2023-2024

AP Physics 1 Summer Assignment

1. The following are ordinary physics problems. Place the answer in scientific notation when appropriate and simplify the units (Scientific notation is used when it takes less time to write than the ordinary number does. As an example 200 is easier to write than 2.00×10^2, but 2.00×10^8 is easier to write than 200,000,000). Do you best to cancel units, and attempt to show the simplified units in the final answer.

a. $T_s = 2\pi\sqrt{\dfrac{4.5\times10^{-2}\,kg}{2.0\times10^3\,kg/s^2}} =$ ________________

b. $K = \dfrac{1}{2}\ 6.6\times10^2\,kg\ (2.11\times10^4\,m/s)^2 =$ ________________

c. $F = \left(9.0\times10^9\,\dfrac{N\cdot m^2}{C^2}\right)\dfrac{3.2\times10^{-9}C\quad 9.6\times10^{-9}C}{0.32m\quad ^2} =$ ________________

d. $\dfrac{1}{R_p} = \dfrac{1}{4.5\times10^2\Omega} + \dfrac{1}{9.4\times10^2\Omega}$ $\qquad R_P =$ ________________

e. $e = \dfrac{1.7\times10^3 J\ -\ 3.3\times10^2 J}{1.7\times10^3 J} =$ ________________

f. $1.33\ \sin 25.0^o\ =\ 1.50\ \sin\theta$ $\qquad \theta =$ ________________

g. $K_{max} =\ 6.63\times10^{-34}J\cdot s\quad 7.09\times10^{14}s\ -2.17\times10^{-19}J =$ ________________

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h. $\gamma = 1\Big/\sqrt{1-\dfrac{2.25\times10^8\,m/s}{3.00\times10^8\,m/s}} =$ ________________

2. Often problems on the exam are done with variables only. Solve for the variable indicated. Don't let the different letters confuse you. Manipulate them algebraically as though they were numbers.

a. $v^2 = v_o{}^2 + 2a(s-s_o)$ $\quad ,a =$ ________________

b. $K = \dfrac{1}{2}kx^2$ $\quad ,x =$ ________________

c. $T_p = 2\pi\sqrt{\dfrac{l}{g}}$ $\quad ,g =$ ________________

d. $F_g = G\dfrac{m_1m_2}{r^2}$ $\quad ,r =$ ________________

e. $mgh = \dfrac{1}{2}mv^2$ $\quad ,v =$ ________________

g. $B = \dfrac{\mu_o}{2\pi}\dfrac{I}{r}$ $\quad ,r =$ ________________

h. $x_m = \dfrac{m\lambda L}{d}$ $\quad ,d =$ ________________

i. $pV = nRT$ $\quad ,T =$ ________________

j. $\sin\theta_c = \dfrac{n_1}{n_2}$ $\quad ,\theta_c =$ ________________

k. $qV = \dfrac{1}{2}mv^2$ $\quad ,v =$ ________________

f. $x = x_o + v_o t + \frac{1}{2}at^2$, $t =$ ______________

l. $\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$, $s_i =$ ______________

3. Science uses the ***KMS*** system (***SI***: System Internationale). ***KMS*** stands for kilogram, meter, second. These are the units of choice of physics. The equations in physics depend on unit agreement. So you must convert to ***KMS*** in most problems to arrive at the correct answer.

kilometers (*km*) to meters (*m*)	and meters to kilometers	gram (*g*) to kilogram (*kg*)
centimeters (*cm*) to meters (*m*)	and meters to centimeters	Celsius (^oC) to Kelvin (*K*)
millimeters (*mm*) to meters (*m*)	and meters to millimeters	atmospheres (*atm*) to Pascals (*Pa*)
nanometers (*nm*) to meters (*m*)	and meters to nanometers	liters (*L*) to cubic meters (m^3)
micrometers (μm) to meters (*m*)		

Other conversions will be taught as they become necessary.

What if you don't know the conversion factors? Colleges want students who can find their own information (so do employers).

a. 4008 *g* = ______________ *kg*

b. 1.2 *km* = ______________ *m*

c. 823 *nm* = ______________ *m*

d. 298 *K* = ______________ ^oC

e. 0.77 *m* = ______________ *cm*

f. $8.8\text{x}10^{-8}$ *m* = ______________ *mm*

g. 1.2 *atm* = ______________ *Pa*

h. 25.0 μm = ______________ *m*

i. 2.65 *mm* = ______________ *m*

j. 8.23 *m* = ______________ *km*

k. 5.4 *L* = ______________ m^3

l. 40.0 *cm* = ______________ *m*

m. $6.23\text{x}10^{-7}$ *m* = ______________ *nm*

n. $1.5\text{x}10^{11}$ *m* = ______________ *km*

6. Solve the following geometric problems.

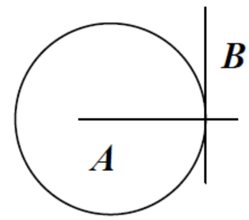
a. Line ***B*** touches the circle at a single point. Line ***A*** extends through the center of the circle.

i. What is line ***B*** in reference to the circle?

____

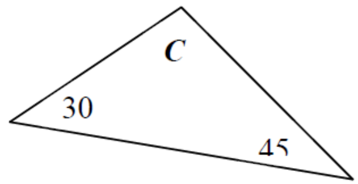
ii. How large is the angle between lines ***A*** and ***B***?

____



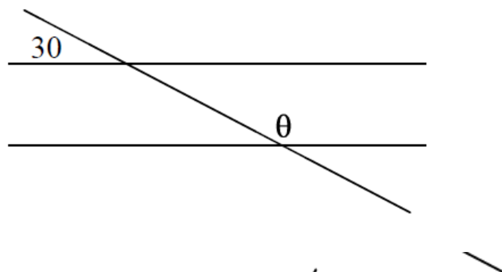
b. What is angle ***C***?

____



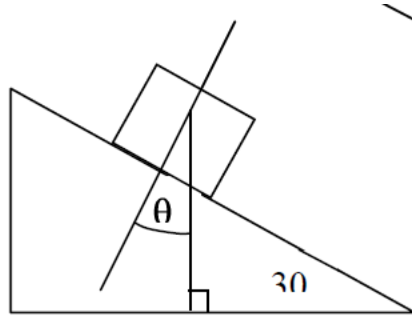
c. What is angle θ ?

____



d. How large is *θ*?

____



e. The radius of a circle is 5.5 *cm*,

i. What is the circumference in <u>meters</u>?

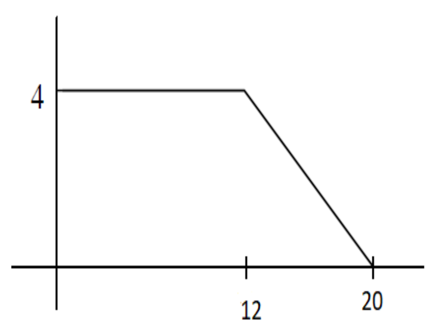
____

ii. What is its area in <u>square meters</u>?

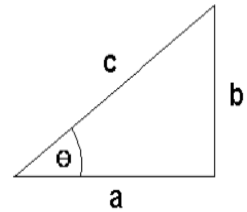
____

f. What is the area under the curve at the right?

____



7. Using the generic triangle to the right, Right Triangle Trigonometry and Pythagorean Theorem solve the following. ***<u>Your calculator must be in degree mode.</u>***



a. $\theta = 55^{\circ}$ and $\boldsymbol{c}$ = 32 *m*, solve for ***a*** and ***b***.

____

b. $\theta = 45^{\circ}$ and $\boldsymbol{a}$ = 15 *m/s*, solve for ***b*** and ***c***.

____

c. ***b*** = 17.8 *m* and $\theta = 65^{\circ}$, solve for ***a*** and ***c***.

____

d. ***a*** = 250 *m* and ***b*** = 180 *m*, solve for θ and ***c***.

____

e. ***a*** =25 *cm* and ***c*** = 32 *cm*, solve for ***b*** and θ.

____

f. ***b*** =104 *cm* and ***c*** = 65 *cm*, solve for ***a*** and θ.

____

Vectors

Most of the quantities in physics are vectors. ***<u>This makes proficiency in vectors extremely important</u>***.

Magnitude: Size or extent. The numerical value.

Direction: Alignment or orientation of any position with respect to any other position.

Scalars: A physical quantity described by a single number and units. A quantity described by **<u>magnitude only</u>**.

Examples: time, mass, and temperature

Vector: A physical quantity with **<u>both a magnitude and a direction</u>**. A directional quantity.

Examples: velocity, acceleration, force

Notation: $\vec{A}$ or $\xrightarrow{\vec{A}}$ <u>Length</u> of the arrow is <u>proportional to the vectors magnitude</u>. <u>Direction</u> the arrow points <u>is the direction of the vector</u>.

Negative Vectors

Negative vectors have the same magnitude as their positive counterpart. They are just pointing in the opposite direction.

$\xrightarrow{\vec{A}}$ $\xleftarrow{-\vec{A}}$

Vector Addition and subtraction

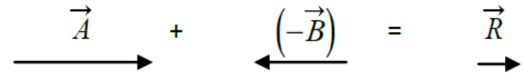
Think of it as vector addition only. The result of adding vectors is called the <u>resultant</u>. $\vec{R}$

$\vec{A}+\vec{B}=\vec{R}$ $\xrightarrow{\vec{A}}$ + $\xrightarrow{\vec{B}}$ = $\xrightarrow{\quad\quad\vec{R}\quad\quad}$

So if ***A*** has a magnitude of 3 and ***B*** has a magnitude of 2, then ***R*** has a magnitude of 3+2=5.

When you need to subtract one vector from another think of the one being subtracted as being a negative vector. Then add them.

$\vec{A} - \vec{B}$ is really $\vec{A} + \left(-\vec{B}\right) = \vec{R}$



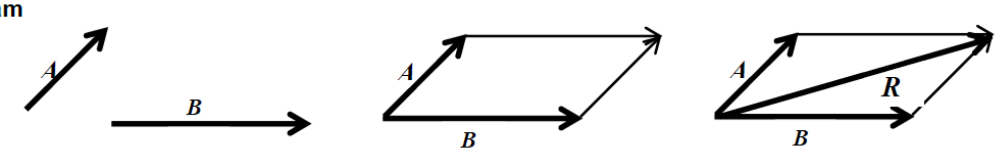
A negative vector has the same length as its positive counterpart, but its direction is reversed. So if ***A*** has a magnitude of 3 and ***B*** has a magnitude of 2, then ***R*** has a magnitude of 3+(-2)=1.

<u>This is very important</u>. In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), they just point in different directions (180° apart).

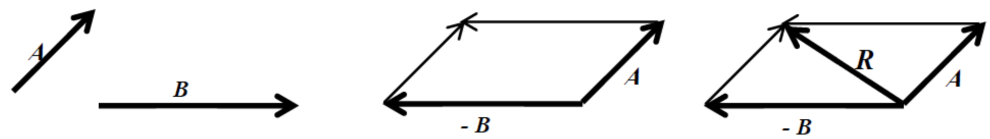
There are two methods of adding vectors

Parallelogram

A + B

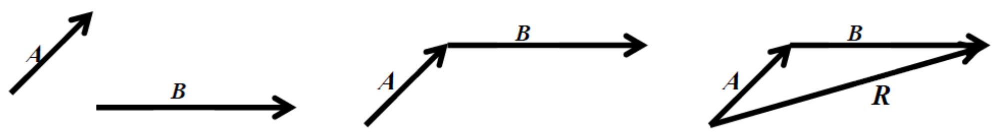


A – B

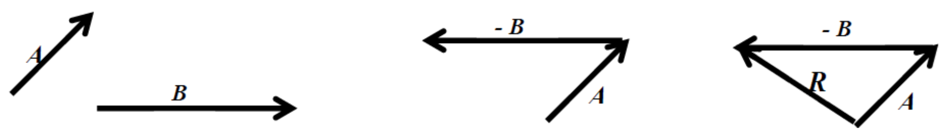


Tip to Tail

A + B



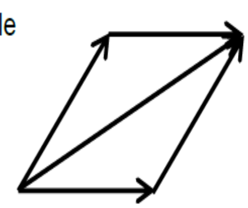
A – B



It is readily apparent that both methods arrive at the exact same solution since either method is essentially a parallelogram. It is useful to understand both systems. In some problems one method is advantageous, while in other problems the alternative method is superior.

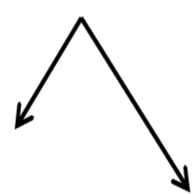
8. Draw the resultant vector using the parallelogram method of vector addition.

Example

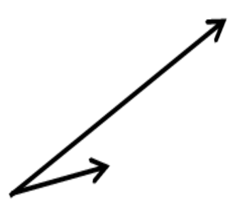


b.

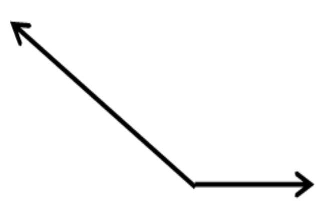
d.



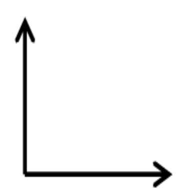
a.



c.

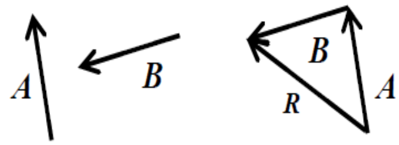


e.

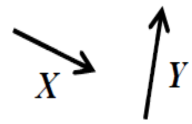


9. Draw the resultant vector using the tip to tail method of vector addition. Label the resultant as vector ***R***

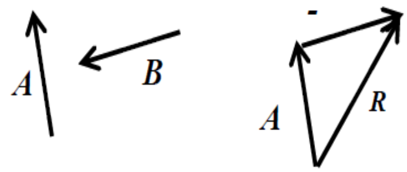
Example 1: ***A*** + ***B***



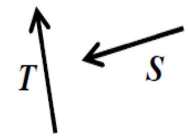
a. ***X*** + ***Y***



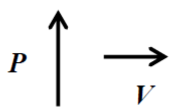
Example 2: ***A*** – ***B***



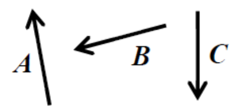
b. ***T*** – ***S***



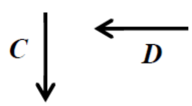
c. ***P*** + ***V***



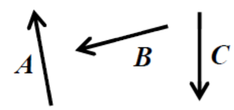
e. ***A*** + ***B*** + ***C***



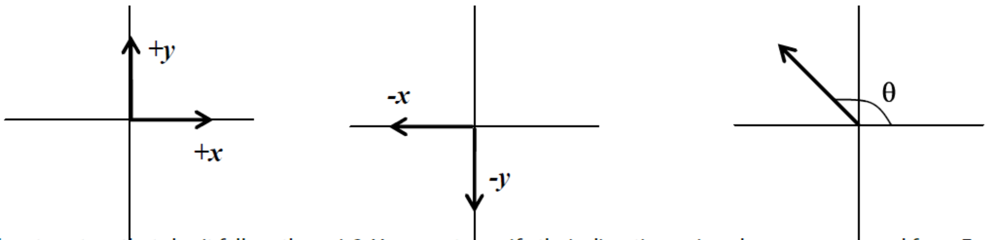
d. ***C*** – ***D***



f. ***A*** – ***B*** – ***C***



Direction: What does positive or negative direction mean? How is it referenced? The answer is the coordinate axis system. **In physics a coordinate axis system is used to give a problem a frame of reference.** Positive direction is a vector moving in the positive ***x*** or positive ***y*** direction, while a negative vector moves in the negative ***x*** or negative ***y*** direction (This also applies to the ***z*** direction, which will be used sparingly in this course).

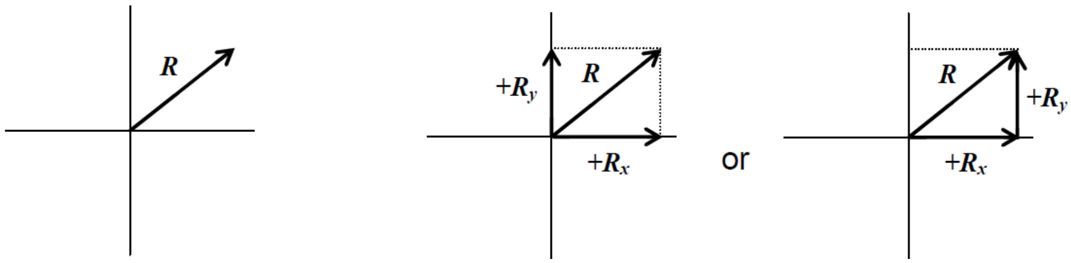


What about vectors that don't fall on the axis? You must specify their direction using degrees measured from East.

Component Vectors

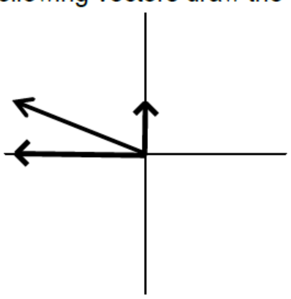
A resultant vector is a vector resulting from the sum of two or more other vectors. Mathematically the resultant has the same magnitude and direction as the total of the vectors that compose the resultant. Could a vector be described by two or more other vectors? Would they have the same total result?

This is the reverse of finding the resultant. You are given the resultant and must find the component vectors on the coordinate axis that describe the resultant.

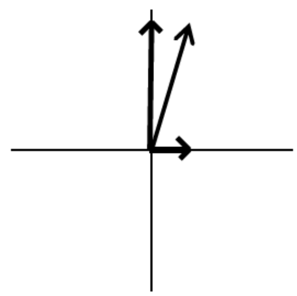


10. For the following vectors draw the component vectors along the ***x*** and ***y*** axis.

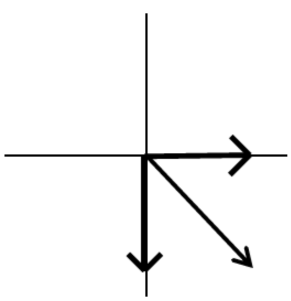
a.



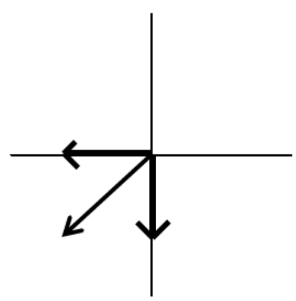
b.



c.



d.

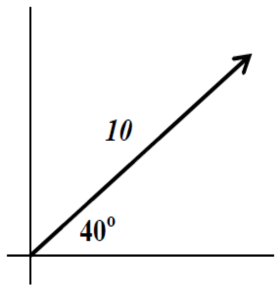


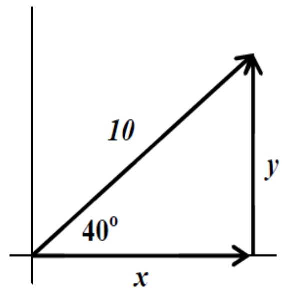
Obviously the quadrant that a vector is in determines the sign of the ***x*** and ***y*** component vectors.

AP Physics 1 Summer Assignment

Trigonometry and Vectors

Given a vector, you can now draw the ***x*** and ***y*** component vectors. The sum of vectors ***x*** and ***y*** describe the vector exactly. Again, any math done with the component vectors will be as valid as with the original vector. The advantage is that math on the ***x*** and/or ***y*** axis is greatly simplified since direction can be specified with plus and minus signs instead of degrees. But, how do you mathematically find the length of the component vectors? Use trigonometry.





