**AP Calculus BC Summer Packet** **Name** _____

Dear Students of AP Calculus BC,

This set of problems is suggested practice to refresh key skills for AP Calculus BC in the fall. Proficiency with these topics from Precalculus Honors will support your success in AP Calculus BC. The best way to use this packet is to do the work independently and for accuracy. Focus on understanding, not speed! If you need help, you can access your previous notes from Precalculus HN or use helpful sites like Khan Academy. You may find it helpful to work with a friend. If this works for you, great! However, please keep in mind those topics that you needed help on so that you can get additional practice when the year begins. You can also get help from your teacher or the ILC once school begins. The answers are provided for you to check your solutions.

We suggest that you complete this packet a week or two before school begins. It helps you know what you have retained over time and what you will need to refresh. If you feel that you'd like to do this packet closer to the end of this current school year, please plan to re-visit the packet prior to the start of the year.

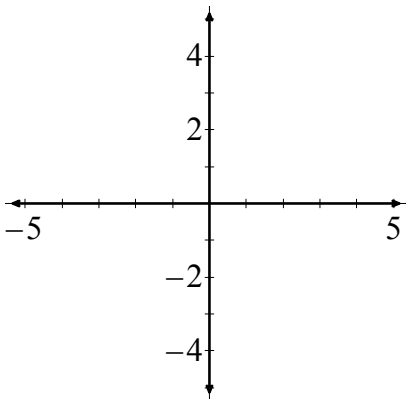
We are looking forward to a great year together getting to know you and exploring Calculus together!

