POPE JOHN PAUL II
PREPARATORY SCHOOL

AP Calculus AB and BC

Required Summer Assignment

May 2024

Congratulations on your completion of Pre-Calculus. In the fall, both AP Calculus AB and BC courses will pick up with new methods for differentiating and evaluating limits, so it is important that you maintain your skills and understanding of the topics in this summer assignment. After a test over the summer homework, we will continue our study of Derivative Calculus by studying the various applications of derivatives before moving into Integral Calculus, and then a review of all Calculus topics for the AP exam in May 2025.

For all of you to be successful next year, it is imperative that each of you stay abreast of the mathematics that was taught in Algebra 2 and Pre-Calculus. *<u>You may work together</u> and you may look up techniques online as long as you can defend and understand your own work.*

Authorized Aid on Summer Homework Assignment

You may seek assistance from any source as long as the end result is the answer "yes" to the following questions:

1. Do I understand how to work the problem?
2. Can I rework the problem myself?
3. Can I explain it to another person?
4. Can I work another problem similar to it?
5. Do I know the concepts required to work this problem?
6. Could I extend my knowledge of this problem to apply it to another application?
7. Could I use my knowledge of this problem to complete a problem on a test that requires the use the concepts developed in this problem?

Your summer assignment must be submitted on Google Classroom by the second meeting of the first full week of school. You can work on blank pages in Notability or you may work on paper and submit pictures of your work. You are strongly encouraged to submit your work throughout the summer as you finish each page of problems instead of waiting until the last minute. On **<u>June 12, June 26, July 10, and July 24</u>**, we will check for any submissions to each page and send you solutions to any pages that you have completed. Your work will be returned and you will check, correct, and resubmit your work. You will have an assessment covering the summer homework problems in the first few weeks of school. We will not be going over the summer assignment in class. Your completion grade will be assigned from the work and corrections that you submit to Google Classroom.

We are excited to have you in class next year. Calculus is amazing and beautiful and fun, but it requires hard work and a lot of thinking. If you have any questions you may send a private comment on Google Classroom or email Mrs. Peper at Amanda.peper@popeprep.org.

Have a great summer and see you in Calculus!

Mrs. Cash and Mrs. Peper

Google Classroom Codes

BC Calculus: g6svo72 **AB Calculus:** ybkbn3c

Find all solutions in $[0,2\pi)$

a. $\sin x = \dfrac{\sqrt{2}}{2}$

b. $3\cos x - 1 = 2$

c. $2\sin(2x) = 1$

d. $0 = (2\sin x + 1)\left(2\cos x - \sqrt{2}\right)$

2. **Simplify the following rational expressions into a single fraction with no common factors**

   a. $\dfrac{x}{x+2} + \dfrac{3}{x-4}$

   b. $\dfrac{x^2 + x - 2}{x^2 - 1}$

   c. $\dfrac{x^2 + x - 6}{x^2 - 3x + 2}$

   d. $\left(\dfrac{x}{x+2} + 3\right) \div \dfrac{x+1}{x-1}$

3. **Simplify into a single expression using only positive exponents. Do not rationalize the denominators.**