$\cos\theta = \frac{adj}{hyp}$	$\sin\theta = \frac{opp}{hyp}$
$adj = hyp\cos\theta$	$opp = hyp\sin\theta$
$x = hyp\cos\theta$	$y = hyp\sin\theta$
$x = 10\cos 40^{o}$	$y = 10\sin 40^{o}$
$x = 7.66$	$y = 6.43$

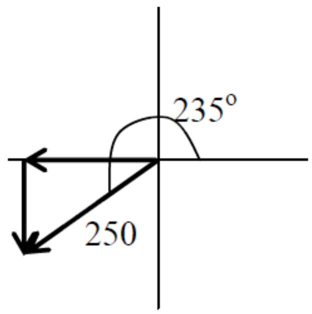
11. Solve the following problems. You will be converting from a polar vector, where direction is specified in ***degrees measured counterclockwise from east***, to component vectors along the ***x*** and ***y*** axis. Remember the plus and minus signs on you answers. They correspond with the quadrant the original vector is in.

    Hint: Draw the vector first to help you see the quadrant. Anticipate the sign on the ***x*** and ***y*** vectors. Do not bother to change the angle to less than 90°. Using the number given will result in the correct + and – signs.

    The first number will be the magnitude (length of the vector) and the second the degrees from east.

    Your calculator must be in degree mode.

Example: 250 at 235°



$$x = hyp\cos\theta$$
$$x = 250\cos 235^{o}$$
$$x = -143$$

$$y = hyp\sin\theta$$
$$y = 250\sin 235^{o}$$
$$y = -205$$

b. 6.50 at 345°

x=
y=

a. 89 at 150°

x=
y=

c. 0.00556 at 60°

x=
y=

d. 7.5×10^4 at 180°

x=
y=

f. 990 at 320°

x=
y=

e. 12 at 265°

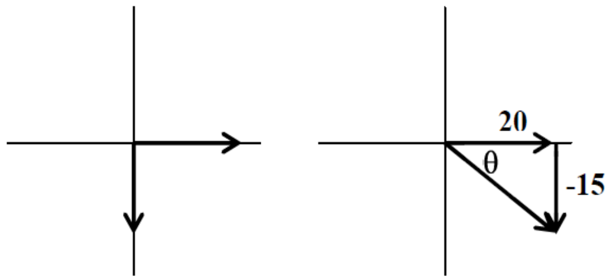
x=
y=

g. 8653 at 225°

x=
y=

12. Given two component vectors solve for the resultant vector. This is the opposite of number 11 above. Use Pythagorean Theorem to find the hypotenuse, then use inverse (arc) tangent to solve for the angle.

Example: ***x*** = 20, ***y*** = -15



$$R^2 = x^2 + y^2 \qquad \tan\theta = \frac{opp}{adj}$$

$$R = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\frac{opp}{adj}$$

$$R = \sqrt{20^2 + 15^2} \qquad \theta = \tan^{-1}\frac{y}{x}$$

$$R = 25$$

$$360^o - 36.9^o = 323.1^o$$

a. ***x*** = 600, ***y*** = 400

r=
θ=

c. ***x*** = -32, ***y*** = 16

r=
θ=

b. ***x*** = -0.75, ***y*** = -1.25

r=
θ=

d. ***x*** = 0.0065, ***y*** = -0.0090

r=
θ=

How are vectors used in Physics?

They are used everywhere!

Speed

Speed is a scalar. It only has magnitude (numerical value).

$\boldsymbol{v_s}$ = 10 *m/s* means that an object is going 10 meters every second. But, we do not know where it is going.

Velocity

Velocity is a vector. It is composed of both magnitude and direction. Speed is a part (numerical value) of velocity.

v = 10 *m/s* north, or ***v*** = 10 *m/s* in the ***+x*** direction, etc.

There are three types of speed and three types of velocity

Instantaneous speed / velocity: The speed or velocity at an instant in time. You look down at your speedometer and it says 20 *m/s*. You are traveling at 20 *m/s* at that instant. Your speed or velocity could be changing, but at that moment it is 20 *m/s*.

Average speed / velocity: If you take a trip you might go slow part of the way and fast at other times. If you take the total distance traveled divided by the time traveled you get the average speed over the whole trip. If you looked at your speedometer from time to time you would have recorded a variety of instantaneous speeds. You could go 0 *m/s* in a gas station, or at a light. You could go 30 *m/s* on the highway, and only go 10 *m/s* on surface streets. But, while there are many instantaneous speeds there is only one average speed for the whole trip.

b. A car travels 35 *km* west and 75 *km* east. What is its displacement?

c. A car travels 35 *km* west, 90 *km* north. What distance did it travel?

d. A car travels 35 *km* west, 90 *km* north. What is its displacement?

e. A bicyclist pedals at 10 *m/s* in 20 *s*. What distance was traveled?

f. An airplane flies 250.0 *km* at 300 *m/s*. How long does this take?

g. A skydiver falls 3 *km* in 15 *s*. How fast are they going?

h. A car travels 35 *km* west, 90 *km* north in two hours. What is its average speed?

Constant speed / velocity: If you have cruise control you might travel the whole time at one constant speed. If this is the case then you average speed will equal this constant speed.

A trick question

Will an object traveling at a constant speed of 10 *m/s* also always have constant velocity?

Not always. If the object is turning around a curve or moving in a circle it can have a constant speed of 10 *m/s*, but since it is turning, its direction is changing. And if direction is changing then velocity must change, since velocity is made up of speed and direction.

Constant velocity must have both constant magnitude and constant direction.

Rate

Speed and velocity are rates. A rate is a way to quantify anything that takes place during a time interval. Rates are easily recognized. They always have time in the denominator.

10 *m/s* 10 meters / second

The very first Physics Equation

Velocity and Speed both share the same equation. Remember speed is the numerical (magnitude) part of velocity. Velocity only differs from speed in that it specifies a direction.

$v = \frac{x}{t}$ *v* stands for velocity ***x*** stands for displacement ***t*** stands for time

Displacement is a vector for distance traveled in a straight line. It goes with velocity. Distance is a scalar and goes with speed. Displacement is measured from the origin. It is a value of how far away from the origin you are at the end of the problem. The direction of a displacement is the shortest straight line from the location at the beginning of the problem to the location at the end of the problem.

How do distance and displacement differ? Supposes you walk 20 meters down the **+ *x*** axis and turn around and walk 10 meters down the **–** ***x*** axis.

The distance traveled does not depend on direction since it is a scalar, so you walked 20 + 10 = 30 meter.

Displacement only cares about your distance from the origin at the end of the problem. +20 – 10 = 10 meter.

13. Attempt to solve the following problems. Take heed of the following.

Always use the *MKS* system: Units must be in meters, kilograms, seconds.

On the all tests, including the AP exam you must:

1. List the original equation used.
2. Show correct substitution.
3. Arrive at the correct answer with correct units.

Distance and displacement are measured in meters (*m*)

Speed and velocity are measured in meters per second (*m/s*)

Time is measured in seconds (*s*)

Example: A car travels 1000 meters in 10 seconds. What is its velocity?

$v = \frac{x}{t}$ $v = \frac{1000m}{10s}$ $v = 100m/s$

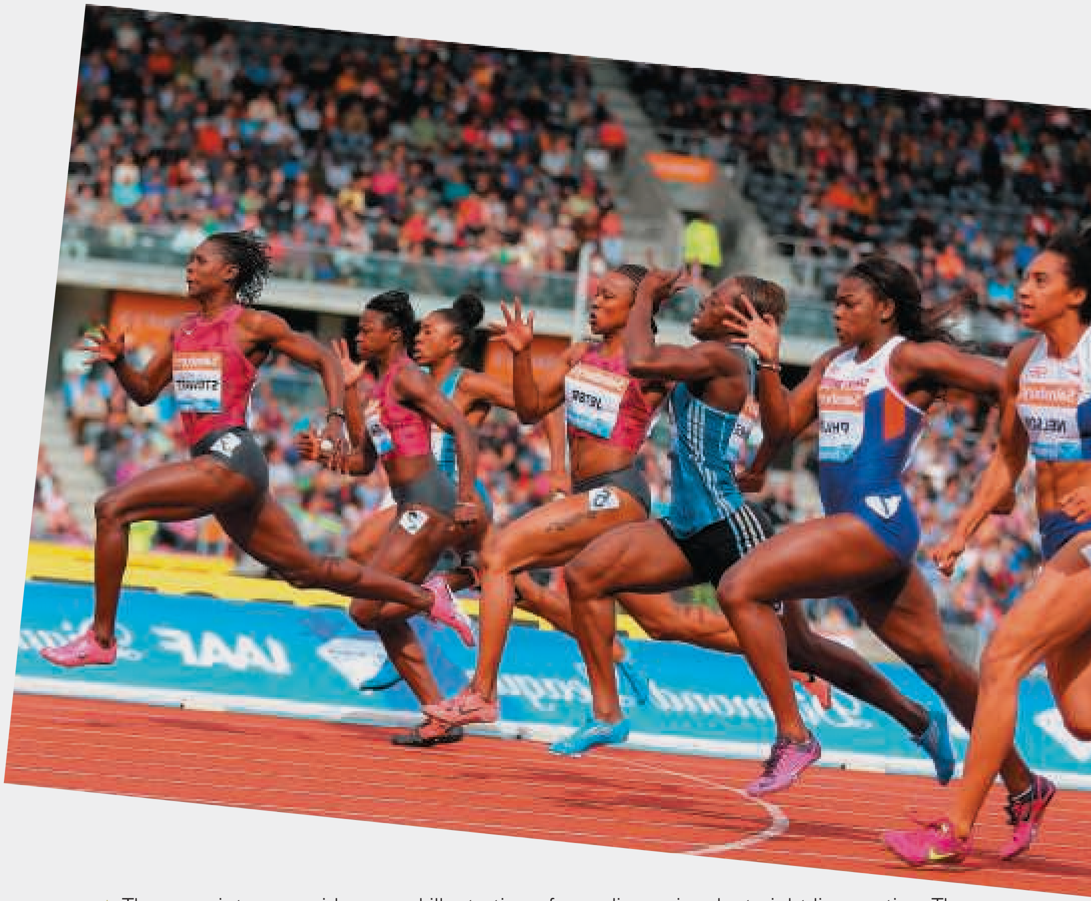
a. A car travels 35 *km* west and 75 km east. What distance did it travel?

2

One-Dimensional Kinematics

Big Ideas

1. Position is measured relative to a coordinate system.
2. Velocity is the rate of change of position with time.
3. Acceleration is the rate of change of velocity with time.
4. Speed increases (decreases) when velocity and acceleration are in the same (opposite) direction.
5. Equations of motion relate position, velocity, acceleration, and time.
6. Free fall is motion with a constant downward acceleration of magnitude *g*.



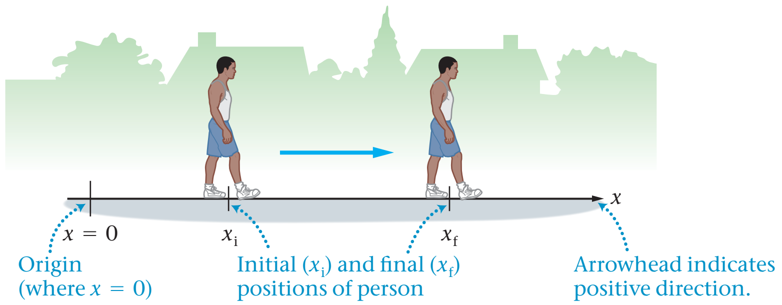
▲ These sprinters provide a good illustration of one-dimensional, straight-line motion. They accelerated as they left the starting blocks, and now they maintain a constant velocity as they approach the finish line.

We begin our study of physics with **mechanics**, the area of physics most apparent to us in our everyday lives. Every time you raise an arm, stand up or sit down, throw a ball, or open a door, your actions are governed by the laws of mechanics. In this chapter we focus on kinematics—the basic properties of motion—in one dimension.

2-1 Position, Distance, and Displacement

In physics, the terms *position, distance,* and *displacement* have specific meanings. This section gives the physics definitions of these terms and shows how they are used to describe the motion of a particle.

Position The first step in describing the motion of a particle is to set up a **coordinate system** that defines its location—that is, its **position.** An example of a coordinate system in one dimension is shown in **FIGURE 2-1**. This is simply an x axis, with an origin (where $x = 0$) and an arrow pointing in the positive direction—the direction in which x increases. In setting up a coordinate system, we are free to choose the origin and the positive direction as we like, but once we make a choice, we must be consistent with it throughout any calculations that follow.



▸ **FIGURE 2-1 A one-dimensional coordinate system** You are free to choose the origin and positive direction as you like, but once your choice is made, stick with it.

The "particle" in Figure 2-1 is a person who has moved to the right from an initial position, x_i, to a final position, x_f. Because the positive direction is to the right, it follows that x_f is greater than x_i; that is, $x_f > x_i$.

Distance Now that we've seen how to set up a coordinate system, let's use one to investigate the situation shown in **FIGURE 2-2**. Suppose you leave your house, drive to the grocery store, and then return home. The **distance** you've covered in your trip is $4.3\text{ mi} + 4.3\text{ mi} = 8.6\text{ mi}$. In general, distance is defined as the length of a trip:

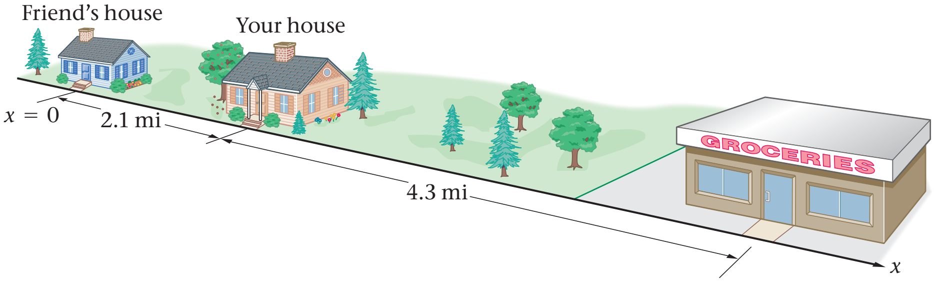
Definition of Distance

distance = total length of travel

SI unit: meter, m

Using SI units, we find that the distance in this case is

$$8.6\text{ mi} = (8.6\text{ mi})\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 1.4 \times 10^4\text{ m}$$



▸ **FIGURE 2-2 One-dimensional coordinates** The locations of your house, your friend's house, and the grocery store in terms of a one-dimensional coordinate system.

In a car, the distance traveled is indicated by the odometer. Distance is always positive, and is a scalar quantity (see Chapter 1) with no associated direction.

Displacement Another useful way to characterize a particle's motion is in terms of its **displacement**, Δx, which is simply the change in position:

Definition: Displacement, Δx

displacement = change in position = final position − initial position

displacement = $\Delta x = x_f - x_i$ 2-1

SI unit: meter, m

The SI units of displacement are meters—the same as for distance—but displacement and distance are really quite different. For example, in the round trip from your house to the grocery store and back, the distance traveled is 8.6 mi, whereas the displacement is zero because $x_f = x_i = 2.1$ mi, and hence $\Delta x = x_f - x_i = 0$.

Notice that we use the delta notation, Δx, as a convenient shorthand for the quantity $x_f - x_i$. (See Appendix A for a complete discussion of delta notation.) Also, notice that Δx can be positive (if the final position is to the right of the initial position, $x_f > x_i$), negative (if the final position is to the left of the initial position, $x_f < x_i$), or zero (if the final and initial positions are the same, $x_f = x_i$). In fact, the displacement is a one-dimensional vector, as defined in Chapter 1, and its direction (right or left) is given by its sign (positive or negative, respectively).

For example, suppose you go from your house to the grocery store and then to your friend's house in Figure 2-2. On this trip the distance is 10.7 mi, but the displacement is

$$\Delta x = x_f - x_i = (0) - (2.1 \text{ mi}) = -2.1 \text{ mi}$$

The minus sign means your displacement is in the negative direction; that is, to the left.

QUICK EXAMPLE 2-1 DISPLACEMENT AND DISTANCE

Predict/Calculate

Referring to Figure 2-2, suppose you take a trip from your friend's house to the grocery store and then to your house. **(a)** Is the displacement for this trip positive, negative, or zero? Explain. **(b)** Find the displacement for this trip. **(c)** What is the distance covered in this trip?

REASONING AND SOLUTION

Displacement is the final position minus the initial position; it can be positive or negative. Distance is the length of travel, which is always positive.

1. **Part (a)** The displacement is positive because the final position is to the right (positive direction) of the initial position.
2. **Part (b)** Determine the initial position for the trip, using Figure 2-2: $x_i = 0$
3. Determine the final position for the trip, using Figure 2-2: $x_f = 2.1$ mi
4. Subtract x_i from x_f to find the displacement. Notice that the result is positive, as expected: $\Delta x = x_f - x_i = 2.1 \text{ mi} - 0 = 2.1 \text{ mi}$
5. **Part (c)** Add the distances for the various parts of the trip: $2.1 \text{ mi} + 4.3 \text{ mi} + 4.3 \text{ mi} = 10.7 \text{ mi}$

Enhance Your Understanding (Answers given at the end of the chapter)

1. For each of the following questions, give an example if your answer is yes. Explain why not if your answer is no. **(a)** Is it possible to take a trip in which the distance covered is less than the magnitude of the displacement? **(b)** Is it possible to take a trip in which the distance covered is greater than the magnitude of the displacement?

Section Review

- Distance is the total length of a trip.
- Displacement is the change in position; displacement = $\Delta x = x_f - x_i$.

PHYSICS IN CONTEXT
Looking Back

In this chapter we make extensive use of the sign conventions for one-dimensional vectors introduced in Chapter 1—positive for one direction, negative for the opposite direction.

Big Idea 1 Position and displacement are measured relative to a coordinate system. The coordinate system must include an origin and a positive direction. Displacement has a direction and a magnitude, and hence it is a vector. Distance has only a numerical value, and hence it is a scalar.

2-2 Average Speed and Velocity

The next step in describing motion is to consider how rapidly an object moves. For example, how much time does it take for a major-league fastball to reach home plate? How far does an orbiting satellite travel in one hour? These are examples of some of the most basic questions regarding motion, and in this section we learn how to answer them.

Average Speed The simplest way to characterize the rate of motion is with the **average speed**:

$$\text{average speed} = \frac{\text{distance}}{\text{elapsed time}} \qquad \text{2-2}$$

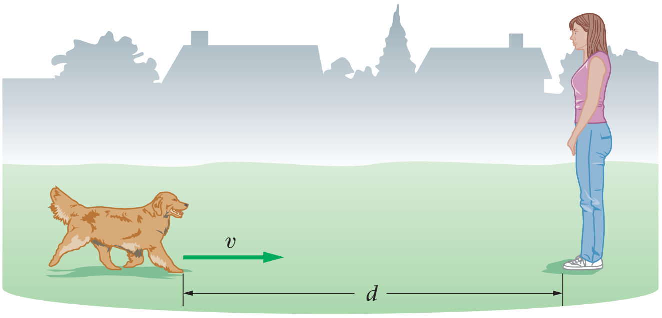
The dimensions of average speed are distance per time or, in SI units, meters per second, m/s. Both distance and elapsed time are positive; thus average speed is always positive.

EXAMPLE 2-2 DOG RUN

A dog trots back to its owner with an average speed of 1.40 m/s from a distance of 2.3 m. How much time does it take for the dog to reach its owner?

PICTURE THE PROBLEM

The dog moves in a straight line through a distance of $d = 2.3$ m. The average speed of the dog is $v = 1.40$ m/s.



REASONING AND STRATEGY

Equation 2-2 $\left(\text{average speed} = \dfrac{\text{distance}}{\text{elapsed time}}\right)$ relates average speed, distance, and elapsed time. We can solve for the elapsed time by rearranging this equation.

Known Average speed, $v = 1.40$ m/s; distance, $d = 2.3$ m.
Unknown Elapsed time = ?