***No Calculator unless otherwise stated. Show all work.***

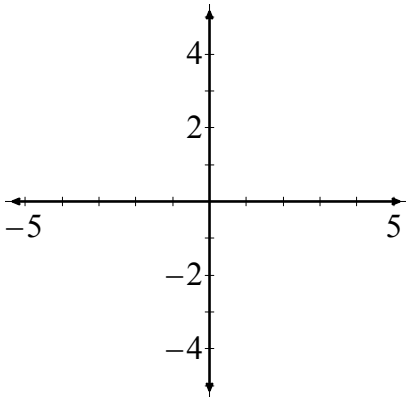
*I. Essential Graphs*

1. Sketch each graph. You should know the graphs of these functions.

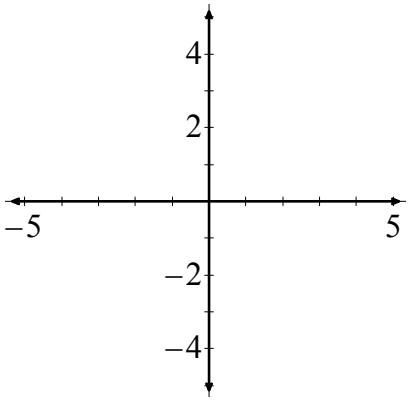
a. $f(x) = \sqrt{x}$



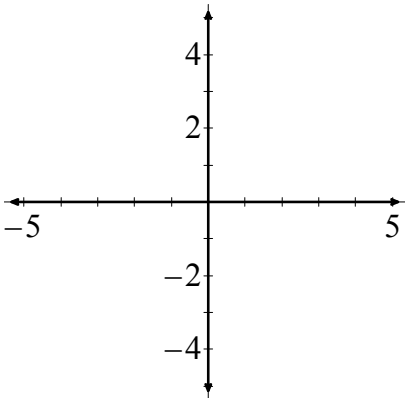

b. $g(x) = e^x$



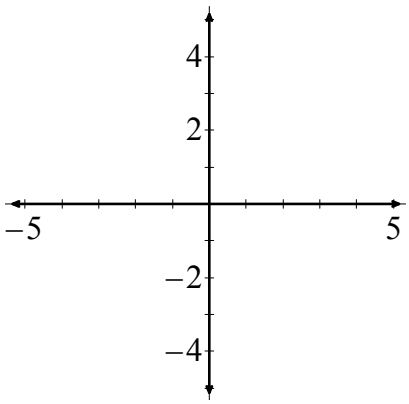

c. $f(x) = \ln x$



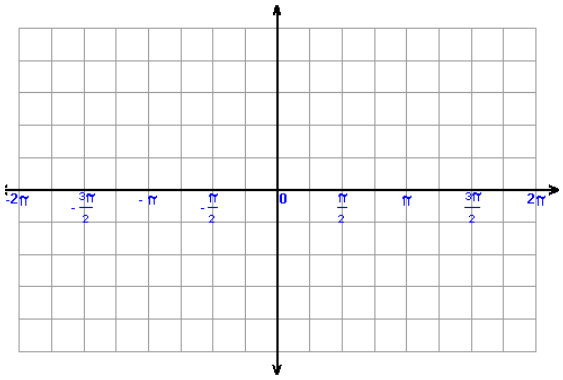

d. $y = \frac{1}{x}$



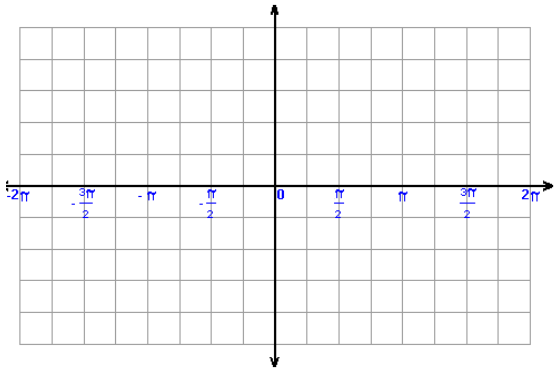

e. $y = \sqrt{4 - x^2}$



f. $f(x) = \sin x$




g. $g(x) = \cos x$




2. Which of the functions above are even functions? Which are odd function?

*II. Algebra Skills:*

1. Write the equation of the line, in point-slope form, with a slope of $\frac{2}{3}$ and passing through the point (–4, 1).

2. Write the equation of the line, in point-slope form, that passes through (–2, –7) and perpendicular to the line $x - 3y = 5$.

3. Given $f(x) = x^2 - 1$ and $g(x) = 2x - 3$, find the following:

a. $g(f(-3)) =$ b. $f(g(x)) =$ c. $g(f(x^3))$

4. Simplify: $\left(\frac{2x^3y^{-1}}{3y^6\sqrt{x}}\right)^{-2}$

5. Expand: $\ln\sqrt{\frac{x^3y}{z^5}}$

6. Divide the polynomials: $(2x^4 - x^3 + 3x^2 - 5x + 10) \div (2x - 3)$

7. Rationalize the numerator: $\frac{\sqrt{x+2}-3}{x-7}$

8. Given $f(x) = 2(x-1)^3$, find $f^{-1}(x)$.

9. Given $f(x) = \begin{cases} \sqrt[3]{x+2} & x < 0 \\ -x^2 + 4x & 0 \le x < 3 \\ |5-x| & x \ge 3 \end{cases}$

a. $f(-3) =$

b. $f\big(f(3)\big) =$

c. $\lim_{x \to 3} f(x)$

10. Solve the following equations for x.

a. $\frac{x^2-3x-10}{x(x+3)} = 0$

b. $\ln(x+5) = 1$

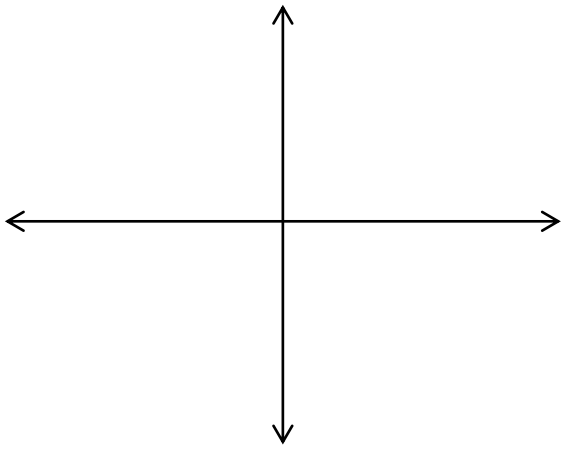
c. $\frac{3}{x} + \frac{x}{x+2} = 2$

11. Solve the following inequalities graphically. Write your answer in interval notation.

a. $x^2e^x > 4e^x$

b. $x^2(x-3)(x-4)^2 \leq 0$

12. Sketch the region bounded by $y = 2 - x^2$ and $y = 2x - 1$ and label any points of intersection. No Calculator.



III. Calculator Skills:

# 84 PLUS CE QUICK REFERENCE SHEET

## GRAPH FUNCTIONS IN APPROPRIATE WINDOW

- ❑ Enter equation into [y=]
- ❑ Use [zoom] 6: Standard Window
- ❑ To set a custom view screening use [window] and change the setting for Xmin=, Xmax=, Ymin=, Ymax=


## SOLVE EQUATIONS NUMERICALLY - INTERSECTION

This method will use a solve by graphing approach by graphing both sides of the equation separately to find the point(s) of intersection.