   a. $x(1-2x)^{-3/2} + (1-2x)^{-1/2}$

   b. $\sqrt{3x-2} + \dfrac{x}{\sqrt{3x-2}}$

   c. $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

4. **Simplify to a single expression**

   a. $3^{2\log_3 5}$

   b. $\ln\sqrt{e}$

   c. $2\ln e^4$

   d. $\ln\frac{1}{e}$

   e. $\ln\sqrt[3]{e}$

   f. $\ln 2 + \ln 3$

   g. $\log_2 5 + \log_2\left(x^2 - 1\right) - \log_2(x-1)$

5. **Solve for x**

   a. $\ln e^3 = x$

   b. $\ln e^x = 4$

   c. $\ln x + \ln x = 0$

   d. $\ln 1 - \ln e = x$

   e. $\ln 6 + \ln x - \ln 2 = 3$

   f. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

6. **For the function** R **whose graph is shown, state the following.**

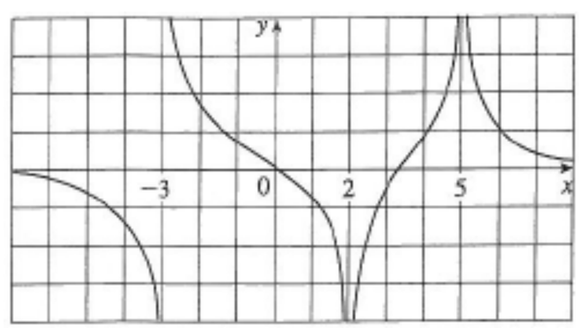
   a. $\lim_{x\to 2} R(x)$

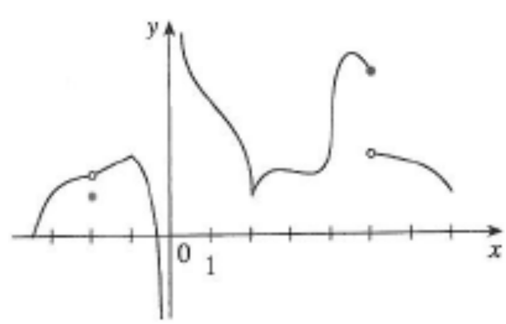
   b. $\lim_{x\to 5} R(x)$

   c. $\lim_{x\to -3^-} R(x)$

   d. $\lim_{x\to -3^+} R(x)$

   e. The equations of the vertical asymptotes





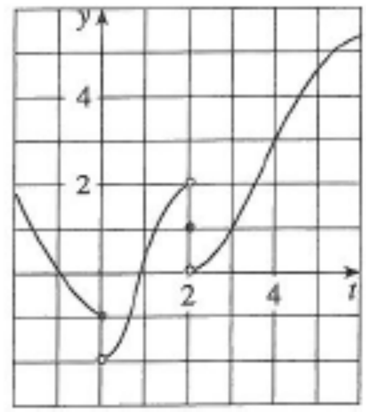
7. **The graph of** g **is given in the bottom right.**

   a. At what numbers is g discontinuous? Why?

   b. At what numbers is g not differentiable? Why?

8. **For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.**

   a. $\lim_{t\to 0^-} g(t)$
   b. $\lim_{t\to 0^+} g(t)$
   c. $\lim_{t\to 0} g(t)$
   d. $\lim_{t\to 2^-} g(t)$
   e. $\lim_{t\to 2^+} g(t)$
   f. $\lim_{t\to 2} g(t)$
   g. $g(2)$
   h. $\lim_{t\to 4} g(t)$



9. **Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists:**

$$f(x)=\begin{cases} 2-x, & \text{for } x<-1 \\ x, & \text{for } -1\le x<1 \\ (x-1)^2, & \text{for } x\ge 1 \end{cases}$$

10. **Sketch a graph of** $f(x)=\begin{cases} 4-x^2, & x\le 2 \\ x-2, & x>2 \end{cases}$.

   a. Find $\lim_{x\to 2^-} f(x)$

   b. Find $\lim_{x\to 2^+} f(x)$

   c. Does $\lim_{x\to 2} f(x)$ exist?

11. **Is $f(x)$ continuous at $x=0$? At $x=2$?**
    Use the definition of continuity to explain why or why not.

$$f(x)=\begin{cases} 1+x^2, & x\le 0 \\ 2-x, & 0<x\le 2 \\ (x-2)^2, & x>2 \end{cases}$$

12. **For what value of the constant c is $f(x)$ continuous on $(-\infty,\infty)$ if** $f(x)=\begin{cases} cx+1, & x\le 3 \\ cx^2-1, & x>3 \end{cases}$?

13. **[SET UP ONLY, DO NOT EVALUATE THE LIMIT]**
    Use the alternate definition of the derivative $\lim_{x\to a}\dfrac{f(x)-f(a)}{x-a}$ **to find $f'(a)$.**

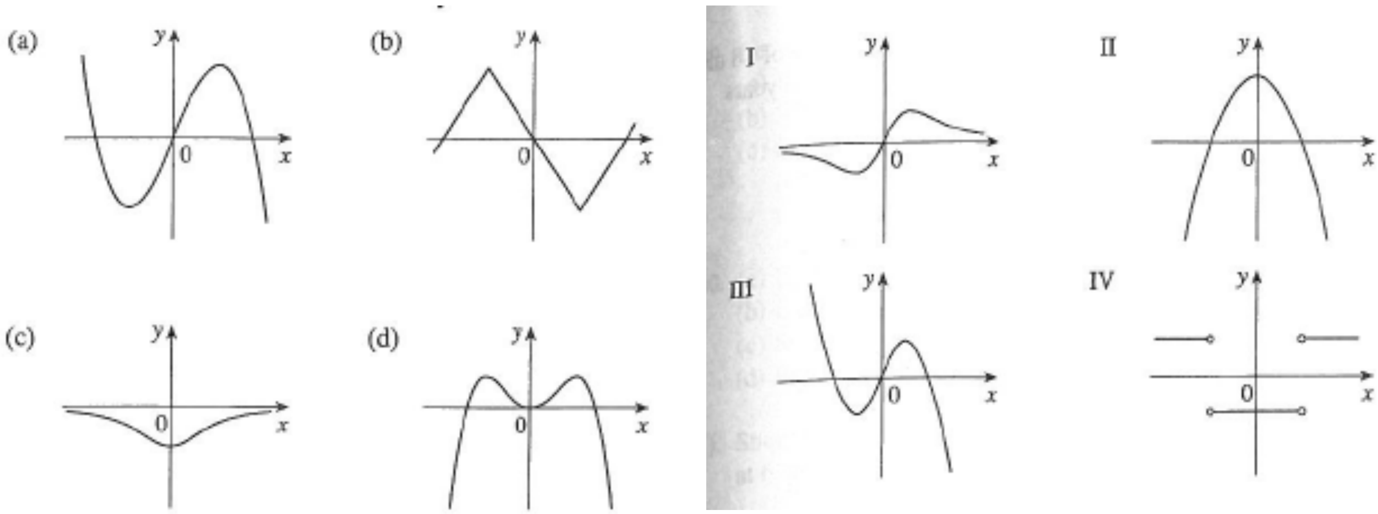
   a. $f(x)=3-2x+4x^2$, at $a=1$

   b. $f(x)=\dfrac{1}{\sqrt{x+2}}$, at $a=7$

FROM THIS POINT FORWARD, you are taking lots of derivatives.
Leave no negative exponents in your final answers.