SOLUTION

1. Rearrange Equation 2-2 to solve for the elapsed time: $\text{elapsed time} = \dfrac{\text{distance}}{\text{average speed}} = \dfrac{d}{v}$
2. Substitute numerical values to find the time: $\text{elapsed time} = \dfrac{2.3\ \cancel{\text{m}}}{1.40\ \cancel{\text{m}}/\text{s}} = \dfrac{2.3}{1.40}\ \text{s} = 1.6\ \text{s}$

INSIGHT

As this example shows, Equation 2-2 is not just a formula for calculating the average speed. It relates speed, time, and distance. Any one of these quantities can be determined if the other two are known.

PRACTICE PROBLEM

A dog trots with an average speed of 1.44 m/s for 1.9 s. What is the distance it covers?
[**Answer:** distance = (average speed) (elapsed time) = (1.44 m/s) (1.9 s) = 2.7 m]

Some related homework problems: Problem 11, Problem 13

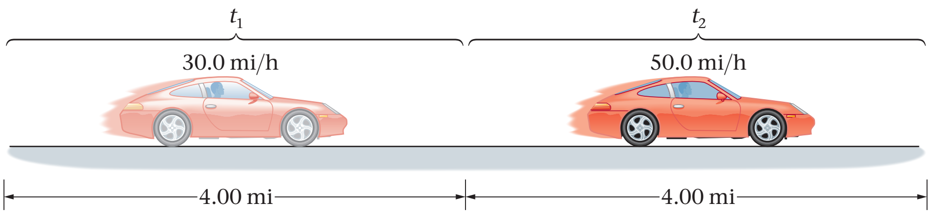
Next, we calculate the average speed for a trip consisting of two parts of equal length, each traveled with a different speed.

CONCEPTUAL EXAMPLE 2-3 AVERAGE SPEED PREDICT/EXPLAIN

You drive 4.00 mi at 30.0 mi/h and then another 4.00 mi at 50.0 mi/h. **(a)** Is your average speed for the 8.00-mi trip greater than, less than, or equal to 40.0 mi/h? **(b)** Which of the following is the *best explanation* for your prediction?

I. The average of 30.0 mi/h and 50.0 mi/h is 40.0 mi/h, and hence this is the average speed for the trip.
II. You go farther during the 50.0 mi/h part of the trip, and hence your average speed is greater than 40.0 mi/h.
III. You spend more time during your trip traveling at 30.0 mi/h, and hence your average speed is less than 40.0 mi/h.

CONTINUED



REASONING AND DISCUSSION

At first glance it might seem that the average speed is definitely 40.0 mi/h. On further reflection, however, it is clear that it takes more time to travel 4.00 mi at 30.0 mi/h than it does to travel 4.00 mi at 50.0 mi/h. Therefore, you will be traveling at the lower speed for a greater period of time, and hence your average speed will be *less* than 40.0 mi/h—that is, closer to 30.0 mi/h than to 50.0 mi/h.

ANSWER

(a) The average speed is less than 40.0 mi/h. **(b)** The best explanation is III.

To confirm the conclusion of Conceptual Example 2-3, we simply apply the definition of average speed to find its value for the entire trip. We already know that the distance traveled is 8.00 mi; what we need now is the total elapsed time. The elapsed time on the first 4.00 mi is

$$t_1 = \frac{4.00\text{ mi}}{30.0\text{ mi/h}} = \left(\frac{4.00}{30.0}\right)\text{h} = 0.133\text{ h}$$

The time required to cover the second 4.00 mi is

$$t_2 = \frac{4.00\text{ mi}}{50.0\text{ mi/h}} = \left(\frac{4.00}{50.0}\right)\text{h} = 0.0800\text{ h}$$

Therefore, the elapsed time for the entire trip is

$$t_1 + t_2 = 0.133\text{ h} + 0.0800\text{ h} = 0.213\text{ h}$$

This gives the following average speed:

$$\text{average speed} = \frac{8.00\text{ mi}}{0.213\text{ h}} = 37.6\text{ mi/h} < 40.0\text{ mi/h}$$

It's important to note that a "guess" will never give a detailed result like 37.6 mi/h; a systematic, step-by-step calculation is required.

Average Velocity There is another physical quantity that is often more useful than the average speed. It is the **average velocity**, v_{av}, and it is defined as displacement per time:

Definition: Average Velocity, v_{av}

$$\text{average velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad \text{2-3}$$

SI unit: meter per second, m/s

Not only does the average velocity tell us, on average, how fast something is moving, it also tells us the *direction* the object is moving. For example, if an object moves in the positive direction, then $x_f > x_i$ and the average velocity is positive, $v_{av} > 0$. On the other hand, if an object moves in the negative direction, then $x_f < x_i$ and $v_{av} < 0$. As with displacement, the average velocity is a one-dimensional vector, and its direction

Balls Take High and Low Tracks

PROBLEM-SOLVING NOTE

"Coordinate" the Problem

The first step in solving a physics problem is to produce a simple sketch of the system. Your sketch should include a coordinate system, along with an origin and a positive direction. Next, you should identify quantities that are given in the problem, such as initial position, initial velocity, acceleration, and so on. These preliminaries will help you produce a mathematical representation of the problem.

is given by its sign. Average velocity gives more information than average speed; hence it is used more frequently in physics.

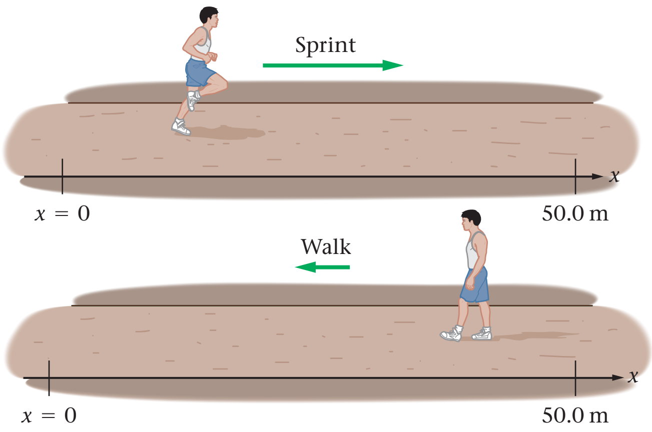
In the next Example, pay close attention to the sign of each quantity.

EXAMPLE 2-4 SPRINT TRAINING

An athlete sprints in a straight line for 50.0 m in 8.00 s, and then walks slowly back to the starting line in 40.0 s. If the "sprint direction" is taken to be positive, what are **(a)** the average sprint velocity, **(b)** the average walking velocity, and **(c)** the average velocity for the complete round trip?

PICTURE THE PROBLEM

In our sketch we set up a coordinate system with the sprint going in the positive *x* direction, as described in the problem. For convenience, we choose the origin to be at the starting line. The finish line, then, is at $x = 50.0$ m.



REASONING AND STRATEGY

In each part of the problem we are asked for the average velocity, and we are given information for times and distances. All that is needed, then, is to determine $\Delta x = x_f - x_i$ and $\Delta t = t_f - t_i$ in each case, and to apply Equation 2-3 $\left(v_{av} = \frac{\Delta x}{\Delta t}\right)$

Known Sprint distance = 50.0 m; sprint time = 8.00 s; walking distance = 50.0 m; walking time = 40.0 s.

Unknown **(a)** Average sprint velocity, $v_{av} = ?$ **(b)** Average walking velocity, $v_{av} = ?$ **(c)** Average velocity for round trip, $v_{av} = ?$

SOLUTION

Part (a)

1. Apply $v_{av} = \frac{\Delta x}{\Delta t}$ to the sprint, with $x_f = 50.0\text{ m}$, $x_i = 0$, $t_f = 8.00\text{ s}$, and $t_i = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{50.0\text{ m} - 0}{8.00\text{ s} - 0} = \frac{50.0}{8.00}\text{ m/s} = 6.25\text{ m/s}$$

Part (b)

2. Apply $v_{av} = \frac{\Delta x}{\Delta t}$ to the walk. In this case, $x_f = 0$, $x_i = 50.0\text{ m}$, $t_f = 48.0\text{ s}$, and $t_i = 8.00\text{ s}$:

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 50.0\text{ m}}{48.0\text{ s} - 8.00\text{ s}} = -\frac{50.0}{40.0}\text{ m/s} = -1.25\text{ m/s}$$

Part (c)

3. For the round trip, $x_f = x_i = 0$; thus $\Delta x = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{0}{48.0\text{ s}} = 0$$

INSIGHT

The sign of the velocities in parts (a) and (b) indicates the direction of motion: positive for motion to the right, negative for motion to the left. In addition, notice that the average *speed* for the entire 100.0-m trip $(100.0\text{ m}/48.0\text{ s} = 2.08\text{ m/s})$ is nonzero, even though the average velocity vanishes.

PRACTICE PROBLEM

If the average velocity during the walk is -1.50 m/s, how much time does it take the athlete to walk back to the starting line? [**Answer:** $\Delta t = \Delta x/v_{av} = (-50.0\text{ m})/(-1.50\text{ m/s}) = 33.3\text{ s}$]

Some related homework problems: Problem 9, Problem 14, Problem 16

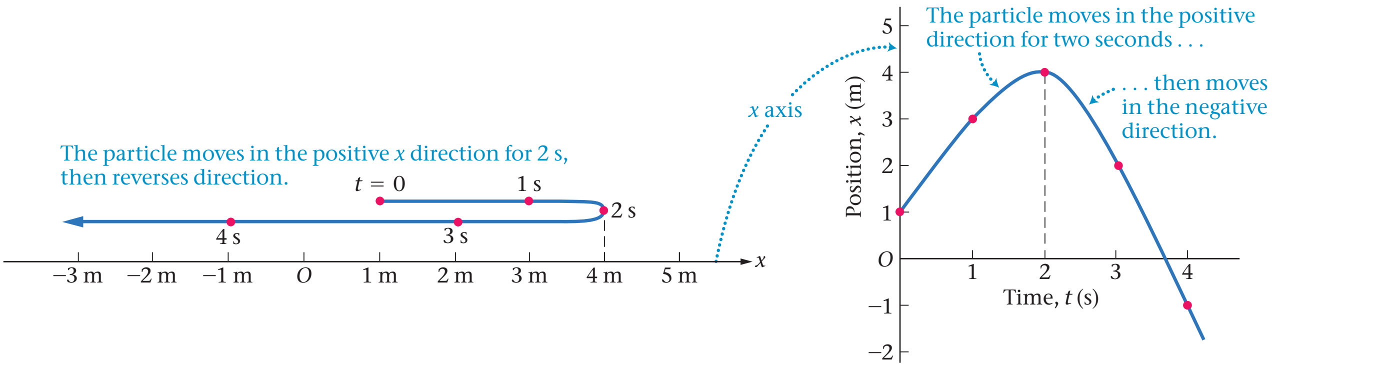
TABLE 2-1 Time and Position Values for Figure 2-3

t (s)	*x* (m)
0	1
1	3
2	4
3	2
4	−1

Graphical Interpretation of Average Velocity

It's often useful to "visualize" a particle's motion by sketching its position as a function of time. For example, suppose a particle moves back and forth along the *x* axis, with the positions and times listed in Table 2-1. This data is plotted in **FIGURE 2-3 (a)**, which is certainly a better way to "see" the motion than a table of numbers.

Even so, this way of showing a particle's position and time is a bit messy, so let's replot the same information with a different type of graph. In **FIGURE 2-3 (b)** we again plot the motion shown in Figure 2-3 (a), but this time with the vertical axis representing the position, *x*, and the horizontal axis representing time, *t*. This is referred to as an ***x*-versus-*t* graph**, and it makes it much easier to visualize a particle's motion, as we shall see.



(a) The particle's path shown on a coordinate axis

(b) The same path as a graph of position x versus time t

▲ **FIGURE 2-3 Two ways to visualize one-dimensional motion (a)** Position plotted for different times. Although the path is shown with a "U" shape for clarity, the particle actually moves straight back and forth along the x axis. **(b)** Plot of position (vertical) versus time (horizontal).

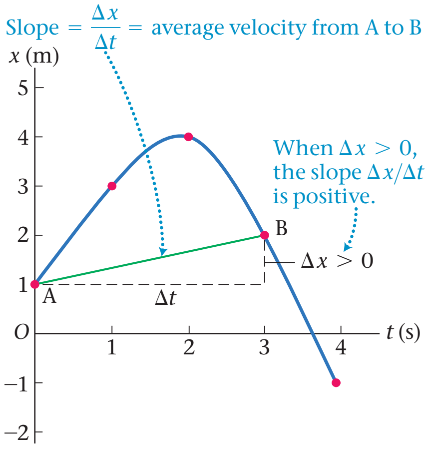
Slope Is Equal to Average Velocity An x-versus-t plot leads to a particularly useful interpretation of average velocity. To see how, suppose you would like to know the average velocity of the particle in Figure 2-3 from $t = 0$ to $t = 3$ s. From our definition of average velocity in Equation 2-3, we know that $v_{av} = \Delta x/\Delta t$, or $v_{av} = (2\text{ m} - 1\text{ m})/(3\text{ s} - 0) = +0.3\text{ m/s}$ for this particle. To relate this to the x-versus-t plot, draw a straight line connecting the position at $t = 0$ (call this point A) and the position at $t = 3$ s (point B). The result is shown in **FIGURE 2-4 (a)**.

The slope of the straight line from A to B is equal to the rise over the run, which in this case is $\Delta x/\Delta t$. But $\Delta x/\Delta t$ is the average velocity. Thus, we conclude the following:

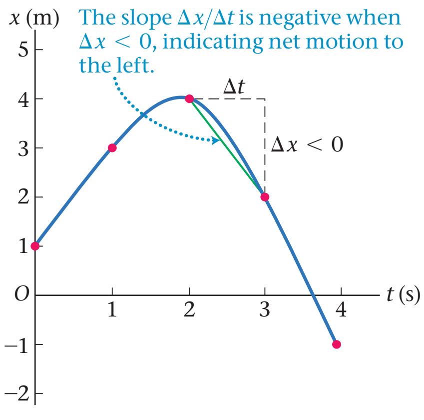
> The *slope* of a line connecting two points on an x-versus-t plot is equal to the *average velocity* during that time interval.

As an additional example, let's calculate the average velocity of this particle between times $t = 2$ s and $t = 3$ s. A line connecting the corresponding points is shown in **FIGURE 2-4 (b)**. The first thing we notice about this line is that it has a negative slope; thus $v_{av} < 0$ and the particle is moving to the left. We can also see that this line is inclined more steeply than the line in Figure 2-4 (a); hence the magnitude of its slope (its speed) is greater. In fact, if we calculate the slope of this line we find that $v_{av} = -2$ m/s for this time interval.

Thus, connecting points on an x-versus-t plot gives an immediate "feeling" for the average velocity over a given time interval. This type of graphical analysis will be particularly useful in the next section.



(a) Average velocity between $t = 0$ and $t = 3$ s



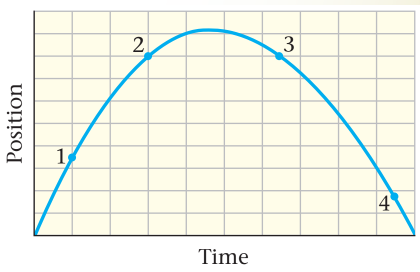
(b) Average velocity between $t = 2$ s and $t = 3$ s

▲ **FIGURE 2-4 Average velocity on an *x*-versus-*t* graph** The slope of a straight line between any two points on an x-versus-t graph equals the average velocity between those points. Positive slopes indicate net motion to the right; negative slopes indicate net motion to the left.

Enhance Your Understanding

(Answers given at the end of the chapter)

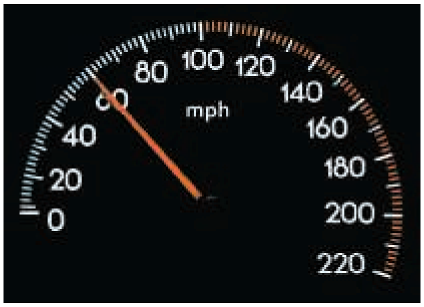
2. The position of an object as a function of time is shown in **FIGURE 2-5**. For the time intervals between **(a)** points 1 and 2, **(b)** points 2 and 3, and **(c)** points 3 and 4, state whether the average velocity is positive, negative, or zero.



▲ **FIGURE 2-5**

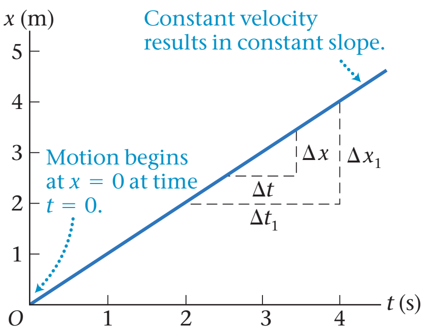
Section Review

- Average speed is distance divided by time.
- Average velocity is displacement divided by time; $v_{av} = \dfrac{\Delta x}{\Delta t}$.
- The slope of a line connecting two points on an x-versus-t plot is the average velocity between those two points.



▲ **FIGURE 2-6** A speedometer indicates a car's instantaneous speed but gives no information about its *direction*. Thus, the speedometer is truly a "speed meter," not a velocity meter.

Big Idea 2 Velocity is the rate of change of position with time, and speed is the magnitude of velocity. Velocity has a magnitude and a direction, and hence it is a vector; speed has only a magnitude, and hence it is a scalar.



▲ **FIGURE 2-7 Constant velocity corresponds to constant slope on an *x*-versus-*t* graph** The slope $\Delta x_1/\Delta t_1$ is equal to (4 m − 2 m)/(4 s − 2 s) = (2 m)/(2 s) = 1 m/s. Because the *x*-versus-*t* plot is a straight line, the slope $\Delta x/\Delta t$ is also equal to 1 m/s for any value of Δt.

TABLE 2-2 *x*-versus-*t* Values for Figure 2-8

t (s)	*x* (m)
0	0
0.25	9.85
0.50	17.2
0.75	22.3
1.00	25.6
1.25	27.4
1.50	28.1
1.75	28.0
2.00	27.4

2-3 Instantaneous Velocity

Though average velocity is a useful way to characterize motion, it can miss a lot. For example, suppose you travel by car on a long, straight highway, covering 92 mi in 2.0 hours. Your average velocity is 46 mi/h. Even so, there may have been only a few times during the trip when you were actually driving at 46 mi/h.

To have a more accurate representation of your trip, you should average your velocity over shorter periods of time. If you calculate your average velocity every 15 minutes, you have a better picture of what the trip was like. An even better, more realistic picture of the trip is obtained if you calculate the average velocity every minute or every second. Ideally, when you deal with the motion of any particle, it's desirable to know the velocity of the particle at each *instant* of time.

This idea of a velocity corresponding to an instant of time is what is meant by the **instantaneous velocity**. Mathematically, we define the instantaneous velocity as follows:

Definition: Instantaneous Velocity, *v*

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad \text{2-4}$$

SI unit: meter per second, m/s

In this expression the notation $\lim_{\Delta t \to 0}$ means "evaluate the average velocity, $\Delta x/\Delta t$, over shorter and shorter time intervals, approaching zero in the limit." Notice that the instantaneous velocity can be positive, negative, or zero, just like the average velocity—and just like the average velocity, the instantaneous velocity is a one-dimensional vector. The magnitude of the instantaneous velocity is called the **instantaneous speed**. In a car, the speedometer gives a reading of the vehicle's instantaneous speed. **FIGURE 2-6** shows a speedometer reading 60 mi/h.