- ❑ Enter both left and right sides into $Y_1 =$ and $Y_2 =$
- ❑ Go to [graph] and adjust window as necessary so that you can see the intersection.
- ❑ Use [2nd] [trace] 5: Intersect
- ❑ Use arrows to move cursor close to a point of intersection, press [enter] three times (*first curve, second curve, guess*)
- ❑ Repeat the step above to find other points of intersection

## FINDING THE VALUE FOR $f(x)$ GIVEN THE X-VALUE

- ❑ Use [alpha] [trace] 1: $Y_1$ (*insert number*)

## FINDING A MAXIMUM OR MINIMUM

- ❑ Use [2nd] [trace] 3: minimum OR 4: maximum
- ❑ Use arrows to move cursor to the left of the min/max, then press [enter].
- ❑ Use arrows to move cursor to the right of the min/max, then press [enter].
- ❑ Press [enter] one more time when it says guess.

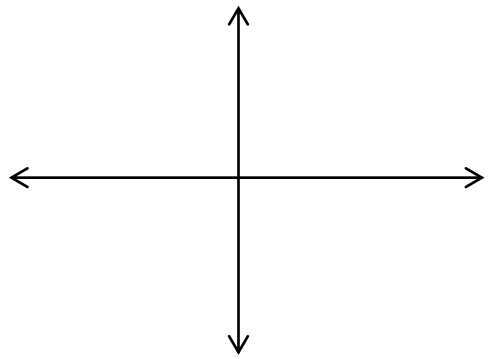
**** Be sure your calculator is in RADIANS****

Go to [mode]


## FINDING AN X-INTERCEPT

- ❑ Use [2nd] [trace] 2: zero
- ❑ Use arrows to move cursor to the left of x-intercept, then press [enter].
- ❑ Use arrows to move cursor to the right of x-intercept, then press [enter].
- ❑ Press [enter] one more time when it says guess.

1. Using your calculator, find the points of intersection of the graphs of $y = \ln(x - 1)$ and $y = x^2 - 3x$. Draw the graph and label intersection points. Round to 3 decimal places.



2. **Show all equations, but use a graphing calculator to solve this problem.** ***Round answers to at least 3 decimal places.***

A closed can in the shape of a right circular cylinder is designed to hold 12 oz. soda.
If the height of the cylinder is $x$ in., write an equation in terms of $x$ to represent the amount of material needed to construct the **closed** can. $\left(1 \text{ fl. oz.} = 1.805 \text{ in}^3\right)$ Find the minimum amount of material needed.

3. Given the velocity of an object is given by $v(t) = 2t^2 \sin t - 2t - 1$,

a. Find all values of $t$ on the interval $0 \leq t \leq 5$ where the velocity is zero.

b. Find all values of $t$ on the interval $0 \leq t \leq 5$ where the velocity is equal to 2.

c. Find the maximum velocity on the interval $0 \leq t \leq 5$

IV. Trigonometry

**KNOW the following identities:**

$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

$\sin 2x = 2\sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$ $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

1. Evaluate (give answers to j. – l. in radians)

a. $\cos\left(-\frac{4\pi}{3}\right)$ b. $\sin\frac{7\pi}{4}$ c. $\tan 120°$

d. $\sin\frac{3\pi}{2}$ e. $\cos 210°$ f. $\csc\left(\frac{10\pi}{3}\right)$

g. $\tan\frac{3\pi}{2}$ h. $\sec\left(\frac{-5\pi}{4}\right)$ i. $\tan 3\pi$

j. $\sin^{-1}\left(-\frac{1}{2}\right)$ k. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ l. $\tan^{-1}(-\sqrt{3})$

