Pascack Valley Regional High School District

Pascack Hills High School, Montvale, New Jersey 07645
Pascack Valley High School, Hillsdale, New Jersey 07642

Course Name: Advanced Placement Calculus BC

Born On: July, 2011
Previous Revision: August, 2020
Current Revision: August, 2023
Board Approval: 8/28/23

Course Description:

BC Calculus is a full year course that is based on the standards and requirements of the College Board's *Advanced Placement Calculus BC* curriculum as published on the AP website. The primary objective of this course is to prepare students for the BC Calculus advanced placement test administered in May. Concurrent with this objective is fostering in students a deep conceptual understanding of calculus as taught to them from an analytical, numerical and graphical point of view. The philosophy of the course is to offer students instruction strictly at the honors level while following curriculum guidelines so that students are thoroughly prepared for the AP test. Students who successfully complete the course and achieve grades on the AP test of high qualification, may receive up to two semesters of college credit for mathematics.

All mathematics courses in the Pascack Valley Regional High School District are designed to address multiple learning styles and needs, and accommodations and modifications are made for students with disabilities, multilingual students, students at risk of failure, gifted and talented students, and students with 504 plans. *AP Calculus BC* builds on concepts learned and skills developed in *Honors Precalculus*, while also spiraling in those concepts and skills to reinforce and strengthen students' algebraic foundation. Additionally, *AP Calculus BC* anticipates higher-level mathematics that will be learned in college-level math and applied math courses, and enrichment opportunities are provided to challenge students and engage them in rich, interesting mathematics. Students are encouraged to analyze data using tools and models to make valid and reliable claims (9.4.12.IML.3), and various technologies are integrated throughout the curriculum, including scientific calculators, graphing calculators, specialized software, and various Internet programs and subscriptions. These tools enrich the curriculum by giving students' access to additional mathematical representations, and they also help to differentiate by providing students with additional options to engage with mathematical tasks.

The Pascack Valley Regional High School Mathematics Department integrates 21st century life and career skills across its courses, with the dual goal of informing students about careers and fields of study that use mathematics (9.3.ST.5, 9.3.ST-ET.5 and 9.3.ST-SM.2), and helping students improve the quantitative, mathematical, and statistical reasoning skills they will need to be effective producers and consumers of quantitative information in their everyday lives (9.2.12.CAP.2). Mathematics courses address the *New Jersey Student Learning Standards for Career Readiness, Life Literacies and Key Skills*, with a particular emphasis on demonstrating the ability to reflect, analyze and use creative skills and ideas (9.4.12.CI.1), investigating new challenges and opportunities for personal growth, advancement and transition (9.4.12.CI.3), identifying problem-solving strategies used in the development of an innovative product or practice (9.4.12.CT.1), and explaining the potential benefits of collaborating to enhance critical thinking and problem solving (9.4.12.CT.2). Mathematics courses also address the *New Jersey Student Learning Standards for English Language Arts Companion Standards*, with a particular focus on following complex multistep procedures (RST.9-10.3/RST.11-12.3), determining the meaning of symbols, key terms, and other domain-specific words and phrases (RST.9-10.4/RST.11-12.3), and translating quantitative or technical information expressed in words into visual forms and translating information expressed visually or mathematically into words (RST.9-10.7). Similarly, the mathematics department seeks to support students by providing them with opportunities to use quantitative, statistical, and mathematical reasoning in interdisciplinary contexts, in contexts that are meaningful to students, and in contexts that

attend to the contributions and perspectives of historically marginalized groups. Specifically, mathematics courses will look to incorporate, when appropriate, contributions and experiences of people from the LGBTQ+ community and individuals with disabilities, and references to issues of social and cultural relevance, including climate change.