Calculating the Instantaneous Velocity

As Δt becomes smaller, Δx becomes smaller as well, but the ratio $\Delta x/\Delta t$ approaches a constant value. To see how this works, consider first the simple case of a particle moving with a constant velocity of +1 m/s. If the particle starts at $x = 0$ at $t = 0$, then its position at $t = 1$ s is $x = 1$ m, its position at $t = 2$ s is $x = 2$ m, and so on. Plotting this motion in an *x*-versus-*t* plot gives a straight line, as shown in **FIGURE 2-7**.

Now, suppose we want to find the instantaneous velocity at $t = 3$ s. To do so, we calculate the average velocity over small intervals of time centered at 3 s, and let the time intervals become arbitrarily small, as shown in the Figure. Because the *x*-versus-*t* plot is a straight line, it's clear that $\Delta x/\Delta t = \Delta x_1/\Delta t_1$, no matter how small the time interval Δt. As Δt becomes smaller, so does Δx, but the ratio $\Delta x/\Delta t$ is simply the slope of the line, 1 m/s. Thus, the instantaneous velocity at $t = 3$ s is 1 m/s.

Of course, in this example the instantaneous velocity is 1 m/s for any instant of time, not just $t = 3$ s. Therefore:

> When the velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.

In general, a particle's velocity varies with time, and the *x*-versus-*t* plot is not a straight line. An example is shown in **FIGURE 2-8**, with the corresponding numerical values of *x* and *t* given in Table 2-2.

In this case, what is the instantaneous velocity at, say, $t = 1.00$ s? As a first approximation, let's calculate the average velocity for the time interval from $t_i = 0$ to $t_f = 2.00$ s. Notice that this time interval is centered at $t = 1.00$ s. From Table 2-2 we see that $x_i = 0$ and $x_f = 27.4$ m; thus $v_{av} = 13.7$ m/s. The corresponding straight line connecting these two points is the lowest straight line in Figure 2-8.

The next three lines, in upward progression, refer to time intervals from 0.250 s to 1.75 s, 0.500 s to 1.50 s, and 0.750 s to 1.25 s, respectively. The corresponding average velocities, given in Table 2-3, are 12.1 m/s, 10.9 m/s, and 10.2 m/s. Table 2-3 also gives results for even smaller time intervals. In particular, for the interval from 0.900 s to 1.10 s

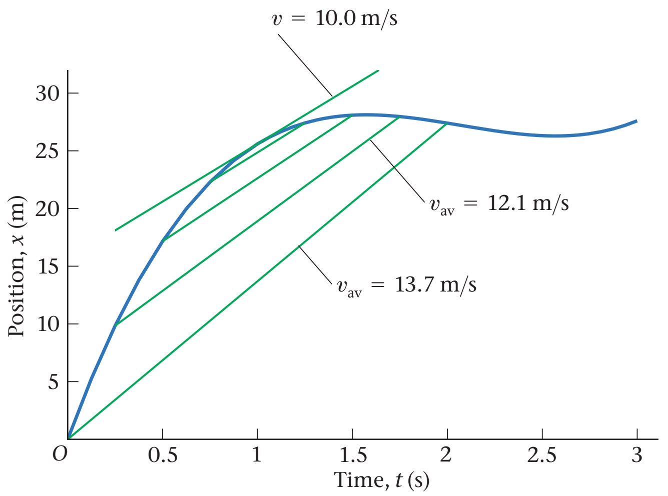


FIGURE 2-8 Instantaneous velocity An x-versus-t plot for motion with variable velocity. The instantaneous velocity at $t = 1$ s is equal to the slope of the tangent line at that time. The average velocity for a small time interval centered on $t = 1$ s approaches the instantaneous velocity at $t = 1$ s as the time interval goes to zero.

TABLE 2-3 Calculating the Instantaneous Velocity at $t = 1$ s for Figure 2-8

t_i (s)	t_f (s)	Δt (s)	x_i (m)	x_f (m)	Δx (m)	$v_{av} = \Delta x/\Delta t$ (m/s)
0	2.00	2.00	0	27.4	27.4	13.7
0.250	1.75	1.50	9.85	28.0	18.2	12.1
0.500	1.50	1.00	17.2	28.1	10.9	10.9
0.750	1.25	0.50	22.3	27.4	5.10	10.2
0.900	1.10	0.20	24.5	26.5	2.00	10.0
0.950	1.05	0.10	25.1	26.1	1.00	10.0

the average velocity is 10.0 m/s. Smaller intervals also give 10.0 m/s. Thus, we can conclude that the instantaneous velocity at $t = 1.00$ s is $v = 10.0$ m/s.

Tangent Lines and the Instantaneous Velocity The uppermost straight line in Figure 2-8 is the tangent line to the x-versus-t curve at the time $t = 1.00$ s; that is, it is the line that touches the curve at just a single point. Its slope is 10.0 m/s. Clearly, the average-velocity lines have slopes that approach the slope of the tangent line as the time intervals become smaller. This is an example of the following general result:

> The instantaneous velocity at a given time is equal to the slope of the tangent line at that point on an x-versus-t graph.

Thus, a visual inspection of an x-versus-t graph gives information not only about the location of a particle, but also about its velocity.

CONCEPTUAL EXAMPLE 2-5 INSTANTANEOUS VELOCITY

Referring to Figure 2-8, is the instantaneous velocity at $t = 0.500$ s **(a)** greater than, **(b)** less than, or **(c)** the same as the instantaneous velocity at $t = 1.00$ s?

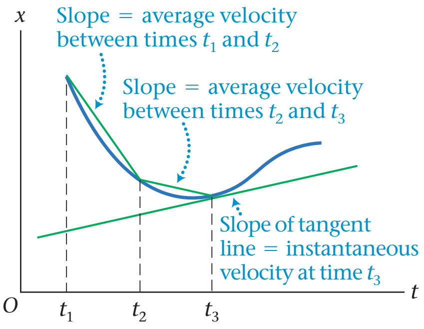
REASONING AND DISCUSSION

From the x-versus-t graph in Figure 2-8 it is clear that the slope of a tangent line drawn at $t = 0.500$ s is greater than the slope of the tangent line at $t = 1.00$ s. It follows that the particle's velocity at 0.500 s is greater than its velocity at 1.00 s.

ANSWER

(a) The instantaneous velocity is greater at $t = 0.500$ s.

In the remainder of the book, when we say *velocity* it is to be understood that we mean *instantaneous* velocity. If we want to refer to the average velocity, we will specifically say average velocity.



▲ **FIGURE 2-9 Graphical interpretation of average and instantaneous velocity** Average velocities correspond to the slope of straight-line segments connecting different points on an *x*-versus-*t* graph. Instantaneous velocities are given by the slope of the tangent line at a given time.

Graphical Interpretation of Average and Instantaneous Velocity

Let's summarize the graphical interpretations of average and instantaneous velocity on an *x*-versus-*t* graph:

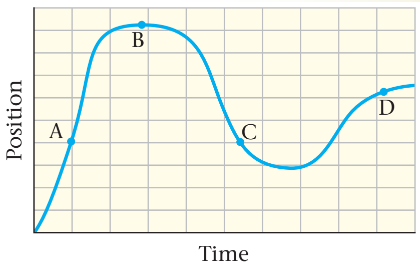
- Average velocity is the slope of the straight line connecting two points corresponding to a given time interval.
- Instantaneous velocity is the slope of the tangent line at a given instant of time.

These relationships are illustrated in **FIGURE 2-9**.

Enhance Your Understanding

(Answers given at the end of the chapter)

3. **FIGURE 2-10** shows the position-versus-time graph for an object. Rank the instantaneous velocity of the object at points A, B, C, and D in order of increasing velocity, from most negative to most positive. Indicate ties where appropriate.



▲ **FIGURE 2-10**

Section Review

- Instantaneous velocity is the limit of average velocity over shorter and shorter time intervals; $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$. The magnitude of the instantaneous velocity is the instantaneous speed.
- The instantaneous velocity on an *x*-versus-*t* plot is the slope of a tangent line at a given time.

2-4 Acceleration

Just as velocity is the rate of change of *displacement* with time, **acceleration** is the rate of change of *velocity* with time. Thus, an object accelerates whenever its velocity *changes,* no matter what the change—it accelerates when its velocity increases, it accelerates when its velocity decreases. Of all the concepts discussed in this chapter, perhaps none is more central to physics than acceleration. Galileo, for example, showed that falling bodies move with constant acceleration. Newton showed that acceleration and force are directly related, as we shall see in Chapter 5. Thus, it is particularly important to have a clear, complete understanding of acceleration before leaving this chapter.

Average Acceleration We begin with the definition of **average acceleration**, which is the change in velocity divided by the change in time:

Definition: Average Acceleration, a_{av}

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad \text{2-5}$$

SI unit: meter per second per second, m/s^2

Notice that the dimensions of average acceleration are the dimensions of velocity per time, or (meters per second) per second:

$$\frac{\text{meters per second}}{\text{second}} = \frac{m/s}{s} = \frac{m}{s^2}$$

This is generally spoken as "meters per second squared." For example, the acceleration of gravity on the Earth's surface is approximately 9.81 m/s^2, which means that the velocity of a falling object changes by 9.81 meters per second (m/s) every second (s). In addition, we see that the average acceleration can be positive, negative, or zero. In fact, it is a one-dimensional vector, just like displacement, average velocity, and instantaneous velocity. Typical magnitudes of acceleration are given in Table 2-4. A real-world example of linear acceleration is shown in FIGURE 2-11.

▲ **FIGURE 2-11** The space shuttle *Columbia* accelerates upward on the initial phase of its journey into orbit. The astronauts on board experienced an approximately linear acceleration that may have been as great as 20 m/s^2.

EXERCISE 2-6 AVERAGE ACCELERATION

a. A certain car can go from 0 to 60.0 mi/h in 7.40 s. What is the average acceleration of this car in meters per second squared?

b. An airplane has an average acceleration of 2.19 m/s^2 during takeoff. If the airplane starts at rest, how much time does it take for it to reach a speed of 174 mi/h?

REASONING AND SOLUTION

(a) Average acceleration is the change in velocity divided by the elapsed time, $a_{av} = \Delta v/\Delta t$.

(b) The equation for average acceleration can be rearranged to solve for the elapsed time, $\Delta t = \Delta v/a_{av}$.

a. $\text{average acceleration} = a_{av} = (60.0\ \text{mi/h})/(7.40\ \text{s})$

$$= (60.0\ \text{mi/h})\left(\frac{1609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)/(7.40\ \text{s})$$

$$= (26.8\ \text{m/s})/(7.40\ \text{s}) = 3.62\ \text{m/s}^2$$

b. $\Delta t = \Delta v/a_{av} = (174\ \text{mi/h})\left(\frac{1609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)/(2.19\ \text{m/s}^2)$

$$= (77.8\ \text{m/s})/(2.19\ \text{m/s}^2) = 35.5\ \text{s}$$

Big Idea 3 Acceleration is the rate of change of velocity with time. Acceleration has both a magnitude and a direction, and hence it is a vector.

Instantaneous Acceleration

We considered the limit of smaller and smaller time intervals to find an instantaneous velocity, and we can do the same to define the **instantaneous acceleration**:

Definition: Instantaneous Acceleration, *a*

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{2-6}$$

SI unit: meter per second per second, m/s^2

As you might expect, the instantaneous acceleration is a vector, just like the average acceleration, and its direction in one dimension is given by its sign. For simplicity, when we say acceleration in this text, we are referring to the instantaneous acceleration.

One final note before we go on to some examples. If the acceleration is constant, it has the same value at all times. Therefore:

> When acceleration is constant, the instantaneous and average accelerations are the same.

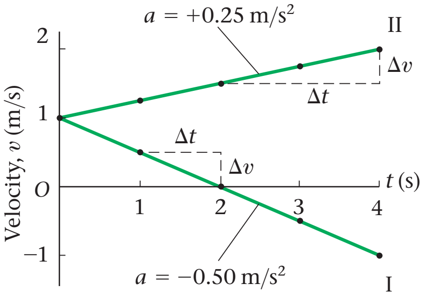
We shall make use of this fact when we return to the special case of constant acceleration in the next section.

TABLE 2-4 Typical Accelerations (m/s^2)

Ultracentrifuge	3×10^6
Bullet fired from a rifle	4.4×10^5
Batted baseball	3×10^4
Click beetle righting itself	400
Acceleration required to deploy air bags	60
Bungee jump	30
High jump	15
Acceleration of gravity on Earth	9.81
Emergency stop in a car	8
Airplane during takeoff	5
An elevator	3
Acceleration of gravity on the Moon	1.62

Graphical Interpretation of Acceleration

To see how acceleration can be interpreted graphically, suppose a particle has a constant acceleration of $-0.50\ m/s^2$. This means that the velocity of the particle *decreases* by 0.50 m/s each second. Thus, if its velocity is 1.0 m/s at $t = 0$, then at $t = 1$ s its velocity is 0.50 m/s, at $t = 2$ s its velocity is 0, at $t = 3$ s its velocity is -0.50 m/s, and so on. This is illustrated by curve I in FIGURE 2-12, where we see that a plot of v versus



► **FIGURE 2-12** ***v*-versus-*t* plots for motion with constant acceleration** Curve I represents the movement of a particle with constant acceleration $a = -0.50\ m/s^2$. Curve II represents the motion of a particle with constant acceleration $a = +0.25\ m/s^2$.

t results in a straight line with a negative slope. Curve II in Figure 2-12 has a positive slope, corresponding to a constant acceleration of $+0.25\ \text{m/s}^2$. Thus, in terms of a v-versus-t plot, a constant acceleration results in a straight line with a slope equal to the acceleration.

CONCEPTUAL EXAMPLE 2-7 SPEED AS A FUNCTION OF TIME

The speed of a particle with the v-versus-t graph shown by curve II in Figure 2-12 increases steadily with time. Consider, instead, a particle whose v-versus-t graph is given by curve I in Figure 2-12. As a function of time, does the speed of this particle **(a)** increase, **(b)** decrease, or **(c)** decrease and then increase?

REASONING AND DISCUSSION

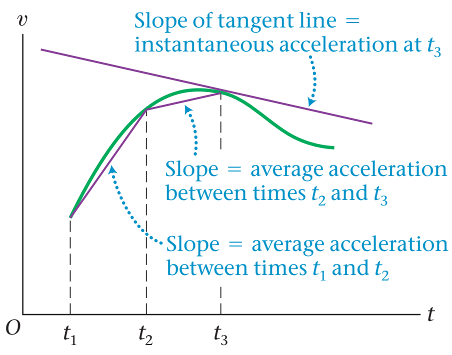
Recall that speed is the *magnitude* of velocity. In curve I of Figure 2-12 the speed starts out at 1.0 m/s, then *decreases* to 0 at $t = 2$ s. After $t = 2$ s the speed *increases* again. For example, at $t = 3$ s the speed is 0.50 m/s, and at $t = 4$ s the speed is 1 m/s.

Did you realize that the particle represented by curve I in Figure 2-12 changes direction at $t = 2$ s? It certainly does. Before $t = 2$ s the particle moves in the positive direction; after $t = 2$ s it moves in the negative direction. At precisely $t = 2$ s the particle is momentarily at rest. However, regardless of whether the particle is moving in the positive direction, moving in the negative direction, or instantaneously at rest, it still has the same constant acceleration. Acceleration has to do only with the way the velocity is *changing* at a given moment.

ANSWER

(c) The speed decreases and then increases.

The graphical interpretations for velocity presented in Figure 2-9 apply equally well to acceleration, with just one small change: Instead of an x-versus-t graph, we use a v-versus-t graph, as in **FIGURE 2-13**. Thus, the average acceleration in a v-versus-t plot is the slope of a straight line connecting points corresponding to two different times. Similarly, the instantaneous acceleration is the slope of the tangent line at a particular time.



▶ **FIGURE 2-13 Graphical interpretation of average and instantaneous acceleration** Average accelerations correspond to the slopes of straight-line segments connecting different points on a v-versus-t graph. Instantaneous accelerations are given by the slope of the tangent line at a given time.

EXAMPLE 2-8 ACCELERATION OF A BICYCLE

A cyclist riding in a straight line has an initial velocity of 3.5 m/s, and accelerates at $-1.0\ \text{m/s}^2$ for 2.0 s. The cyclist then coasts with zero acceleration for 3.0 s, and finally accelerates at $1.5\ \text{m/s}^2$ for an additional 2.0 s. **(a)** What is the final velocity of the cyclist? **(b)** What is the average acceleration of the cyclist for these seven seconds?

PICTURE THE PROBLEM

We begin by sketching a v-versus-t plot for the cyclist. The basic idea is that each interval of constant acceleration is represented by a straight line of the appropriate slope. Therefore, we draw a straight line with the slope $-1.0\ \text{m/s}^2$ from $t = 0$ to $t = 2.0$ s, a line with zero slope from $t = 2.0$ s to $t = 5.0$ s, and a line with the slope $1.5\ \text{m/s}^2$ from $t = 5.0$ s to $t = 7.0$ s. The line connecting the initial and final points determines the average acceleration.

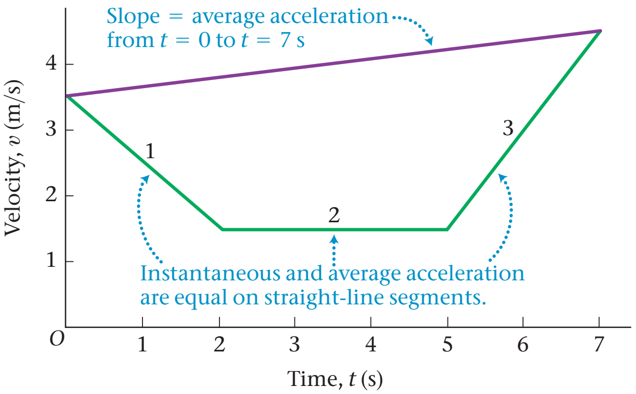
CONTINUED

REASONING AND STRATEGY

During each period of constant acceleration the change in velocity is $\Delta v = a_{av}\Delta t = a\Delta t$.

a. Adding the individual changes in velocity gives the total change, $\Delta v = v_f - v_i$. Because v_i is known, this expression can be solved for the final velocity, $v_f = \Delta v + v_i$.

b. The average acceleration can be calculated using Equation 2-5, $a_{av} = \Delta v/\Delta t$. Notice that Δv has been obtained in part (a), and the total time interval is $\Delta t = 7.0$ s.



Known Initial velocity, $v_i = 3.5$ m/s; accelerations, $a_1 = -1.0\ \text{m/s}^2$, $a_2 = 0$, $a_3 = 1.5\ \text{m/s}^2$.

Unknown **(a)** Final velocity, $v_f = $? **(b)** Average acceleration, $a_{av} = $?

SOLUTION

Part (a)

1. Find the change in velocity during each of the three periods of constant acceleration:

$$\Delta v_1 = a_1\Delta t_1 = (-1.0\ \text{m/s}^2)(2.0\ \text{s}) = -2.0\ \text{m/s}$$
$$\Delta v_2 = a_2\Delta t_2 = (0)(3.0\ \text{s}) = 0$$
$$\Delta v_3 = a_3\Delta t_3 = (1.5\ \text{m/s}^2)(2.0\ \text{s}) = 3.0\ \text{m/s}$$

2. Sum the change in velocity for each period to obtain the total Δv:

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3 = -2.0\ \text{m/s} + 0 + 3.0\ \text{m/s} = 1.0\ \text{m/s}$$

3. Use Δv to find v_f, recalling that $v_i = 3.5$ m/s:

$$\Delta v = v_f - v_i$$
$$v_f = \Delta v + v_i = 1.0\ \text{m/s} + 3.5\ \text{m/s} = 4.5\ \text{m/s}$$

Part (b)

4. The average acceleration is $\Delta v/\Delta t$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{1.0\ \text{m/s}}{7.0\ \text{s}} = 0.14\ \text{m/s}^2$$

INSIGHT

The average acceleration for these seven seconds is *not* the average of the individual accelerations, $-1.0\ \text{m/s}^2$, 0, and $1.5\ \text{m/s}^2$. The reason is that different amounts of time are spent with each acceleration.