2. Solve the following over the interval $0 \leq x \leq 2\pi$:

a. $\tan x = -1$

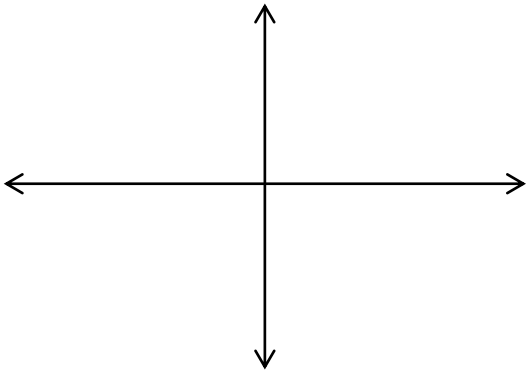
b. $\csc x = -2$

c. $2\sin^2 x + \cos x = 1$

3. Solve for $x$ over $-\frac{\pi}{2} \leq x \leq \pi$ : $\sin(x) - \cos(2x) = 0$

**V. Limits**

1. Sketch the graph and find the stated limits: $f(x) = \frac{2x}{x-3}$

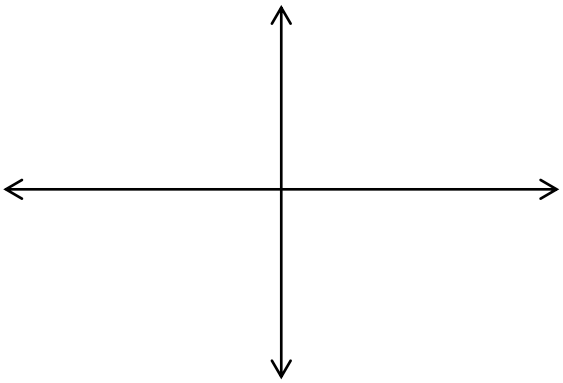


a. $\lim_{x \to 3^+} f(x)$

b. $\lim_{x \to 3} f(x)$

c. $\lim_{x \to \infty} f(x)$

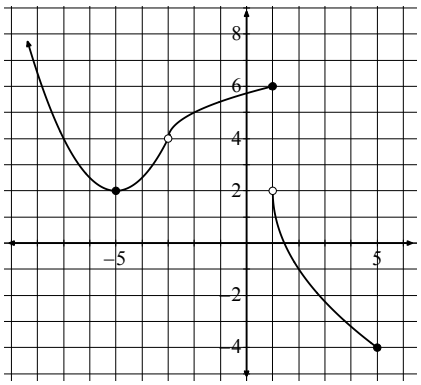
2. Sketch the graph and find the stated limits: $f(x) = \frac{1}{(x+1)^2}$



a. $\lim_{x \to -1} f(x)$

b. $\lim_{x \to \infty} f(x)$

3. Given the graph of $f(x)$ at the right, answer the following:




a. $f(1) =$

b. $\lim_{x \to -3} f(x) =$

c. $\lim_{x \to 1} f(x) =$

d. $\lim_{x \to -\infty} f(x) =$

e. For what values of $x$ is $f(x)$ discontinuous on the open interval $-5 < x < 5$?

**For #4 – 9, evaluate the following limits without using a calculator:**

4. $\lim_{x\to 2}\dfrac{x^3-8}{x-2}$

5. $\lim_{x\to\infty}\dfrac{\sin x}{x}$

6. $\lim_{x\to 0^-}\dfrac{|x|}{x}$

7. $\lim_{x\to\infty}\cos x$

8. $\lim_{x\to\infty}\arctan x$

9. $h(x)=\begin{cases}\sin x, & x<\dfrac{\pi}{2}\\ 1, & \dfrac{\pi}{2}\le x\le 2\\ 1-x, & x>2\end{cases}$

a. $\lim_{x\to\frac{\pi}{2}} h(x)$ b. $\lim_{x\to 2} h(x)$ c. $\lim_{x\to-\frac{\pi}{2}} h(x)$

10. If $f(x)=\dfrac{1}{\sqrt{x}}$ find $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ (Show all work, no "short cuts".)

