebra Concepts

Slope-intercept form of a line $y = mx + b$

Standard form of a line $Ax + By = C$

Point-slope form of a line $y - y_1 = m(x - x_1)$

Slope of a line $m = \frac{y_2 - y_1}{x_2 - x_1}$

Part 2 – Exponents & Roots

Properties of Exponents

$a^m \cdot a^n = a^{m+n}$	**Ex:** $x^5 \cdot x^2 = x^7$	$\frac{a^m}{a^n} = a^{m-n}$	**Ex:** $\frac{x^8}{x^5} = x^3$	$a^0 = 1 \quad a \neq 0$	
$(a^m)^n = a^{m \cdot n}$	**Ex:** $(x^5)^2 = x^{10}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	**Ex:** $\left(\frac{2}{x}\right)^3 = \frac{8}{x^3}$	$a^{-n} = \frac{1}{a^n}$	**Ex:** $\frac{x^{-2}}{1} = \frac{1}{x^2}$
$(ab)^m = a^m b^m$	**Ex:** $(4xy^2)^3 = 64x^3y^6$			$\frac{1}{a^{-n}} = a^n$	**Ex:** $\frac{1}{x^{-2}} = x^2$

Properties of Radicals

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$\sqrt{a} \cdot \sqrt{a} = \left(\sqrt{a}\right)^2 = a$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Rationalizing the denominator: $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

Common Errors

$(a + b)^2 \neq a^2 + b^2$

$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$

$\sqrt{a^2 + b^2} \neq a + b$

AP Pre-Calculus Review of Skills

Part 3 – Factoring & Solving Quadratic Equations

Factoring Methods

GCF
Guess and check
Grouping
AC Method
Box Method
Difference of two squares
Sum/difference of cubes

FACTORING SPECIAL PRODUCTS

Difference of Two Squares Patterns

$a^2 - b^2 = (a + b)(a - b)$ **Example:** $9x^2 - 25 = (3x + 5)(3x - 5)$

Perfect Square Trinomial Pattern

$a^2 + 2ab + b^2 = (a + b)^2$ **Example:** $x^2 + 14x + 49 = (x + 7)^2$

$a^2 - 2ab + b^2 = (a - b)^2$ **Example:** $x^2 - 12x + 36 = (x - 6)^2$

FACTORING MORE SPECIAL PRODUCTS

Sum of Two Cubes Pattern

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ **Example:** $(x^3 + 1) = (x + 1)(x^2 - x + 1)$

Difference of Two Cubes Pattern

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ **Example:** $(x^3 - 8) = (x - 2)(x^2 + 2x + 4)$

Solving Quadratic Equations

By Finding Square Roots

EX: Solve $x^2 - 8 = 0$

$$x^2 = 8$$

$$x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

$$x = 2\sqrt{2} \text{ and } x = -2\sqrt{2}$$

Factoring

EX: Solve $3x^2 - 5x - 12 = 0$

Factor: $(3x + 4)(x - 3) = 0$

Zero
Product
Property: $3x + 4 = 0$ and $x - 3 = 0$

$$x = -\frac{4}{3}, 3$$

Quadratic Formula

Words The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EX: Solve $x^2 + 5x + 6 = 0$.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-5 \pm 1}{2}$$

$$x = \frac{-5+1}{2} = \frac{-4}{2} = -2 \text{ and } x = \frac{-5-1}{2} = \frac{-6}{2} = -3$$

$$x = -2, -3$$

By Graphing

EX: Solve $x^2 + x - 6 = 0$

Graph $y = x^2 + x - 6$

The x-coordinate of the vertex is $-\frac{b}{2a} = -\frac{1}{2}$, and the axis of symmetry is $x = -\frac{1}{2}$.

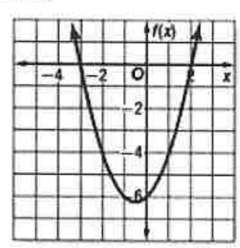
Make table of values using x values around $-\frac{1}{2}$

x	-1	$-\frac{1}{2}$	0	1	2
$f(x)$	-6	$-6\frac{1}{4}$	-6	-4	0

Plot the points:

From the table and the graph
We can see the zeros of the function
Are 2 and -3.

$$x = 2, -3$$



AP Pre-Calculus Review of Skills

Combining Functions & Compositions of Functions

Let $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x}$

Combining Functions

$f(x) + g(x)$ Ex: $(f+g)x = \frac{1}{x-2} + \sqrt{x}$

$f(x) - g(x)$ Ex: $(f-g)x = \frac{1}{x-2} - \sqrt{x}$

Given two functions f and g, the **composite function**, $f \circ g$, (also called the **composition of f and g**) is defined by

$$(f \circ g)(x) = f(g(x))$$

Part 7- Right Triangles & Trigonometry

For the right triangle pictured:

SOHCAHTOA

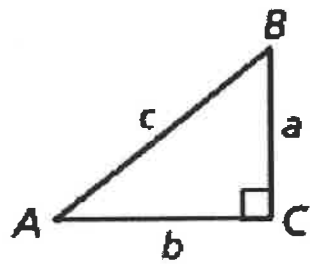
Pythagorean Theorem

$a^2 + b^2 = c^2$

$\sin A = \frac{opp}{hyp} = \frac{a}{c}$ $\sin^{-1}\left(\frac{a}{c}\right) = \angle A$

$\cos A = \frac{adj}{hyp} = \frac{b}{c}$ $\cos^{-1}\left(\frac{b}{c}\right) = \angle A$

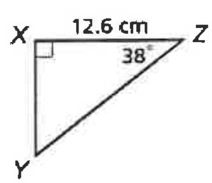
$\tan A = \frac{opp}{adj} = \frac{a}{b}$ $\tan^{-1}\left(\frac{a}{b}\right) = \angle A$



Using Trigonometric Equations

Example: Find the length of $\overline{YZ}$.

YZ is the hypotenuse. You are given XZ, which is adjacent to the given angle, $\angle Z$. Since the adjacent side and hypotenuse are involved, use a cosine ratio.



$\cos Z = \frac{\text{adj. leg}}{\text{hyp.}} = \frac{XZ}{YZ}$

$\cos 38° = \frac{12.6}{YZ}$

$YZ = \frac{12.6}{\cos 38°}$

$YZ \approx 15.99$ cm