PRACTICE PROBLEM

What is the average acceleration of the cyclist between $t = 3.0$ s and $t = 6.0$ s?
[**Answer:** $a_{av} = \Delta v/\Delta t = (3.0\ \text{m/s} - 1.5\ \text{m/s})/(6.0\ \text{s} - 3.0\ \text{s}) = 0.50\ \text{m/s}^2$]

Some related homework problems: Problem 32, Problem 34

Relating the Signs of Velocity and Acceleration to the Change in Speed In one dimension, nonzero velocities and accelerations are either positive or negative, depending on whether they point in the positive or negative direction of the coordinate system chosen. Thus, the velocity and acceleration of an object may have the same or opposite signs. (Of course, in two or three dimensions the relationship between velocity and acceleration can be much more varied, as we shall see in the next several chapters.) This leads to the following two possibilities in one dimension:

- When the velocity and acceleration of an object have the same sign (point in the same direction), the speed of the object increases.
- When the velocity and acceleration of an object have opposite signs (point in opposite directions), the speed of the object decreases.

These two possibilities are illustrated in **FIGURE 2-14**. Notice that when a particle's speed increases, it means either that its velocity becomes more positive, as in **FIGURE 2-14 (a)**, or more negative, as in **FIGURE 2-14 (d)**. In either case, it is the magnitude of the velocity—the speed—that increases. **FIGURE 2-15** shows an example where the velocity and the acceleration are definitely in the same direction.

When a particle's speed decreases, it is often said to be *decelerating*. A common misconception is that deceleration implies a negative acceleration. This is not true. Deceleration can be caused by a positive or a negative acceleration, depending on the direction of the initial velocity. For example, the car in **FIGURE 2-14 (b)** has a positive velocity and

PHYSICS IN CONTEXT
Looking Ahead

The distinctions developed in this chapter between velocity and acceleration play a key role in our understanding of Newton's laws of motion in Chapters 5 and 6.

Big Idea 4 Speed increases when velocity and acceleration are in the same direction; speed decreases when velocity and acceleration are in opposite directions.

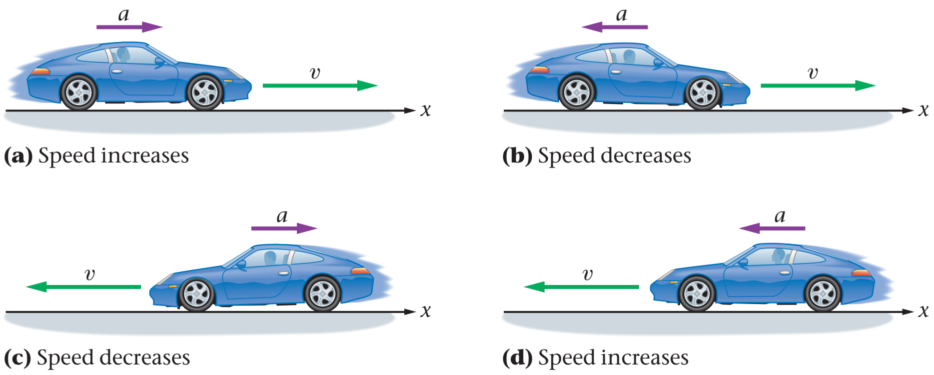


FIGURE 2-14 Cars accelerating or decelerating A car's speed increases when its velocity and acceleration point in the same direction, as in cases **(a)** and **(d)**. When the velocity and acceleration point in opposite directions, as in cases **(b)** and **(c)**, the car's speed decreases.

FIGURE 2-15 The winner of this race was traveling at a speed of 313.91 mi/h at the end of the quarter-mile course. The winning time was just 4.607 s, and hence the *average* acceleration during this race was approximately three times the acceleration of gravity (which is covered in Section 2-7).

a negative acceleration, while the car in **FIGURE 2-14 (c)** has a negative velocity and a positive acceleration. In both cases, the speed of the car decreases. Again, all that is required for deceleration in one dimension is that the velocity and acceleration have *opposite signs*; that is, they must point in *opposite directions*, as in parts (b) and (c) of Figure 2-14.

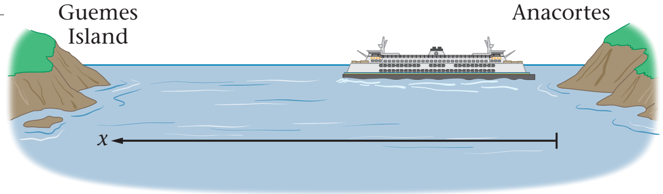
Velocity-versus-time plots for the four situations shown in Figure 2-14 are presented in **FIGURE 2-16**. In each of the four plots in Figure 2-16 we assume constant acceleration. Be sure to understand clearly the connection between the v-versus-t plots in Figure 2-16 and the corresponding physical motions indicated in Figure 2-14.

EXAMPLE 2-9 THE FERRY DOCKS

A ferry makes a short run between two docks: one in Anacortes, Washington, the other on Guemes Island. As the ferry approaches the Guemes dock (traveling in the positive x direction), its speed is 6.8 m/s. **(a)** If the ferry slows to a stop in 13.3 s, what is its average acceleration? **(b)** As the ferry returns to the Anacortes dock, its speed is 6.1 m/s. If it comes to rest in 12.9 s, what is its average acceleration?

PICTURE THE PROBLEM

Our sketch shows the locations of the two docks and the positive direction indicated in the problem. The distance between docks is not given, nor is it needed.



REASONING AND STRATEGY

We are given the initial and final velocities (the ferry comes to a stop in each case, so its final speed is zero) and the relevant times. Therefore, we can find the average acceleration using $a_{av} = \Delta v/\Delta t$, being careful to get the signs right.

Known **(a)** Initial velocity, $v_i = 6.8$ m/s; time to stop, $\Delta t = 13.3$ s. **(b)** Initial velocity, $v_i = -6.1$ m/s; time to stop, $\Delta t = 12.9$ s.

Unknown **(a)** Average acceleration, $a_{av} = ?$ **(b)** Average acceleration, $a_{av} = ?$

SOLUTION

Part (a)

1. Calculate the average acceleration, noting that $v_i = 6.8$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 6.8\ \text{m/s}}{13.3\ \text{s}} = -0.51\ \text{m/s}^2$$

CONTINUED

Part (b)

2. In this case, $v_i = -6.1$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - (-6.1 \text{ m/s})}{12.9 \text{ s}} = 0.47 \text{ m/s}^2$$

INSIGHT

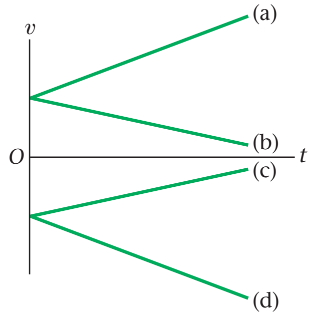
In each case, the acceleration of the ferry is opposite in sign to its velocity; therefore the ferry decelerates.

PRACTICE PROBLEM

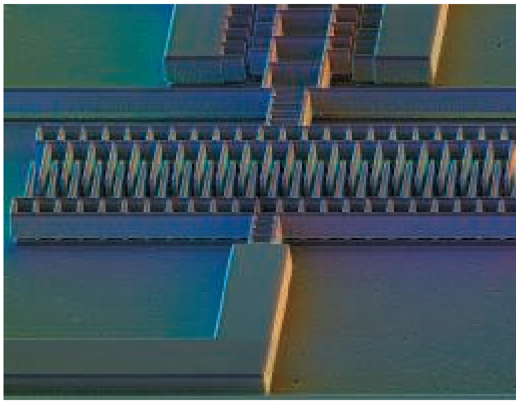
When the ferry leaves Guemes Island, its speed increases from 0 to 4.8 m/s in 9.05 s. What is its average acceleration?
[**Answer:** $a_{av} = -0.53$ m/s^2]

Some related homework problems: Problem 30, Problem 36

RWP* The ability to detect and measure acceleration has become an important capability of many new technologies. A device that measures acceleration is called an *accelerometer*. In recent years a number of accelerometers have been developed that are contained in tiny integrated circuit chips, such as the one shown in **FIGURE 2-17**. These accelerometers allow devices such as smartphones, video game consoles, automotive air bag sensors, aircraft flight stabilization systems, and numerous others to detect acceleration and respond accordingly.



▲ **FIGURE 2-16** ***v*-versus-*t* plots with constant acceleration** Four plots of *v* versus *t* corresponding to the four situations shown in Figure 2-14. Notice that the speed increases in cases **(a)** and **(d)**, but decreases in cases **(b)** and **(c)**.



▲ **FIGURE 2-17 Integrated circuit accelerometer** Microscopic image of the ADXL330, the accelerometer incorporated into the Nintendo *Wii* remote controller. When the chip is accelerated in any direction, it generates an electrical signal that is proportional to the magnitude of the acceleration. It also indicates the direction of acceleration.

Enhance Your Understanding (Answers given at the end of the chapter)

4. At a certain time, object 1 has an initial velocity of −2 m/s and an acceleration of 3 m/s^2. At the same time, object 2 has an initial velocity of 5 m/s and an acceleration of −1 m/s^2. **(a)** Is the speed of object 1 increasing or decreasing? Explain. **(b)** Is the velocity of object 1 increasing or decreasing? Explain. **(c)** Is the speed of object 2 increasing or decreasing? Explain. **(d)** Is the velocity of object 2 increasing or decreasing? Explain.

Section Review

- The average acceleration is the change in velocity divided by the time; $a_{av} = \frac{\Delta v}{\Delta t}$.
- The instantaneous acceleration is the average acceleration over shorter and shorter time intervals; $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$.
- Average acceleration is the slope between two points on a v-versus-t plot; instantaneous acceleration is the slope of a tangent line on a v-versus-t plot.

2-5 Motion with Constant Acceleration

In this section, we derive equations that describe the motion of particles moving with **constant acceleration**. These "equations of motion" can be used to describe a wide range of everyday phenomena. For example, in an idealized world with no air resistance, falling bodies have constant acceleration.

Velocity as a Function of Time As mentioned in the previous section, if a particle has constant acceleration—that is, the same acceleration at every instant of time—then its instantaneous acceleration, a, is equal to its average acceleration, a_{av}. Recalling the definition of average acceleration (Equation 2-5), we have

$$a_{av} = \frac{v_f - v_i}{t_f - t_i} = a$$

*Real World Physics applications are denoted by the acronym RWP.

The initial and final times may be chosen arbitrarily in this equation. For example, let $t_i = 0$ for the initial time, and let $v_i = v_0$ denote the velocity at time zero.* For the final time and velocity we drop the subscripts to simplify the notation; thus we let $t_f = t$ and $v_f = v$. With these identifications we have

$$a_{av} = \frac{v - v_0}{t - 0} = a$$

With a slight rearrangement we find

$$v - v_0 = a(t - 0) = at$$

This yields our first equation of motion:

Constant-Acceleration Equation of Motion: Velocity as a Function of Time

$$v = v_0 + at \qquad \text{2-7}$$

PHYSICS IN CONTEXT
Looking Back

We are careful to check the dimensional consistency of our equations in this chapter. This concept was introduced in Chapter 1.

Equation 2-7 describes a straight line on a v-versus-t plot. The line crosses the velocity axis at the value v_0 and has a slope a, in agreement with the graphical interpretations discussed in the previous section. For example, in curve I of Figure 2-12, the equation of motion is $v = v_0 + at = (1\ \text{m/s}) + (-0.5\ \text{m/s}^2)t$. Also, notice that $(-0.5\ \text{m/s}^2)t$ has the units $(\text{m/s})^2\ (\text{s}) = \text{m/s}$; thus each term in Equation 2-7 has the same dimensions (as it must to be a valid physical equation).

EXERCISE 2-10 VELOCITY WITH CONSTANT ACCELERATION

A ball is thrown straight upward with an initial velocity of +7.3 m/s. If the acceleration of the ball is downward, with the value $-9.81\ \text{m/s}^2$, find the velocity of the ball after **(a)** 0.45 s and **(b)** 0.90 s.

REASONING AND SOLUTION

For constant acceleration, velocity is related to acceleration and time by Equation 2-7 ($v = v_0 + at$). Given the initial velocity ($v_0 = 7.3\ \text{m/s}$) and the acceleration ($a = -9.81\ \text{m/s}^2$), the final velocity can be found by substituting the desired time.

a. Substituting $t = 0.45$ s in $v = v_0 + at$ yields

$$v = 7.3\ \text{m/s} + (-9.81\ \text{m/s}^2)(0.45\ \text{s}) = 2.9\ \text{m/s}$$

The ball is still moving upward at this time.

b. Similarly, using $t = 0.90$ s in $v = v_0 + at$ gives

$$v = 7.3\ \text{m/s} + (-9.81\ \text{m/s}^2)(0.90\ \text{s}) = -1.5\ \text{m/s}$$

The ball is moving downward (negative direction) at this time.

Position as a Function of Time and Velocity How far does a particle move in a given time if its acceleration is constant? To answer this question, recall the definition of average velocity:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Using the same identifications given previously for initial and final times, and letting $x_i = x_0$ and $x_f = x$, we have

$$v_{av} = \frac{x - x_0}{t - 0}$$

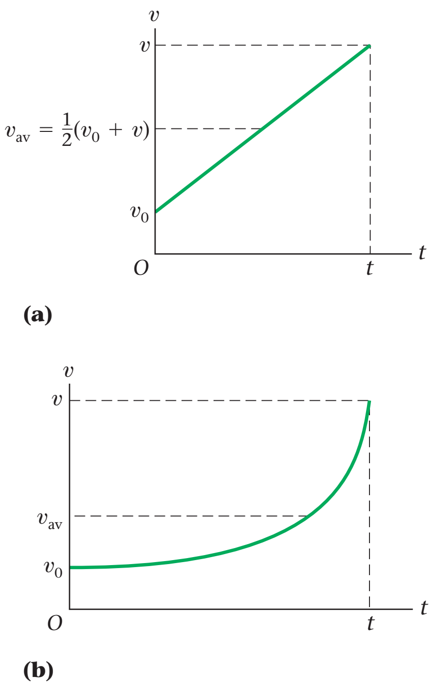
Multiplying through by $(t - 0)$ gives

$$x - x_0 = v_{av}(t - 0) = v_{av}t$$

Finally, a simple rearrangement results in

$$x = x_0 + v_{av}t \qquad \text{2-8}$$

*We often use the subscript i to denote the initial value of a quantity, as in v_i. In the special case where the initial value corresponds to zero time, $t = 0$, we use the subscript 0 to be more specific, as in v_0.



▶ **FIGURE 2-18 Average velocity (a)** When acceleration is constant, the velocity varies linearly with time. As a result, the average velocity, v_{av}, is simply the average of the initial velocity, v_0, and the final velocity, v. **(b)** The velocity curve for nonconstant acceleration is nonlinear. In this case, the average velocity is no longer midway between the initial and final velocities.

Now, Equation 2-8 is fine as it is. In fact, it applies whether the acceleration is constant or not. A more useful expression for the case of constant acceleration is obtained by writing v_{av} in terms of the initial and final velocities. This can be done by referring to **FIGURE 2-18 (a)**. Here the velocity changes linearly (since a is constant) from v_0 at $t = 0$ to v at some later time t. The average velocity during this period of time is simply the average of the initial and final velocities—that is, the sum of the two velocities divided by two:

Constant-Acceleration Equation of Motion: Average Velocity

$$v_{av} = \frac{1}{2}(v_0 + v) \qquad \text{2-9}$$

The average velocity is indicated in the figure. Notice that if the acceleration is not constant, as in **FIGURE 2-18 (b)**, this simple averaging of initial and final velocities is no longer valid.

Substituting the expression for v_{av} from Equation 2-9 into Equation 2-8 yields

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + \frac{1}{2}(v_0 + v)t \qquad \text{2-10}$$

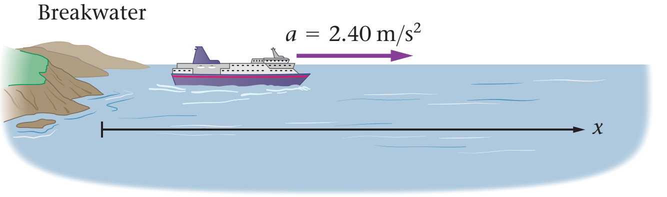
This equation, like Equation 2-7, is valid *only* for constant acceleration. The utility of Equations 2-7 and 2-10 is illustrated in the next Example.

EXAMPLE 2-11 FULL SPEED AHEAD

A boat moves slowly inside a marina (so as not to leave a wake) with a constant speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. **(a)** How fast is the boat moving after accelerating for 5.00 s? **(b)** How far has the boat traveled in these 5.00 s?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the breakwater, and the positive x direction to be the direction of motion. With this choice the initial position is $x_0 = 0$, and the initial velocity is $v_0 = 1.50$ m/s.



REASONING AND STRATEGY

The acceleration is constant, so we can use Equations 2-7 to 2-10. In part (a) we want to relate velocity to time, so we use Equation 2-7, $v = v_0 + at$. In part (b) our knowledge of the initial and final velocities allows us to relate position to time using Equation 2-10, $x = x_0 + \frac{1}{2}(v_0 + v)t$.

Known Velocity at time $t = 0$, $v_0 = 1.50$ m/s; acceleration, $a = 2.40$ m/s^2.

Unknown (a) Velocity at $t = 5.00$ s, $v = ?$ (b) Distance traveled at $t = 5.00$ s, $x = ?$

SOLUTION

Part (a)

1. Use Equation 2-7 ($v = v_0 + at$) to find the final velocity, with $v_0 = 1.50$ m/s and $a = 2.40$ m/s^2:

$$v = v_0 + at = 1.50\text{ m/s} + (2.40\text{ m/s}^2)(5.00\text{ s}) = 1.50\text{ m/s} + 12.0\text{ m/s} = 13.5\text{ m/s}$$

Part (b)

2. Apply Equation 2-10 $\left(x = x_0 + \frac{1}{2}(v_0 + v)t\right)$ to find the distance covered, using the result for v obtained in part (a):

$$x = x_0 + \frac{1}{2}(v_0 + v)t = 0 + \frac{1}{2}(1.50\text{ m/s} + 13.5\text{ m/s})(5.00\text{ s}) = (7.50\text{ m/s})(5.00\text{ s}) = 37.5\text{ m}$$

CONTINUED

INSIGHT

The boat has a constant acceleration between $t = 0$ and $t = 5.00$ s, and hence its velocity-versus-time curve is linear during this time interval. As a result, the average velocity for these 5.00 seconds is the average of the initial and final velocities, $v_{av} = \frac{1}{2}(1.50\text{ m/s} + 13.5\text{ m/s}) = 7.50\text{ m/s}$. Multiplying the average velocity by the time, 5.00 s, gives the distance traveled—which is exactly what Equation 2-10 does in Step 2.