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Content/Topic:	Key Learning Items/Concepts and Pacing Guide	Observable Proficiencies and Skills:	NJSLS	Formative, Summative, Benchmark, and Alternative Assessments	Core Instructional and Supplemental Materials/ Modifications and Accommodations
Unit 1 – Derivatives **Time:** 10-12 weeks *(see column 2 for a more detailed breakdown)*	**Key learning items/concepts:** Chapter 1: Prerequisites (3 days) • Parametric Equations Chapter 3: Derivatives (12 days) • Chain Rule in Leibniz Notation and Function Notation (4 days) • Implicit differentiation (4 days) • Derivatives of trigonometric, inverse trigonometric, exponential and logarithmic functions (4 days) Chapter 4: Applications of Derivatives (25 days) • Finding Extrema on a Closed Interval (3 days) • Mean Value Theorem for Derivatives (3 days) • Using the first and second derivative to find: Critical point(s) and extreme values - When the function is increasing or decreasing - Point(s) of inflection	- *convert between parametric and Cartesian functions* - *graph parametric functions using different restriction of the parameter* - *identify the graphs of common parametric functions (i.e. x = cos(t), y = sin(t)).* - *use the definition of the derivative along with differentiation rules to compute the first and second derivatives of a function* - *use derivatives to determine whether or not a function is differentiable at a point, and if it is the derivative at*	**NJSLS Content Standards** *AP Calculus BC* builds on many of the concepts and skills learned in the New Jersey Student Learning Standards **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B)	Selection of primary sources *Suggestion(s):* Texts: Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic. Reading, Mass.: Addison- Wesley, 1999 (on grade level); Larson, Hostetler, Edwards. Calculus with Analytic Geometry. Houghton Mifflin Company, 7th edition, 2002 (remedial); Stewart, James. Single Variable Calculus. Brooks/Cole Publishing Company, Pacific Grove, CA, 2002 (advanced) Additional Resources: - Anton, Howard. Calculus – A New Horizon. John Wiley and Sons, Inc., New York, 1999 - TI – 83, TI – 84 graphing calculators - Selected websites used for demonstration purposes - College Board Website - Graphmatica **Modifications and Accommodations:** **Students with special needs:** Teachers and support staff will attend to all modifications and accommodations listed in students' IEPs and

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	- When the function is concave up or concave down (4 days) • Optimization problems (3 days) • Using the tangent line to approximate function values (3 days) • Newton's Method (3 days) • Differentials and estimation techniques (3 days) • Related Rates and modeling rates of change (3 days) **Content-specific modifications and accommodations** - just in time support will be incorporated to help students reinforce necessary prerequisite skills - differentiated problem sets can be used to support and challenge students **Interdisciplinary/additional connections** - problems may include applications to science or engineering - position, velocity, and acceleration graphs will be explored	*that point* *- use the derivative to compute the velocity and acceleration functions given the position function - answer questions using either the equation or graph of the position, velocity, or acceleration equations* *- use the first and second derivative to compute the maximums, minimums, and inflection points of functions* *- explain how f, f ', f '' are connected by using equations, graphs, and numerical data* *- use implicit differentiation to solve related rate problem*	repeated reasoning **NJSLS for ELA Companion Standards** RST.9-10.3 RST.9-10.4 RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2	- alternative assessments (A) - Take home exams and investigations (F) - On-line work including AP exam open ended questions (F) - Administration of previous AP exam sections and questions based on relevant topics (F) - curve sketching task (F) -maximize/ minimize task (F)	504s. Teachers will incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning. **Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students' understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.

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Unit 2 – Integrals **Time:** 13-15 weeks *(see column 2 for a more detailed breakdown)*	**Key learning items/concepts:** Chapter 5: The Definite Integral (14 days) • Estimating with finite sums, LRAM, RRAM and MRAM (3 days) • Definition of the definite integral as a Riemann Sum (2 days) • The definite integral interpreted as a signed area (3 days) • The 1st and 2nd Fundamental Theorems of Calculus (3 days) • Approximating the definite integral with the Trapezoidal Rule (3 days) Chapter 6: Differential Equations and Mathematical Modeling (15 days) • Slope fields (3 days) • Antiderivatives as an	- *approximate signed area under a curve using rectangles and trapezoids* - *use calculators to compute this signed area* - *apply the two fundamental theorems of calculus to a variety of problems.* - *utilize integration techniques* - *interpret and find solutions to differential equations given initial conditions* - *create and interpret slope fields* - *use Euler method to approximate values of functions when separation of variables is not possible* - *compute area, net*	**NJSLS Content Standards** *AP Calculus BC* builds on many of the concepts and skills learned in the New Jersey Student Learning Standards **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B)	Selection of primary sources *Suggestion(s):* Texts: Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic. Reading, Mass.: Addison- Wesley, 1999 (on grade level); Larson, Hostetler, Edwards. Calculus with Analytic Geometry. Houghton Mifflin Company, 7th edition, 2002 (remedial); Stewart, James. Single Variable Calculus. Brooks/Cole Publishing Company, Pacific Grove, CA, 2002 (advanced) Additional Resources: - Anton, Howard. Calculus – A New Horizon. John Wiley and Sons, Inc., New York, 1999 - TI – 83, TI – 84 graphing calculators - Selected websites used for demonstration purposes - College Board Website - Graphmatica **Modifications and Accommodations:** **Students with special needs:** Teachers and support staff will attend to all modifications and accommodations listed in students' IEPs and