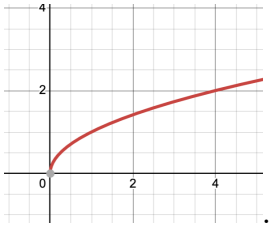
11. Sketch a graph for a function with the given characteristics:

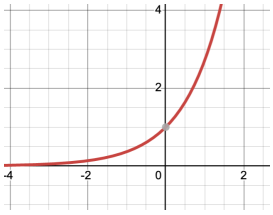
a. $\lim_{x\to 2} f(x)$ does not exist and $f(2)=3$.

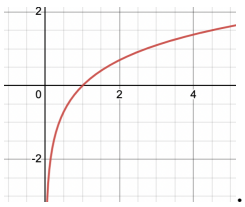
b. $\lim_{x\to 2} f(x)=3$ and $f(2)\neq 3$

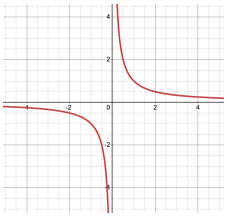
**Answers:**

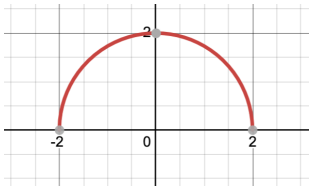
*Essential Graphs*

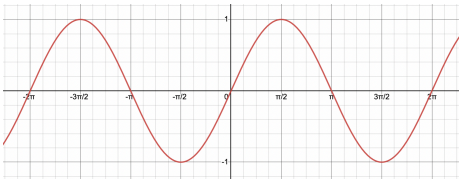
1a. 

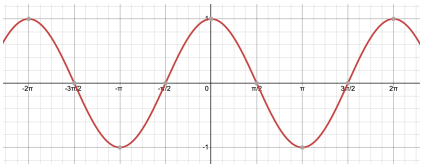

b. 


c. 


d. 

e. 


f. 

g. 

2. Even: $y=\sqrt{4-x^2}, y=cosx$

Odd: $y=\frac{1}{x}, y=sinx$

*Algebra Skills*

1. $y-1=\frac{2}{3}(x+4)$
2. $y+7=-3(x+2)$
3a. 13 b. $4x^2-12x+8$
c. $2x^6-5$
4. $\frac{9}{4x^5}$
5. $\frac{3}{2}lnx+\frac{1}{2}lny-\frac{5}{2}lnz$
6. $8x^3$
7. $x^3+\frac{3}{2}x^2+\frac{15}{4}x+\frac{25}{8}+\frac{27}{8(2x-3)}$
8. $\frac{1}{\sqrt{x+2}+3}$
9. $\sqrt[3]{\frac{x}{2}}+1=f^{-1}(x)$
10. a. -1 b. 4
c. DNE
11. a. $x=5,-2$ b. $x=-5+e$
c. $x=-3,2$
12. a. $(-\infty,-2)\cup(2,\infty)$ b. $[-\infty,3]\cup[4,4]$
13. Intersection points: (1, 1) and (-3, −7)

*Calculator Skills*

1. $(1.121,-2.107)$, $(3.249,0.811)$
2. 43.015 $in^3$
3. a. 1.385, 2.684 b. 1.889, 2.395
c. 2.438

*Trigonometry*

1. a. $-\frac{1}{2}$ b. $-\frac{\sqrt{2}}{2}$
c. $-\sqrt{3}$ d. -1
e. $-\frac{\sqrt{3}}{2}$ f. $-\frac{2\sqrt{3}}{3}$
g. $\emptyset$ h. $-\sqrt{2}$
i. 0 j. $-\frac{\pi}{6}$
k. $\frac{\pi}{4}$ l. $-\frac{\pi}{3}$
2. a. $\frac{3\pi}{4},\frac{7\pi}{4}$ b. $\frac{7\pi}{6},\frac{11\pi}{6}$
c. $\frac{\pi}{3},\frac{4\pi}{3},0,2\pi$
3. $\frac{\pi}{6},\frac{5\pi}{6},-\frac{\pi}{2}$

*Limits*

1. a. $\infty$ b. DNE
c. 2
2. a. DNE $(+\infty)$ b. 0
3. a. 6 b. 4
c. DNE d. $\infty$
e. $x=-3,1$
4. 12
5. 0
6. -1
7. DNE
8. $\frac{\pi}{2}$
9. a. 1 b. DNE
c. -1
10. $\frac{-1}{2x\sqrt{x}}$
11. a. Answers will vary
b. Answers will vary