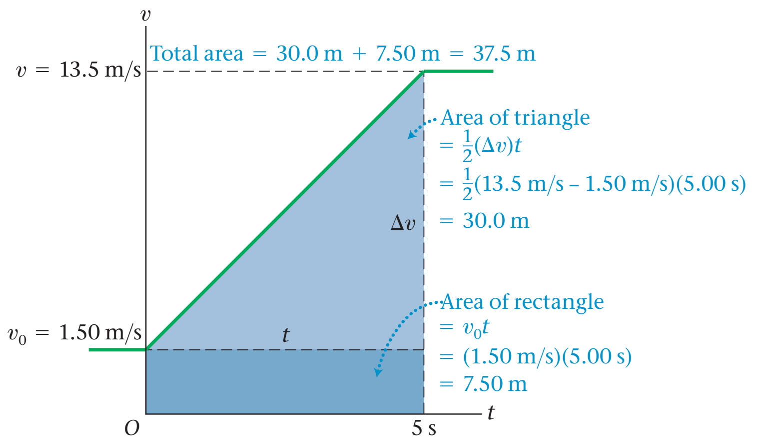
PRACTICE PROBLEM

At what time is the boat's speed equal to 10.0 m/s? [**Answer:** $t = 3.54$ s]

Some related homework problems: Problem 40, Problem 43

Position as a Function of Time and Acceleration The velocity of the boat in Example 2-11 is plotted as a function of time in **FIGURE 2-19**, with the acceleration starting at time $t = 0$ and ending at $t = 5.00$ s. We will now show that the *distance* traveled by the boat from $t = 0$ to $t = 5.00$ s *is equal to the corresponding area under the velocity-versus-time curve*. This is a general result, valid for any velocity curve and any time interval:

> The distance traveled by an object from a time t_1 to a time t_2 is equal to the area under the velocity curve between those two times.



▲ **FIGURE 2-19 Velocity versus time for the boat in Example 2-11** The distance traveled by the boat between $t = 0$ and $t = 5.00$ s is equal to the corresponding area under the velocity curve.

In this case, the area is the sum of the areas of a rectangle and a triangle. The rectangle has a base of 5.00 s and a height of 1.50 m/s, which gives an area of $(5.00\text{ s})(1.50\text{ m/s}) = 7.50\text{ m}$. Similarly, the triangle has a base of 5.00 s and a height of $(13.5\text{ m/s} - 1.50\text{ m/s}) = 12.0\text{ m/s}$, for an area of $\frac{1}{2}(5.00\text{ s})(12.0\text{ m/s}) = 30.0\text{ m}$. Clearly, the total area is 37.5 m, which is the same distance found in Example 2-11.

Staying with Example 2-11 for a moment, let's repeat the calculation of part (b), only this time for the general case. First, we use the final velocity from part (a), calculated with $v = v_0 + at$, in the expression for the average velocity, $v_{av} = \frac{1}{2}(v_0 + v)$. Symbolically, this gives the following result:

$$\frac{1}{2}(v_0 + v) = \frac{1}{2}[v_0 + (v_0 + at)] = v_0 + \frac{1}{2}at \qquad \textit{(constant acceleration)}$$

Next, we substitute this result into Equation 2-10 $\left(x = x_0 + \frac{1}{2}(v_0 + v)t\right)$, which yields

$$x = x_0 + \frac{1}{2}(v_0 + v)t = x_0 + \left(v_0 + \frac{1}{2}at\right)t$$

Simplifying this expression yields the following result:

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad \text{2-11}$$

Here we have an expression for position versus time that is explicitly in terms of the acceleration, a.

Notice that each term in Equation 2-11 has the same dimensions, as they must. For example, the velocity term, v_0t, has the units $(\text{m/s})(\text{s}) = \text{m}$. Similarly, the acceleration term, $\frac{1}{2}at^2$, has the units $(\text{m/s}^2)(\text{s}^2) = \text{m}$.

EXERCISE 2-12 POSITION WITH CONSTANT ACCELERATION

Repeat part (b) of Example 2-11 using Equation 2-11, $x = x_0 + v_0t + \frac{1}{2}at^2$.

REASONING AND SOLUTION

Given the initial position, initial velocity, and acceleration, the position at any given time can be found with $x = x_0 + v_0t + \frac{1}{2}at^2$. In this case, we have

$$x = x_0 + v_0t + \frac{1}{2}at^2 = 0 + (1.50\text{ m/s})(5.00\text{ s}) + \frac{1}{2}(2.40\text{ m/s}^2)(5.00\text{ s})^2 = 37.5\text{ m}$$

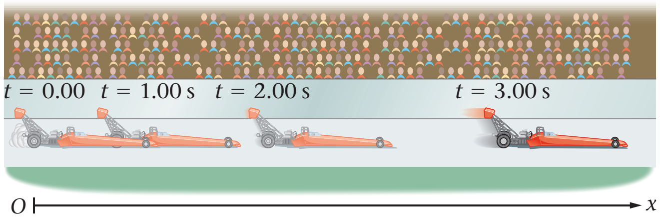
The next Example gives further insight into the physical meaning of Equation 2-11.

EXAMPLE 2-13 PUT THE PEDAL TO THE METAL

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in **(a)** 1.00 s, **(b)** 2.00 s, **(c)** 3.00 s?

PICTURE THE PROBLEM

We set up a coordinate system in which the drag racer starts at the origin and accelerates in the positive x direction. With this choice, it follows that $x_0 = 0$ and $a = +7.40\text{ m/s}^2$. Also, the racer starts from rest, and hence its initial velocity is zero, $v_0 = 0$. Incidentally, the positions of the racer in the sketch have been drawn to scale.



REASONING AND STRATEGY

This problem gives the acceleration, which is constant, and asks for a relationship between position and time. Therefore, we use Equation 2-11, $x = x_0 + v_0t + \frac{1}{2}at^2$.

Known Velocity at time $t = 0$, $v_0 = 0$; acceleration, $a = 7.40\text{ m/s}^2$.
Unknown Distance traveled, $x = ?$, at **(a)** $t = 1.00$ s, **(b)** $t = 2.00$ s, **(c)** $t = 3.00$ s

SOLUTION

Part (a)

1. Evaluate $x = x_0 + v_0t + \frac{1}{2}at^2$ with $a = 7.40\text{ m/s}^2$ and $t = 1.00$ s:

$$x = x_0 + v_0t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}at^2 = \frac{1}{2}at^2$$
$$x = \frac{1}{2}(7.40\text{ m/s}^2)(1.00\text{ s})^2 = 3.70\text{ m}$$

Part (b)

2. From the calculation in part (a), we see that $x = x_0 + v_0t + \frac{1}{2}at^2$ reduces to $x = \frac{1}{2}at^2$ in this situation. Evaluate $x = \frac{1}{2}at^2$ at $t = 2.00$ s:

$$x = \frac{1}{2}at^2 = \frac{1}{2}(7.40\text{ m/s}^2)(2.00\text{ s})^2 = 14.8\text{ m} = 4(3.70\text{ m})$$

Part (c)

3. Evaluate $x = \frac{1}{2}at^2$ at $t = 3.00$ s:

$$x = \frac{1}{2}at^2$$
$$= \frac{1}{2}(7.40\text{ m/s}^2)(3.00\text{ s})^2 = 33.3\text{ m} = 9(3.70\text{ m})$$

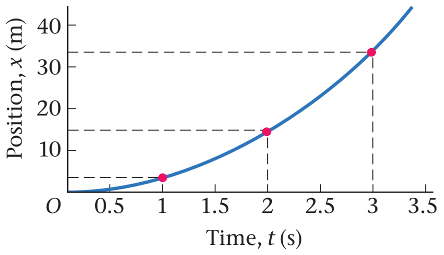
INSIGHT

This Example illustrates one of the key features of accelerated motion—position does not change uniformly with time when an object accelerates. In this case, the distance traveled in the first two seconds is 4 times the distance traveled in the first second, and the distance traveled in the first three seconds is 9 times the distance traveled in the first second. This kind of behavior is a direct result of the fact that x depends on t^2 when the acceleration is nonzero.

PRACTICE PROBLEM

We've seen that in one second the racer travels 3.70 m. How much time does it take for the racer to travel twice this distance, $2(3.70\text{ m}) = 7.40\text{ m}$? [**Answer:** $t = \sqrt{2}\text{ s} = 1.41\text{ s}$]

Some related homework problems: Problem 43, Problem 46



▲ **FIGURE 2-20 Position versus time for Example 2-13** The upward-curving, parabolic shape of this x-versus-t plot indicates a positive, constant acceleration. The dots on the curve show the position of the drag racer in Example 2-13 at the times 1.00 s, 2.00 s, and 3.00 s.

FIGURE 2-20 shows a graph of x versus t for Example 2-13. Notice the parabolic shape of the x-versus-t curve, which is due to the $\frac{1}{2}at^2$ term and is characteristic of constant acceleration. In particular, if acceleration is positive ($a > 0$), then a plot of x versus t curves upward; if acceleration is negative ($a < 0$), a plot of x versus t curves downward. The greater the magnitude of a, the greater the curvature. In contrast, if a particle moves with constant velocity ($a = 0$), the t^2 dependence vanishes, and the x-versus-t plot is a straight line.

Velocity as a Function of Position Our final equation of motion with constant acceleration relates velocity to position. We start by solving for the time, t, in Equation 2-7 ($v = v_0 + at$):

$$v = v_0 + at \quad \text{or} \quad t = \frac{v - v_0}{a}$$

Next, we substitute this result into Equation 2-10, $x = x_0 + \frac{1}{2}(v_0 + v)t$, thus eliminating t:

$$x = x_0 + \frac{1}{2}(v_0 + v)t = x_0 + \frac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right)$$

Noting that $(v_0 + v)(v - v_0) = v_0 v - v_0{}^2 + v^2 - v v_0 = v^2 - v_0{}^2$, we have

$$x = x_0 + \frac{v^2 - v_0{}^2}{2a}$$

Finally, a straightforward rearrangement of terms yields the following:

Constant-Acceleration Equation of Motion: Velocity in Terms of Displacement

$$v^2 = v_0{}^2 + 2a(x - x_0) = v_0{}^2 + 2a\Delta x \qquad \text{2-12}$$

This equation allows us to relate the velocity at one position to the velocity at another position, without knowing how much time is involved. The next Example shows how Equation 2-12 is used.

PHYSICS IN CONTEXT
Looking Ahead

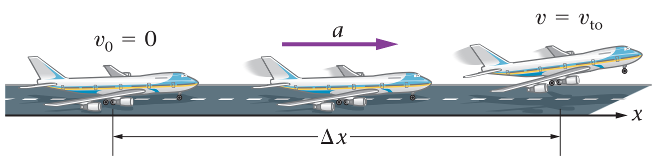
The equations developed for motion with constant acceleration in this chapter are used again with slightly different symbols when we study motion in two dimensions in Chapter 4 and rotational motion in Chapter 10.

EXAMPLE 2-14 TAKEOFF DISTANCE FOR AN AIRLINER

RWP Jets at John F. Kennedy International Airport accelerate from rest at one end of a runway, and must attain takeoff speed before reaching the other end of the runway. **(a)** Plane A has acceleration a and takeoff speed v_{to}. What is the minimum length of runway, Δx_A, required for this plane? Give a symbolic answer. **(b)** Plane B has the same acceleration as plane A, but requires twice the takeoff speed. Find Δx_B and compare with Δx_A. **(c)** Find the minimum runway length for plane A if $a = 2.20\ \text{m/s}^2$ and $v_{to} = 95.0$ m/s. (These values are typical for a 747 jetliner. For purposes of comparison, the shortest runway at JFK International Airport is 04R/22L, which has a length of 2560 m.)

PICTURE THE PROBLEM

In our sketch, we choose the positive x direction to be the direction of motion. With this choice, it follows that the acceleration of the plane is positive, $a = +2.20\ \text{m/s}^2$. Similarly, the takeoff velocity is positive as well, $v_{to} = +95.0$ m/s.



REASONING AND STRATEGY

From the sketch it's clear that we want to express Δx, the distance the plane travels in attaining takeoff speed, in terms of the acceleration, a, and the takeoff speed, v_{to}. Equation 2-12 ($v^2 = v_0{}^2 + 2a\Delta x$), which relates distance to velocity, allows us to do this.

Known Acceleration, $a = 2.20\ \text{m/s}^2$; takeoff velocity, $v_{to} = 95.0$ m/s.
Unknown Takeoff distance, $\Delta x = ?$

SOLUTION

Part (a)

1. Solve $v^2 = v_0{}^2 + 2a(x - x_0) = v_0{}^2 + 2a\Delta x$ for Δx. To find Δx_A, set $v_0 = 0$ and $v = v_{to}$:

$$\Delta x = \frac{v^2 - v_0{}^2}{2a}$$

$$\Delta x_A = \frac{v_{to}{}^2}{2a}$$

CONTINUED

Part (b)

2. To find Δx_B, simply change v_{to} to $2v_{to}$ in part (a): $\Delta x_B = \frac{(2v_{to})^2}{2a} = \frac{4v_{to}^2}{2a} = 4\Delta x_A$

Part (c)

3. Substitute numerical values into the result found in part (a): $\Delta x_A = \frac{v_{to}^2}{2a} = \frac{(95.0\ \text{m/s})^2}{2(2.20\ \text{m/s}^2)} = 2050\ \text{m}$

INSIGHT

This Example illustrates the fact that there are many advantages to obtaining symbolic results before substituting numerical values. In this case, we found that the takeoff distance is proportional to v^2; hence, we conclude immediately that doubling v results in a fourfold increase of Δx.

PRACTICE PROBLEM

Find the minimum acceleration needed for a takeoff speed of $v_{to} = (95.0\ \text{m/s})/2 = 47.5\ \text{m/s}$ on a runway of length $\Delta x = (2050\ \text{m})/4 = 513\ \text{m}$. [**Answer:** $a = v_{to}^2/2\Delta x = 2.20\ \text{m/s}^2$]

Some related homework problems: Problem 40, Problem 90

Enhance Your Understanding (Answers given at the end of the chapter)

5. The equation of motion for an object moving with constant acceleration is $x = 6\ \text{m} - (5\ \text{m/s})t + (4\ \text{m/s}^2)t^2$. **(a)** What is the position of this object at $t = 0$? **(b)** What is the velocity of this object at $t = 0$? **(c)** What is the acceleration of this object?

Section Review

- Motion with constant acceleration can be described by equations of motion relating quantities like position, velocity, time, and acceleration.

2-6 Applications of the Equations of Motion

We devote this section to a variety of examples that further illustrate the use of the constant-acceleration equations of motion. For convenience, all of our constant-acceleration equations of motion are collected in Table 2-5.

Big Idea 5 Equations of motion for constant acceleration relate position, velocity, acceleration, and time.

TABLE 2-5 Constant-Acceleration Equations of Motion

Variables Related	Equation	Number
velocity, time, acceleration	$v = v_0 + at$	2-7
initial, final, and average velocity	$v_{av} = \frac{1}{2}(v_0 + v)$	2-9
position, time, velocity	$x = x_0 + \frac{1}{2}(v_0 + v)t$	2-10
position, time, acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2$	2-11
velocity, position, acceleration	$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x$	2-12

In our first Example, we consider the distance and time needed to brake a vehicle to a complete stop.

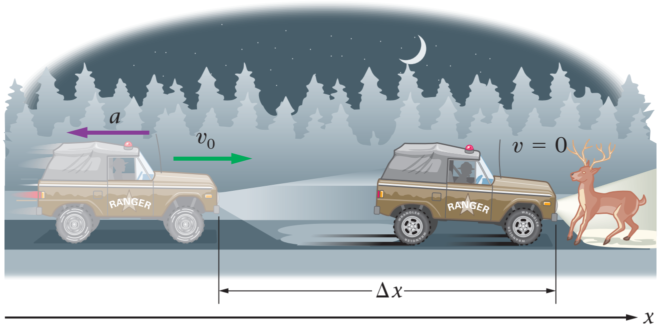
EXAMPLE 2-15 HIT THE BRAKES!

A park ranger driving on a back country road suddenly sees a deer "frozen" in the headlights. The ranger, who is driving at 11.4 m/s, immediately applies the brakes and slows with an acceleration of 3.80 m/s^2. **(a)** If the deer is 20.0 m from the ranger's vehicle when the brakes are applied, how close does the ranger come to hitting the deer? **(b)** How much time is needed for the ranger's vehicle to stop?

CONTINUED

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion. With this choice it follows that $v_0 = +11.4$ m/s. In addition, the fact that the ranger's vehicle is slowing down means its acceleration points in the *opposite* direction to that of the velocity [see Figure 2-14 (b) and (c)]. Therefore, the vehicle's acceleration is $a = -3.80 \text{ m/s}^2$. Finally, when the vehicle comes to rest its velocity is zero, $v = 0$.



REASONING AND STRATEGY

The acceleration is constant, so we can use the equations listed in Table 2-5. In part (a) we want to find a distance when we know the velocity and acceleration, so we use a rearranged version of Equation 2-12 ($v^2 = v_0^2 + 2a\Delta x$). In part (b) we want to find a time when we know the velocity and acceleration, so we use a rearranged version of Equation 2-7 ($v = v_0 + at$).

Known Velocity at time $t = 0$, $v_0 = 11.4$ m/s; acceleration, $a = -3.80 \text{ m/s}^2$.
Unknown **(a)** Distance required to stop, $\Delta x = ?$ **(b)** Time required to stop, $t = ?$

SOLUTION

Part (a)

1. Solve $v^2 = v_0^2 + 2a\Delta x$ for Δx:
$$\Delta x = \frac{v^2 - v_0^2}{2a}$$

2. Set $v = 0$, and substitute numerical values:
$$\Delta x = -\frac{v_0^2}{2a} = -\frac{(11.4 \text{ m/s})^2}{2(-3.80 \text{ m/s}^2)} = 17.1 \text{ m}$$

3. Subtract Δx from 20.0 m to find the distance between the stopped vehicle and the deer:
$$20.0 \text{ m} - 17.1 \text{ m} = 2.9 \text{ m}$$

Part (b)

4. Set $v = 0$ in $v = v_0 + at$ and solve for t:
$$v = v_0 + at = 0$$
$$t = -\frac{v_0}{a} = -\frac{11.4 \text{ m/s}}{(-3.80 \text{ m/s}^2)} = 3.00 \text{ s}$$

INSIGHT

Notice the different ways that t and Δx depend on the initial speed. If the initial speed is doubled, for example, the time needed to stop also doubles. On the other hand, the distance needed to stop increases by a factor of four. This is one reason speed on the highway has such a great influence on safety.

PRACTICE PROBLEM

Show that using $t = 3.00$ s in Equation 2-11 $\left(x = x_0 + v_0t + \frac{1}{2}at^2\right)$ results in the same distance needed to stop.
[**Answer:** $x = x_0 + v_0t + \frac{1}{2}at^2 = 0 + (11.4 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2}(-3.80 \text{ m/s}^2)(3.00 \text{ s})^2 = 17.1$ m, as expected]

Some related homework problems: Problem 55, Problem 56

In Example 2-15, we calculated the distance necessary for a vehicle to come to a complete stop. But how does the speed v vary with distance as the vehicle slows down? The next Conceptual Example deals with this topic.

CONCEPTUAL EXAMPLE 2-16 STOPPING DISTANCE

The ranger in Example 2-15 brakes for 17.1 m. After braking for only half that distance, $\frac{1}{2}(17.1 \text{ m}) = 8.55$ m, is the ranger's speed **(a)** equal to $\frac{1}{2}v_0$, **(b)** greater than $\frac{1}{2}v_0$, or **(c)** less than $\frac{1}{2}v_0$?

REASONING AND DISCUSSION

As pointed out in the Insight for Example 2-15, the fact that the stopping distance, Δx, depends on v_0^2 means that this distance increases by a factor of four when the speed is doubled. For example, the stopping distance with an initial speed of v_0 is four times the stopping distance when the initial speed is $v_0/2$.