14. If $g(x)=1-x^3$, find $g'(0)$ and use it to find an equation of the tangent line to $g(x)$ at $(0,1)$
15. If $f(x)=3x^2-5x$, find $f'(2)$ and use it to find an equation of the tangent line to $f(x)$ at $(2,2)$

16. **Match the graph of each function in a-d with the graph of its derivative I-IV.**



17. **Differentiate each function. (Power Rule)**
   a. $g(x)=5x^8-2x^5+6$
   b. $f(t)=\frac{1}{4}(t^4+8)$
   c. $V(r)=\frac{4}{3}\pi r^3$
   d. $R(t)=5t^{-3/5}$
   e. $R(x)=\frac{\sqrt{10}}{x^7}$
   f. $f(t)=\sqrt{t}-\frac{1}{\sqrt{t}}$
   g. $y=\sqrt[3]{x}$
   h. $g(u)=\sqrt{2}\cdot u+\sqrt{(3u)}$

18. **Differentiate. (Box, Power)**
   a. $y=\sqrt{x}(x-1)$
   b. $g(x)=\frac{3x-1}{2x+1}$
   c. $y=ax^2+bx+c$
   d. $y=A+\frac{B}{x}+\frac{C}{x^2}$

19. **Let $g(x)$ be a differentiable function (so $g'(x)$ exists). Find an expression for the derivative of each function (use box rule):**
   a. $y=xg(x)$
   b. $y=\frac{x}{g(x)}$
   c. $y=\frac{g(x)}{x}$

20. **Differentiate (Trig, Box)**
   a. $y=x\sin x$
   b. $y=\cot x+10\tan x$
   c. $y=2\csc x+5\cos x$
   d. $g(t)=4\sec t+\tan t$
   e. $y=\frac{x}{\cos x}$
   f. $y=\sec\theta\tan\theta$

21. **Find where each curve has a horizontal tangent:**

a. $f(x) = x + 2\sin x$

b. $f(x) = 3x^2 - 12x + 10$

c. $f(x) = 2x^3 - 3x^2$

d. $f(x) = 2\sin x + \sin^2 x$

22. **Differentiate. (Chain)**

a. $y = \sin 4x$

b. $y = (1 - x^2)^{10}$

c. $y = \sqrt{\sin x}$

d. $g(t) = \dfrac{1}{(t^4 + 1)^3}$

e. $f(t) = \sqrt[3]{1 + \tan t}$

23. **Suppose** $F(x) = f(g(x))$ **and** $g(3) = 6,\ g'(3) = 4,\ f'(3) = 2,\ \text{and}\ f'(6) = 7$. **Find** $F'(3)$.

24. **Differentiate**

a. $f(x) = x^2 e^x$

b. $f(u) = e^{1/u}$

c. $y = \cos(e^{\pi x})$

~~d.~~ $\cancel{f(x) = \ln(x^2 + 10)}$

~~e.~~ $\cancel{f(\theta) = \ln(\cos\theta)}$

~~f.~~ $\cancel{f(x) = \cos(\ln x)}$

~~g.~~ $\cancel{f(x) = \log_2(1 - 3x)}$

h. $y = 5^{-1/x}$

i. $y = 4^{3x+1}$

~~j.~~ $\cancel{y = \log x}$

25. **Find an equation of the tangent line AND the normal line (perpendicular) to the curve at the point.**

a. $y = x + \sqrt{x}$ at $x = 1$

b. $y = (1 + 2x)^2$ at $x = 1$

c. $y = \tan x$ at $\left(\dfrac{\pi}{4}, 1\right)$

d. $y = x + \cos x$ at $x = 0$

e. $y = \sec x - 2\cos x$ at $\left(\dfrac{\pi}{3}, 1\right)$

f. $y = e^{2x}\cos \pi x$ at $(0,1)$

g. $y = \dfrac{e^x}{x}$ at $x = 1$

h. $\cancel{y = \ln(\ln x) \text{ at } x = e}$

i. $\cancel{y = \ln(x^3 - 7) \text{ at } x = 2}$

26. **[DON'T DO THESE]**
Prove the first derivative for each equation (implicit).

a. $y = \sin(3x + 4y)$; $\dfrac{dy}{dx} = \dfrac{3\cos(3x + 4y)}{1 - 4\cos(3x + 4y)}$

b. $y = x^2y^3 + x^3y^2$; $\dfrac{dy}{dx} = \dfrac{2xy^3 + 3x^2y^2}{1 - 3x^2y^2 - 2x^3y}$

c. $\cos^2 x + \cos^2 y = \cos(2x + 2y)$; $\dfrac{dy}{dx} = \dfrac{\cos x \sin x - \sin(2x + 2y)}{\sin(2x + 2y) - \cos y \sin y}$