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	indefinite integral (2 days) • Methods of integration: u-Substitution, and the use of elementary forms, integration by parts (3 days) • Solving differential equations by separation of variables (2 days) • Euler's Method (2 days) • Applications of Exponential growth and decay (2 days) • Logistic growth (1 day) Chapter 7: Applications of Definite Integrals (12 days) • Integral of a Rate of Change of a quantity viewed as the net change in the value of that quantity (3 days) • Area between two curves using vertical and horizontal strips (3 days) • Volumes of solids of known cross section (2 days) • Volumes of solids of revolution using e method of disks, washers and cylindrical shells (2 days) • Length of a Curve (2 days) Chapter 8: L'Hopital's Rule,	*change, volume, arc length and service area analytically, graphically, and through the use of technology* *- explain that indeterminate forms of the limit can be solved if it is in or can be placed into the form 0/0 or ∞/∞* *- solve improper integral problems as a set up to series*	repeated reasoning **NJSLS for ELA Companion Standards** RST.9-10.3 RST.9-10.4 RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2	- alternative assessments (A) - Take home exams and investigations (F) - On-line work including AP exam open ended questions (F) - Administration of previous AP exam sections and questions based on relevant topics (F) - Reimann sum task (F)	504s. Teachers will incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning. **Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students' understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.

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	Improper Integrals, Partial Fractions (13 days) • The Indeterminate forms 0/0, (∞)(0), ∞−∞ and L'Hopital's Rule for determining limits (5 days) • Improper integrals including interior discontinuities and p-Integrals (4 days) • Partial Fractions – distinct linear and repeated linear factors (4 days) **Content-specific modifications and accommodations** - technology will be utilized to support students' understanding of area under the curve - differentiated problem sets can be used to support and challenge students **Interdisciplinary/additional connections** - problems may include applications to science or engineering - position, velocity, and acceleration graphs will be explored				

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Unit 3 – Series, Parametrics, Vectors, and Polar Calculus **Time:** 6-8 weeks *(see column 2 for a more detailed breakdown)*	**Key learning items/concepts:** Chapter 9: Infinite Series (15 days) • Geometric series (2 days) • Power series: - Term-by-term differentiation and integration to find power series of new functions (2 days) • Taylor's series, Maclaurin series - creating new series from known series and from the definition (2 days) • Lagrange Error bound (2 days) • Tests for convergence/divergence: - nth term test - Direct Comparison - Ratio Test - Integral Test - Limit Comparison Test - Alternating Series Test (Leibniz's Theorem) (3 days) • Radius and Interval of convergence (2 days) • Endpoint Testing (2 days)	- *use polynomials to approximate the value of a function and the interval over which the approximation can be applied, and determine the error bound of the approximation* - *review graphing techniques of polar and parametric equations* - *apply derivative and integration rules to solving problems involving polar curves, parametric functions, and vectors*	**NJSLS Content Standards** *AP Calculus BC* builds on many of the concepts and skills learned in the New Jersey Student Learning Standards **NJSLS SMP** MP1. Make sense of problems and persevere in solving them MP2. Construct viable arguments and critique the reasoning of others MP3. Reason abstractly and quantitatively MP4. Model with mathematics MP5. Attend to precision MP6. Use appropriate tools strategically MP7. Look for and make use of structure MP8. Look for and express regularity in repeated reasoning	Students will be assessed regularly throughout this course, with a focus on both conceptual understanding and procedural fluency. Assessment tools may include the following: - quizzes (F) - tests (S) - performance tasks (F/S) - projects (S) - homework (F) - discussions (F) - journals (F) - Form A, B, or C benchmark (B) - alternative assessments (A)	Selection of primary sources *Suggestion(s):* Texts: Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. Calculus: Graphical, Numerical, Algebraic. Reading, Mass.: Addison- Wesley, 1999 (on grade level); Larson, Hostetler, Edwards. Calculus with Analytic Geometry. Houghton Mifflin Company, 7th edition, 2002 (remedial); Stewart, James. Single Variable Calculus. Brooks/Cole Publishing Company, Pacific Grove, CA, 2002 (advanced) Additional Resources: - Anton, Howard. Calculus – A New Horizon. John Wiley and Sons, Inc., New York, 1999 - TI – 83, TI – 84 graphing calculators - Selected websites used for demonstration purposes - College Board Website - Graphmatica **Modifications and Accommodations:** **<u>Students with special needs</u>:** Teachers and support staff will attend to all modifications