CONTINUED

To apply this observation to the ranger, suppose that the stopping distance with an initial speed of v_0 is Δx. It follows that the stopping distance for an initial speed of $v_0/2$ is $\Delta x/4$. This means that as the ranger slows from v_0 to 0, it takes a distance $\Delta x/4$ to slow from $v_0/2$ to 0, and the remaining distance, $3\Delta x/4$, to slow from v_0 to $v_0/2$. Thus, at the halfway point the ranger has not yet slowed to half of the initial velocity—the speed at this point is greater than $v_0/2$.

ANSWER

(b) The ranger's speed is greater than $\frac{1}{2}v_0$.

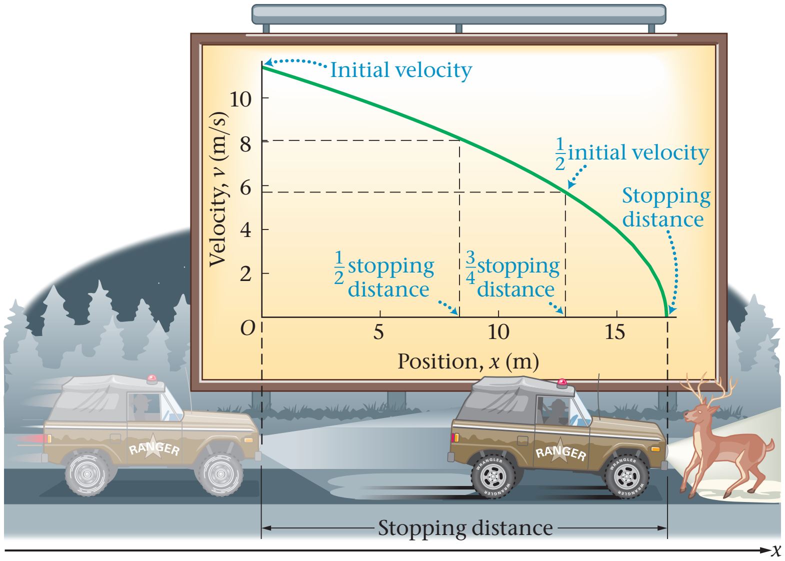
Clearly, v does not decrease uniformly with distance. A plot showing v as a function of x for Example 2-15 is shown in **FIGURE 2-21**. As we can see from the graph, v changes more in the second half of the braking distance than in the first half.

We close this section with a familiar everyday example: a police car accelerating to overtake a speeder. This is the first time we use two equations of motion for two different objects to solve a problem—but it won't be the last. Problems of this type are often more interesting than problems involving only a single object, and they relate to many types of situations in everyday life.

PROBLEM-SOLVING NOTE

Strategize

Before you attempt to solve a problem, it is a good idea to have some sort of plan, or "strategy," for how to proceed. It may be as simple as saying, "The problem asks me to relate velocity and time; therefore I will use Equation 2-7." In other cases the strategy is a bit more involved. Producing effective strategies is one of the most challenging—and creative—aspects of problem solving.



◄ **FIGURE 2-21 Velocity as a function of position for the ranger in Example 2-15** The ranger's vehicle comes to rest with constant acceleration, which means that its velocity decreases uniformly with time. The velocity *does not* decrease uniformly with distance, however. In particular, note how rapidly the velocity decreases in the final one-quarter of the stopping distance.

EXAMPLE 2-17 CATCHING A SPEEDER

A speeder doing 40.0 mi/h (about 17.9 m/s) in a 25 mi/h zone approaches a parked police car. The instant the speeder passes the police car, the police begin their pursuit. If the speeder maintains a constant velocity, and the police car accelerates with a constant acceleration of 4.51 m/s^2, **(a)** how much time does it take for the police car to catch the speeder, **(b)** how far have the two cars traveled in this time, and **(c)** what is the velocity of the police car when it catches the speeder?

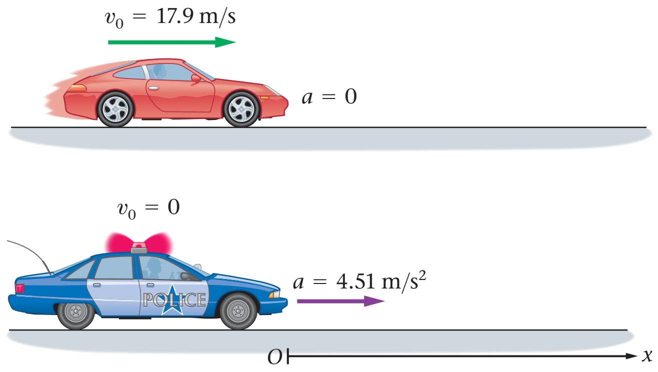
PICTURE THE PROBLEM

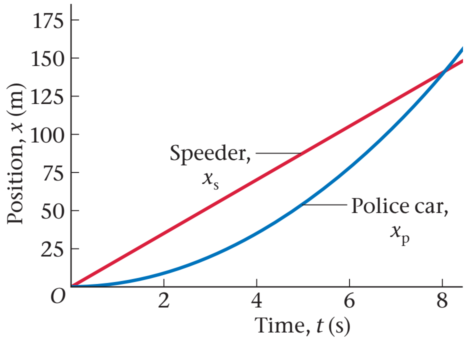
Our sketch shows the two cars at the moment the speeder passes the resting police car. At this instant, which we take to be $t = 0$, both the speeder and the police car are at the origin, $x = 0$. In addition, we choose the positive x direction to be the direction of motion; therefore, the speeder's initial velocity is given by $v_s = +17.9$ m/s, and the police car's initial velocity is zero. The speeder's acceleration is zero, but the police car has an acceleration given by $a = +4.51$ m/s^2. Finally, our figure shows the linear x-versus-t plot for the speeder, and the parabolic x-versus-t plot for the police car.

REASONING AND STRATEGY

To solve this problem, we first write a position-versus-time equation $\left(x = x_0 + v_0t + \frac{1}{2}at^2\right)$ for the police car, x_p, and a separate equation for the speeder, x_s. Next, we find the time it takes the police car to catch the speeder by setting $x_p = x_s$ and solving the resulting equation for t. Once the catch time is determined, it's straightforward to calculate the distance traveled and the velocity of the police car.

CONTINUED





Known Speeder velocity, $v_s = +17.9$ m/s; initial police velocity $= 0$; police acceleration, $a = +4.51$ m/s^2.

Unknown **(a)** Time to catch speeder, $t = ?$ **(b)** Distance to catch speeder? **(c)** Velocity of police when speeder is caught, $v_p = ?$

SOLUTION

Part (a)

1. Use $x = x_0 + v_0 t + \frac{1}{2}at^2$ to write equations of motion for the two vehicles. For the police car, $v_0 = 0$ and $a = 4.51$ m/s^2. For the speeder, $v_0 = 17.9$ m/s $= v_s$ and $a = 0$:

$$x_p = \tfrac{1}{2}at^2$$
$$x_s = v_s t$$

2. Set $x_p = x_s$ and solve for the time:

$$\tfrac{1}{2}at^2 = v_s t \quad \text{or} \quad \left(\tfrac{1}{2}at - v_s\right)t = 0$$

two solutions: $t = 0$ and $t = \dfrac{2v_s}{a}$

3. Clearly, $t = 0$ corresponds to the initial conditions, because both vehicles started at $x = 0$ at that time. The time of interest is obtained by substituting numerical values into the other solution:

$$t = \frac{2v_s}{a} = \frac{2(17.9 \text{ m/s})}{4.51 \text{ m/s}^2} = 7.94 \text{ s}$$

Part (b)

4. Substitute $t = 7.94$ s into the equations of motion for x_p and x_s. Notice that $x_p = x_s$, as expected:

$$x_p = \tfrac{1}{2}at^2 = \tfrac{1}{2}(4.51 \text{ m/s}^2)(7.94 \text{ s})^2 = 142$$
$$x_s = v_s t = (17.9 \text{ m/s})(7.94 \text{ s}) = 142 \text{ m}$$

Part (c)

5. To find the velocity of the police car use Equation 2-7 ($v = v_0 + at$), which relates velocity to time:

$$v_p = v_0 + at = 0 + (4.51 \text{ m/s}^2)(7.94 \text{ s}) = 35.8 \text{ m/s}$$

INSIGHT

When the police car catches up with the speeder, its velocity is 35.8 m/s, which is exactly twice the velocity of the speeder. A coincidence? Not at all. When the police car catches the speeder, both have traveled the same distance (142 m) in the same time (7.94 s); therefore, they have the same average velocity. Of course, the average velocity of the speeder is simply v_s. The average velocity of the police car is $\frac{1}{2}(v_0 + v)$, since its acceleration is constant, and thus $\frac{1}{2}(v_0 + v) = v_s$. Noting that $v_0 = 0$ for the police car, we see that $v = 2v_s$. This result is independent of the acceleration of the police car, as we show in the following Practice Problem.

PRACTICE PROBLEM

Repeat this Example for the case where the acceleration of the police car is $a = 3.25$ m/s^2. [**Answer: (a)** $t = 11.0$ s, **(b)** $x_p = x_s = 197$ m, **(c)** $v_p = 35.8$ m/s]

Some related homework problems: Problem 62, Problem 64

Enhance Your Understanding

(Answers given at the end of the chapter)

6. A submerged alligator swims directly toward two unsuspecting ducks. The equation of motion of the alligator is $x_a = (0.25 \text{ m/s}^2)t^2$, the equation of motion of duck 1 is $x_{d1} = 3 \text{ m} - (1.5 \text{ m/s})t$, and the equation of motion of duck 2 is $x_{d2} = 3 \text{ m} + (1 \text{ m/s})t$. Which duck does the alligator encounter first?

CONTINUED

Section Review

- Equations of motion for constant acceleration are listed in Table 2-5. As an example, the equation relating velocity and time is $v = v_0 + at$.

2-7 Freely Falling Objects

The most famous example of motion with constant acceleration is **free fall**—the motion of an object falling freely under the influence of gravity. It was Galileo (1564–1642) who first showed, with his own experiments, that falling bodies move with constant acceleration. His conclusions were based on experiments done by rolling balls down inclines of various steepness. By using an incline, Galileo was able to reduce the acceleration of the balls, thus producing motion slow enough to be timed with the instruments available at the time.

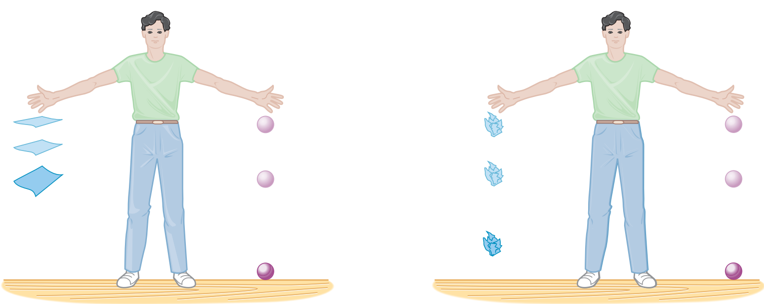
Galileo also pointed out that objects of different weight fall with the *same* constant acceleration—provided air resistance is small enough to be ignored. Whether he dropped objects from the Leaning Tower of Pisa to demonstrate this fact, as legend has it, will probably never be known for certain, but we do know that he performed extensive experiments to support his claim.

Today it is easy to verify Galileo's assertion by dropping objects in a vacuum chamber, where the effects of air resistance are essentially removed. In a standard classroom demonstration, a feather and a coin are dropped in a vacuum, and both fall at the same rate. In 1971, a novel version of this experiment was carried out on the Moon by astronaut David Scott. In the near-perfect vacuum on the Moon's surface he dropped a feather and a hammer and showed a worldwide television audience that they fell to the ground in the same time. Examples of free fall in different contexts are shown in FIGURE 2-22.

To illustrate the effect of air resistance in everyday terms, drop a sheet of paper and a rubber ball from the same height (FIGURE 2-23). The paper drifts slowly to the ground, taking much more time to fall than the ball. Now, wad the sheet of paper into a tight ball and repeat the experiment. This time the ball of paper and the rubber ball reach the ground in nearly the same time. What was different in the two experiments? Clearly, when the sheet of paper was wadded into a ball, the effect of air resistance on it was greatly reduced, so that both objects fell almost as they would in a vacuum.

Characteristics of Free Fall Before considering a few examples, let's discuss exactly what is meant by "free fall." To begin, the word *free* in free fall means free from any effects other than gravity. For example, in free fall we assume that an object's motion is not influenced by any form of friction or air resistance.

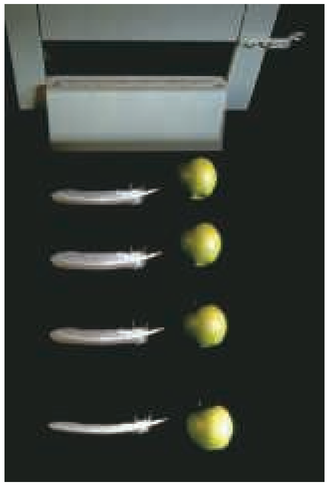
Free fall is the motion of an object subject *only* to the influence of gravity.



(a) Dropping a sheet of paper and a rubber ball

(b) Dropping a wadded-up sheet of paper and a rubber ball

▲ FIGURE 2-23 **Free fall and air resistance**



(a)

(b)

(c)

▲ FIGURE 2-22 **Visualizing Concepts Free fall** **(a)** In the absence of air resistance, all objects fall with the same acceleration, regardless of their mass. **(b)** Whether she is on the way up, at the peak of her flight, or on the way down, this girl is in free fall, accelerating downward with the acceleration of gravity. Only when she is in contact with the blanket does her acceleration change. **(c)** In the absence of air resistance, these lava bombs from the Kilauea volcano on the Big Island of Hawaii would strike the water with the same speed they had when they were blasted into the air. It's an example of the symmetry of free fall.

Big Idea 6 Free fall is motion with a constant downward acceleration of magnitude g, where g is the acceleration due to gravity.

Though free fall is an idealization—which does not apply to many real-world situations—it is still a useful approximation in many other cases. In the following examples we assume that the motion may be considered as free fall.

Next, it should be realized that the word *fall* in free fall does not mean the object is necessarily moving downward. By free fall, we mean *any* motion under the influence of gravity alone. If you drop a ball, it is in free fall. If you throw a ball upward or downward, it is in free fall as soon as it leaves your hand.

> An object is in free fall as soon as it is released, whether it is dropped from rest, thrown downward, or thrown upward.

TABLE 2-6 Values of g at Different Locations on Earth (m/s^2)

Location	Latitude	g
North Pole	90° N	9.832
Oslo, Norway	60° N	9.819
Hong Kong	30° N	9.793
Quito, Ecuador	0°	9.780

Finally, the acceleration produced by gravity on the Earth's surface (sometimes called the gravitational strength) is denoted with the symbol g. As a shorthand name, we will frequently refer to g simply as "the acceleration due to gravity." In fact, as we shall see in Chapter 12, the value of g varies according to one's location on the surface of the Earth, as well as one's altitude above it. Table 2-6 shows how g varies with latitude.

In all the calculations that follow in this book, we shall use $g = 9.81\ m/s^2$ for the acceleration due to gravity. Note, in particular, that g always stands for $+9.81\ m/s^2$, never $-9.81\ m/s^2$. For example, if we choose a coordinate system with the positive direction upward, the acceleration in free fall is $a = -g$. If the positive direction is downward, then free-fall acceleration is $a = g$.

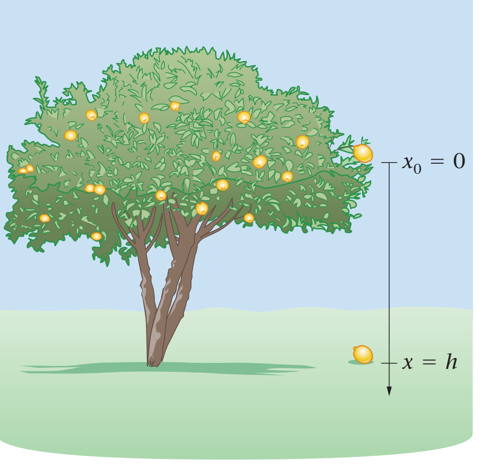
With these comments in mind, we're ready to explore a variety of free-fall examples.

EXAMPLE 2-18 LEMON DROP

A lemon drops from a tree and falls to the ground 3.15 m below. **(a)** How much time does it take for the lemon to reach the ground? **(b)** What is the lemon's speed just before it hits the ground?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the drop height of the lemon, and we let the positive x direction be downward. With these choices, $x_0 = 0$, $a = g$, and the ground is at $x = h = 3.15$ m. Of course, the initial velocity is zero, $v_0 = 0$, because the lemon drops from rest out of the tree.



REASONING AND STRATEGY

We can ignore air resistance in this case and model the motion as free fall. This means we can assume a constant acceleration equal to g and use the equations of motion in Table 2-5. For part (a) we want to find the time of fall when we know the distance and acceleration, so we use Equation 2-11 $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$. For part (b) we can relate velocity to time by using Equation 2-7 $(v = v_0 + at)$, or we can relate velocity to position by using Equation 2-12 $(v^2 = v_0^2 + 2a\Delta x)$. We will implement both approaches and show that the results are the same.

Known Initial position of lemon, $x_0 = 0$; final position of lemon, $x = h = 3.15$ m; initial velocity of lemon, $v_0 = 0$; acceleration of lemon, $a = g = 9.81\ m/s^2$.

Unknown **(a)** Drop time, $t = ?$ **(b)** Landing speed, $v = ?$

SOLUTION

Part (a)

1. Write $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$, $v_0 = 0$, and $a = g$:
$$x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

2. Solve for the time, t, and set $x = h = 3.15$ m:
$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(3.15\ \text{m})}{9.81\ \text{m/s}^2}} = 0.801\ \text{s}$$

Part (b)

3. To find the velocity, use the time found in part (a), $t = 0.801$ s, in $v = v_0 + at$:
$$v = v_0 + gt = 0 + (9.81\ \text{m/s}^2)(0.801\ \text{s}) = 7.86\ \text{m/s}$$

4. We can also find the velocity without knowing the time by using $v^2 = v_0^2 + 2a\Delta x$ with $\Delta x = 3.15$ m:
$$v^2 = {v_0}^2 + 2a\Delta x = 0 + 2g\Delta x$$
$$v = \sqrt{2g\Delta x} = \sqrt{2(9.81\ \text{m/s}^2)(3.15\ \text{m})} = 7.86\ \text{m/s}$$

CONTINUED

INSIGHT
We could just as well solve this problem with the origin at ground level, the drop height at $x = h$, and the positive x direction upward, which means that $a = -g$. The results are the same, of course.

PRACTICE PROBLEM — PREDICT/CALCULATE
Consider the distance the lemon drops in half the time required to reach the ground—that is, in the time $t = (0.801\ \text{s})/2$. **(a)** Is this distance greater than, less than, or equal to half the distance to the ground? Explain. **(b)** Find the distance the lemon drops in this time. [**Answer: (a)** The distance is less than half the distance to the ground, because the average speed of the lemon in the first half of its drop is less than its average speed in the second half of its drop. **(b)** The distance dropped is $x = \frac{1}{2}gt^2 = \frac{1}{2}(9.81\ \text{m/s}^2)\left(\frac{0.801\ \text{s}}{2}\right)^2 = 0.787\ \text{m}$, which is only one-quarter the distance to the ground.]