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	Chapter 10: Parametric Functions, Vectors and Polar Calculus (10 days) • Derivatives of Parametric Functions (2 days) • Component form of a vector as applied to motion analysis (2 days) • Polar graphing (2 days) • The Calculus of Polar Curves - Slope of a Curve - Area Between Polar Curves - Length of a Polar Curve (4 days) **Content-specific modifications and accommodations** - technology will be utilized to support students' understanding of new concepts - differentiated problem sets can be used to support and challenge students **Interdisciplinary/additional connections** - problems may include applications to science or engineering		**NJSLS for ELA Companion Standards** RST.9-10.3 RST.9-10.4 RST.9-10.7 RST.11-12.3 RST.11-12.4 **NJSLS-CLKS** **- 21st Century Life and Careers** 9.4.12.CI.1 9.4.12.CI.3 9.4.12.CT.1 9.4.12.CT.2 **- Technology** 9.4.12.IML.3 **- Career Education** 9.2.12.CAP.2 9.3.ST.5 9.3.ST-ET.5 9.3.ST-SM.2 **NJSLS – CSDT** 8.1.12.DA.1 8.1.12.DA.5 8.1.12.DA.6 8.1.12.AP.1 8.2.12.ETW.2	- Take home exams and investigations (F) - On-line work including AP exam open ended questions (F) - Administration of previous AP exam sections and questions based on relevant topics (F)	and accommodations listed in students' IEPs and 504s. Teachers will incorporate manipulatives, extra time, alternative assessments, scaffolding, spiraling, technology, and flexible grouping to support student learning. **Multilingual students:** Teachers and support staff will work to support multilingual students in their first language and in English, providing materials and/or resources to support students' understanding. Students will be given additional time, as appropriate, and translation tools will be utilized as needed. **Students at risk of school failure:** Formative and summative data will be used to monitor student success, and students at risk of failure will receive additional supports and services, which may include parent consultation, extra help, and differentiation strategies, including small group instruction, group work, scaffolding, and spiraling. **Gifted and Talented Students:** Students who excel in their mastery of course standards will be further challenged with more complex tasks, extensions of concepts and skills, and extended problem solving and critical thinking opportunities.

Activities and Experiences:

At times, students will

- Be taught through the lecture method, where they will work individually and follow the teacher's direction, guidance and modeling.
- Participate in question-and-answer sessions with the whole class or while working in smaller groups.
- Have experiences where they explain or describe their mathematical problem solving strategies to their classmates. This might be written on the blackboard, on a transparency, or on an individual white-board, and then explained to the entire class. Another option might be to work with a small group of students forming a problem-solving team.
- Take notes related to presentations, group-work, textbook readings, or Internet activities.
- Experience open-book tests based either on their own notes, or their textbook material or using online resources.
- Complete pre-designed outlines guiding their note-taking on new or review material
- Work independently on paper-and-pencil practice of sample examples or homework assignments.
- Read about mathematics in their assigned textbooks or other teacher- approved resources.
- Communicate mathematically through written, oral, symbolic, and visual forms of expression. These may consist of journal entries, responding to open-ended questions, creating and analyzing graphs or individual and group projects and presentations.
- Have opportunities to brainstorm, working from the known to the unknown, seeing relationships and patterns, making connections and finally generalizing and discovering rules.
- Have opportunities where they will explain and describe their mathematical thinking and problem-solving process(es).
- Regularly and routinely use graphing calculators, computers and other mathematical tools to enhance mathematical thinking, understanding, and power. This may mean using a graphing calculator and software such as *Excel* or *Geometer's Sketchpad*.
- Explore and discover mathematical relationships and patterns using simulations based on hands-on laboratory-type explorations, using online Internet resources, or computer-generated simulations (i.e., using *Excel* or *Geometer's Sketchpad*.)
- Use real-life data to problem solve. These exercises may be based on textbook material, student-collected data from local surveys, teacher- created material, online resources, and Internet or e-mail generated projects.

- Be given assignments to work individually, in pairs or in groups to problem solve. These may include cooperative-learning jigsaw groupings where each team specializes in one aspect of a topic and then students re-group and teach their new knowledge to other classmates. This may include a teacher-created Webquest where students follow teacher-selected Internet sites to learn about a curriculum-related topic.
- Experience activities based on mathematical games and original projects.
- Be given real-life problems where they have the opportunity to select and apply various methods and tools to reach their conclusions.
- Be given opportunities to research the history of calculus to make connections between calculus and other subject areas and careers.
- Create rubrics to determine criteria for projects or open-ended questions.
- Take tests that include both individual paper-and-pencil assessments, and group-assignments. They also may be assessed through alternative methods such as individual Chapter-Review Projects, interviews, open-ended questions, matching or short-answer questions, or individual/team projects
- Take sample AP tests simulating regular test taking situations