Some related homework problems: Problem 70, Problem 73

Free Fall from Rest The special case of free fall from rest occurs so frequently, and in so many different contexts, that it deserves special attention. If we take x_0 to be zero, and positive to be downward, then position as a function of time is $x = x_0 + v_0t + \frac{1}{2}gt^2 = 0 + 0 + \frac{1}{2}gt^2$, or

$$x = \frac{1}{2}gt^2 \quad \text{2-13}$$

Similarly, velocity as a function of time is

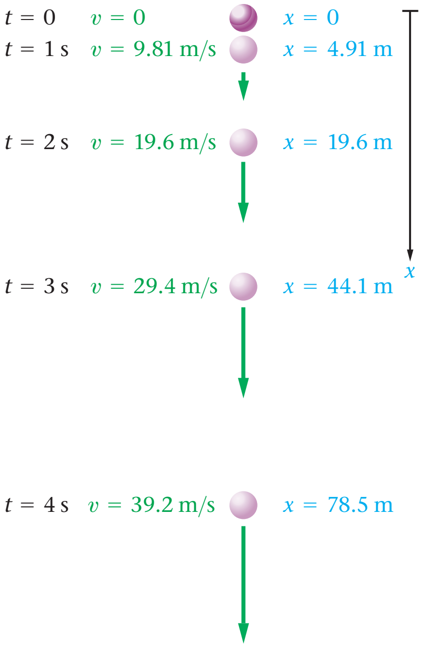
$$v = gt \quad \text{2-14}$$

In addition, velocity as a function of position is

$$v = \sqrt{2gx} \quad \text{2-15}$$

The behavior of these functions is illustrated in **FIGURE 2-24**. Notice that position increases with time squared, whereas velocity increases linearly with time.

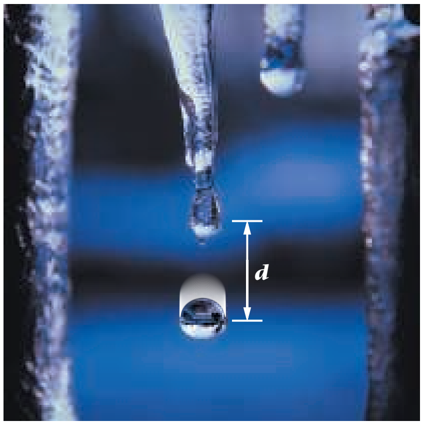
Next we consider two objects that drop from rest, one after the other, and discuss how their separation varies with time.



▶ **FIGURE 2-24 Free fall from rest** Position and velocity are shown as functions of time. It is apparent that velocity depends linearly on t, whereas position depends on t^2.

CONCEPTUAL EXAMPLE 2-19 FREE-FALL SEPARATION

Drops of water detach from the tip of an icicle and fall from rest. When one drop separates, the drop ahead of it has already fallen through a distance d, as shown below. As these two drops continue to fall, does their separation **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION
It might seem that the separation between the drops will remain the same, because both are in free fall. This is not so. The drop that has a head start always has a greater velocity than the one that comes next. Therefore, the first drop covers a greater distance in any interval of time, and as a result, the separation between the drops increases.

ANSWER
(a) The separation between the drops increases.

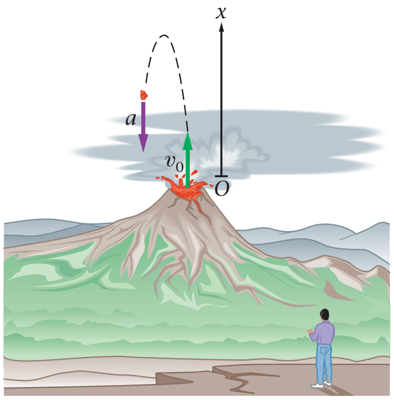
An erupting volcano shooting out fountains of lava is an impressive sight. In the next Example we show how a simple timing experiment can determine the initial velocity of the erupting lava.

EXAMPLE 2-20 BOMBS AWAY: CALCULATING THE SPEED OF A LAVA BOMB

RWP A volcano shoots out blobs of molten lava, called lava bombs, from its summit. A geologist observing the eruption uses a stopwatch to time the flight of a particular lava bomb that is projected straight upward. If the time for the bomb to rise and fall back to its launch height is 4.75 s, and its acceleration is 9.81 m/s^2 downward, what is its initial speed?

PICTURE THE PROBLEM

Our sketch shows a coordinate system with upward as the positive x direction. For clarity, we offset the upward and downward trajectories slightly in our sketch. In addition, we choose $t = 0$ to be the time at which the lava bomb is launched. With these choices it follows that $x_0 = 0$ and the acceleration is $a = -g = -9.81$ m/s^2. The initial speed to be determined is v_0.



REASONING AND STRATEGY

Once again, we can ignore air resistance and model the motion of the lava bomb as free fall—this time with an initial upward velocity. We know that the lava bomb starts at $x = 0$ at the time $t = 0$ and returns to $x = 0$ at the time $t = 4.75$ s. This means that we know the bomb's position, time, and acceleration ($a = -g$), from which we would like to determine the initial velocity. A reasonable approach is to use Equation 2-11 $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$ and solve it for the one unknown it contains, v_0.

Known Flight time, $t = 4.75$ s; acceleration, $a = -g = -9.81$ m/s^2.
Unknown Initial velocity of the lava bomb, $v_0 = ?$

SOLUTION

1. Write out $x = x_0 + v_0 t + \frac{1}{2}at^2$ with $x_0 = 0$ and $a = -g$. Factor out a time, t, from the two remaining terms:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{1}{2}gt^2 = (v_0 - \frac{1}{2}gt)t$$

2. Set x equal to zero, because this is the position of the lava bomb at $t = 0$ and $t = 4.75$ s:

$$x = (v_0 - \frac{1}{2}gt)t = 0$$

Two solutions: (i) $t = 0$ (ii) $v_0 - \frac{1}{2}gt = 0$

3. The first solution is simply the initial condition; that is, $x = 0$ at $t = 0$. Solve the second solution for the initial speed:

$$v_0 - \frac{1}{2}gt = 0 \quad \text{or} \quad v_0 = \frac{1}{2}gt$$

4. Substitute numerical values for g and the time the lava bomb lands:

$$v_0 = \frac{1}{2}gt = \frac{1}{2}(9.81 \text{ m/s}^2)(4.75 \text{ s}) = 23.3 \text{ m/s}$$

INSIGHT

A geologist can determine a lava bomb's initial speed by simply observing its flight time. Knowing the lava bomb's initial speed can help geologists determine how severe a volcanic eruption will be, and how dangerous it might be to people in the surrounding area.

PRACTICE PROBLEM

A second lava bomb is projected straight upward with an initial speed of 25 m/s. What is its flight time?
[**Answer:** $t = 5.1$ s]

Some related homework problems: Problem 71, Problem 78

PROBLEM-SOLVING NOTE

Check Your Solution

Once you have a solution to a problem, check to see whether it makes sense. First, make sure the units are correct; m/s for speed, m/s^2 for acceleration, and so on. Second, check the numerical value of your answer. If you are solving for the speed of a diver dropping from a 3.0-m diving board and you get an unreasonable value like 200 m/s ($\approx$450 mi/h), chances are good that you've made a mistake.

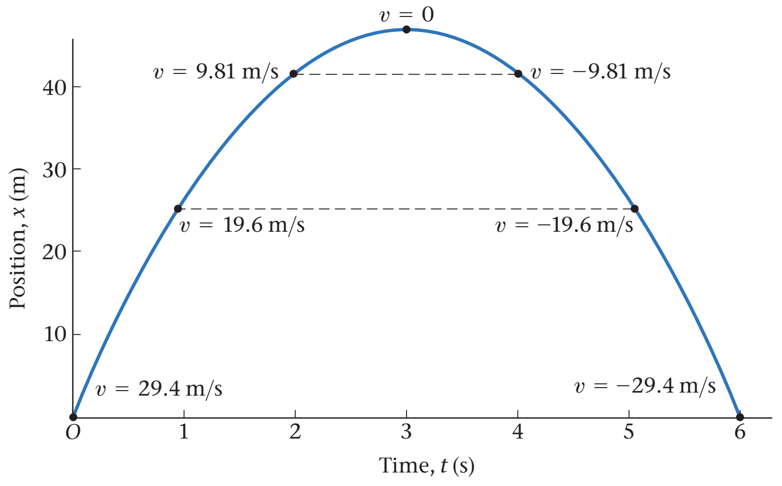
What is the speed of a lava bomb when it returns to Earth; that is, when it returns to the same level from which it was launched? Physical intuition might suggest that, in the absence of air resistance, it should be the same as its initial speed. To show that this hypothesis is indeed correct, write out Equation 2-7 ($v = v_0 + at$) for this case:

$$v = v_0 - gt$$

Substituting numerical values, we find

$$v = v_0 - gt = 23.3 \text{ m/s} - (9.81 \text{ m/s}^2)(4.75 \text{ s}) = -23.3 \text{ m/s}$$

Thus, the velocity of the lava bomb when it lands is just the negative of the velocity it had when launched upward. Or, put another way, when the lava bomb lands, it has the same speed as when it was launched; it's just traveling in the opposite direction.



◀ **FIGURE 2-25 Position and velocity of a lava bomb** This lava bomb is in the air for 6 seconds. Note the symmetry about the midpoint of the bomb's flight.

It's instructive to verify this result symbolically. Recall from Example 2-20 that $v_0 = \frac{1}{2}gt$, where t is the time the bomb lands. Substituting this result into Equation 2-7, we find

$$v = \frac{1}{2}gt - gt = -\frac{1}{2}gt = -v_0$$

The advantage of the symbolic solution lies in showing that the result is not a fluke—no matter what the initial velocity, no matter what the acceleration, the bomb lands with the velocity $-v_0$.

The Symmetry of Free Fall The preceding results suggest a symmetry relating the motion on the way up to the motion on the way down. To make this symmetry more apparent, we first solve for the time when the lava bomb lands. Using the result $v_0 = \frac{1}{2}gt$ from Example 2-20, we find

$$t = \frac{2v_0}{g} \qquad \textit{(time of landing)}$$

Next, we find the time when the velocity of the lava bomb is zero, which is at its highest point. Setting $v = 0$ in Equation 2-7 $(v = v_0 + at)$, we have $v = v_0 - gt = 0$, or

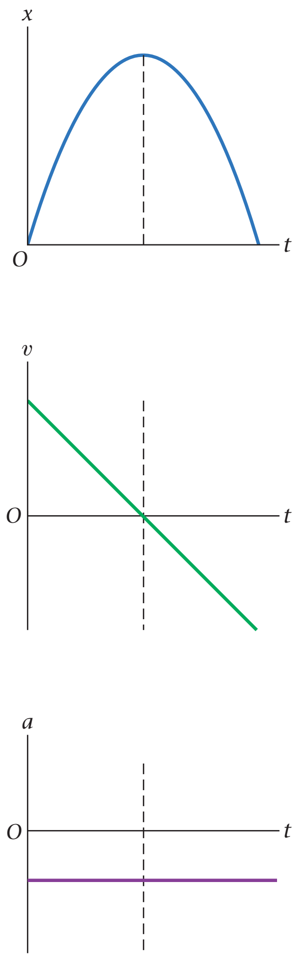
$$t = \frac{v_0}{g} \qquad \textit{(time when } v = 0\textit{)}$$

This is exactly half the time required for the lava to make the round trip. Thus, the velocity of the lava bomb is zero and the height of the bomb is greatest exactly halfway between launch and landing.

This symmetry is illustrated in **FIGURE 2-25**. In this case we consider a lava bomb that is in the air for 6.00 s, moving without air resistance. Note that at $t = 3.00$ s the lava bomb is at its highest point and its velocity is zero. At times equally spaced before and after $t = 3.00$ s, the lava bomb is at the same height and has the same speed, but is moving in opposite directions. As a result of this symmetry, a movie of the lava bomb's flight would look the same whether run forward or in reverse.

FIGURE 2-26 shows the time dependence of position, velocity, and acceleration for an object in free fall without air resistance after being thrown upward. As soon as the object is released, it begins to accelerate downward—as indicated by the negative slope of the velocity-versus-time plot—though it isn't necessarily moving downward. For example, if you throw a ball *upward,* it begins to accelerate *downward* the moment it leaves your hand. It continues moving upward, however, until its speed diminishes to zero. Because gravity is causing the downward acceleration, and gravity doesn't turn off just because the ball's velocity goes through zero, the ball continues to accelerate downward even when it is momentarily at rest.

Similarly, in the next Example we consider a sandbag that falls from an ascending hot-air balloon. This means that before the bag is in free fall it was moving upward—just like a ball thrown upward. And just like the ball, the sandbag continues moving upward for a brief time before momentarily stopping and then moving downward.



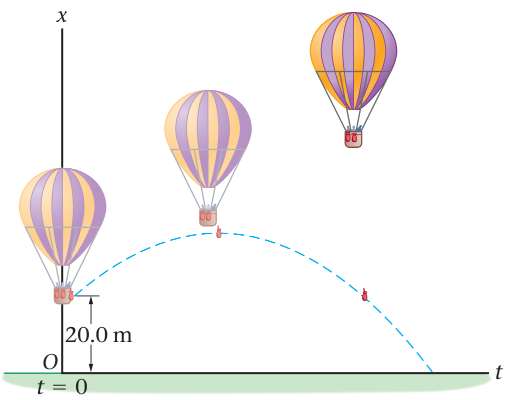
▲ **FIGURE 2-26 Position, velocity, and acceleration of a lava bomb as functions of time** The fact that the *x*-versus-*t* plot is curved indicates an acceleration; the downward curvature shows that the acceleration is negative. This is also clear from the *v*-versus-*t* plot, which has a negative slope. The constant slope of the straight line in the *v*-versus-*t* plot indicates a constant acceleration, as shown in the *a*-versus-*t* plot.

EXAMPLE 2-21 LOOK OUT BELOW! A SANDBAG IN FREE FALL

A hot-air balloon is rising straight upward with a constant speed of 6.5 m/s. When the basket of the balloon is 20.0 m above the ground, a bag of sand tied to the basket comes loose. **(a)** How much time elapses before the bag of sand hits the ground? **(b)** What is the greatest height of the bag of sand during its fall to the ground?

PICTURE THE PROBLEM

We choose the origin to be at ground level and positive to be upward. This means that, for the bag, we have $x_0 = 20.0\text{ m}$, $v_0 = 6.5\text{ m/s}$, and $a = -g$. Our sketch also shows snapshots of the balloon and bag of sand at three different times, starting at $t = 0$ when the bag comes loose. Notice that the bag is moving upward with the balloon at the time it comes loose. It therefore continues to move upward for a short time after it separates from the basket, exactly as if it had been thrown upward.



REASONING AND STRATEGY

The effects of air resistance on the sandbag can be ignored. As a result, we can use the equations in Table 2-5 with a constant acceleration $a = -g$.

In part (a) we want to relate position and time—knowing the initial position and initial velocity—so we use Equation 2-11 $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$. To find the time the bag hits the ground, we set $x = 0$ and solve for t.

For part (b) we have no expression that gives the maximum height of a particle—so we will have to come up with something on our own. We can start with the fact that $v = 0$ at the greatest height, since it is there the bag momentarily stops as it changes direction. Therefore, we can find the time t when $v = 0$ by using Equation 2-7 ($v = v_0 + at$), and then substitute t into Equation 2-11 to find x_{max}.

Known Drop height, $x_0 = 20.0\text{ m}$; upward initial velocity of basket, $v_0 = 6.5\text{ m/s}$; acceleration due to gravity, $a = -g = -9.81\text{ m/s}^2$.

Unknown **(a)** Time to reach the ground, $t = ?$ **(b)** Maximum height of sandbag, $x_{max} = ?$

SOLUTION

Part (a)

1. Apply $x = x_0 + v_0 t + \frac{1}{2}at^2$ to the bag of sand. Do this by letting $a = -g$ and setting $x = 0$, which corresponds to ground level:

$$x = x_0 + v_0 t - \frac{1}{2}gt^2 = 0$$

2. Notice that we have a quadratic equation for t, with $A = -\frac{1}{2}g = -\frac{1}{2}(9.81\text{ m/s}^2)$, $B = v_0 = 6.5\text{ m/s}$, and $C = x_0 = 20.0\text{ m}$. Solve this equation for t. The positive solution, 2.78 s, applies to this problem: (Quadratic equations and their solutions are discussed in Appendix A. In general, one can expect two solutions to a quadratic equation.)

$$t = \frac{-v_0 \pm \sqrt{{v_0}^2 - 4\left(-\frac{1}{2}g\right)(x_0)}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-(6.5\text{ m/s}) \pm \sqrt{(6.5\text{ m/s})^2 + 2(9.81\text{ m/s}^2)(20.0\text{ m})}}{(-9.81\text{ m/s}^2)}$$

$$= \frac{-(6.5\text{ m/s}) \pm 20.8\text{ m/s}}{(-9.81\text{ m/s}^2)} = 2.78\text{ s}, -1.46\text{ s}$$

Part (b)

3. Apply $v = v_0 + at$ to the bag of sand, and solve for the time when the velocity equals zero:

$$v = v_0 + at = v_0 - gt$$

$$v_0 - gt = 0 \quad \text{or} \quad t = \frac{v_0}{g} = \frac{6.5\text{ m/s}}{9.81\text{ m/s}^2} = 0.66\text{ s}$$

4. Use $t = 0.66\text{ s}$ in $x = x_0 + v_0 t + \frac{1}{2}at^2$ to find the maximum height:

$$x_{max} = 20.0\text{ m} + (6.5\text{ m/s})(0.66\text{ s}) - \frac{1}{2}(9.81\text{ m/s}^2)(0.66\text{ s})^2$$
$$= 22\text{ m}$$

INSIGHT

Thus, the bag of sand continues to move upward for 0.66 s after it separates from the basket, reaching a maximum height of 22 m above the ground. It then begins to move downward, and hits the ground 2.78 s after detaching from the basket.

PRACTICE PROBLEM

What is the velocity of the sandbag just before it hits the ground? [**Answer:** $v = v_0 - gt = (6.5\text{ m/s}) - (9.81\text{ m/s}^2) \times (2.78\text{ s}) = -20.8\text{ m/s}$; the minus sign indicates the bag is moving downward, as expected.]

Some related homework problems: Problem 83, Problem 84

Enhance Your Understanding

(Answers given at the end of the chapter)

7. On a distant, airless planet, an astronaut drops a rock to test the planet's gravitational pull. The astronaut finds that in the first second of falling (from $t = 0$ to $t = 1$ s) the rock drops a distance of 1 m. How far does the rock drop from $t = 1$ s to $t = 2$ s? **(a)** 1 m, **(b)** 2 m, **(c)** 3 m, **(d)** 4 m.

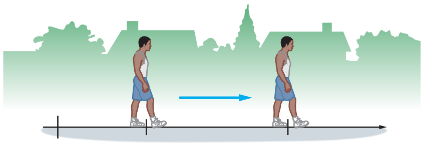
Section Review

- Free fall is motion with a constant downward acceleration of magnitude $g = 9.81\ \text{m/s}^2$.

CHAPTER 2 REVIEW

CHAPTER SUMMARY

2-1 POSITION, DISTANCE, AND DISPLACEMENT



Position

Position is the location of an object as measured on a coordinate system.

Distance

Distance is the total length of travel, from beginning to end. Distance is always positive.

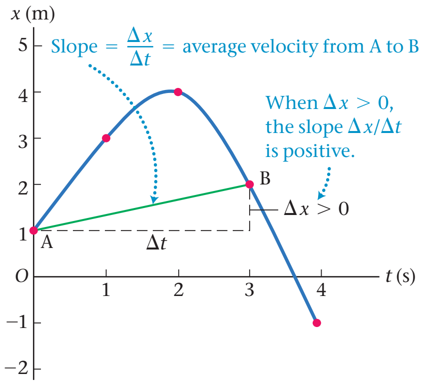
Displacement

Displacement, Δx, is the change in position; that is, $\Delta x = x_f - x_i$.

Positive and Negative Displacement

The *sign* of the displacement indicates the *direction* of motion.

2-2 AVERAGE SPEED AND VELOCITY



Average Speed

Average speed is *distance* divided by elapsed time.

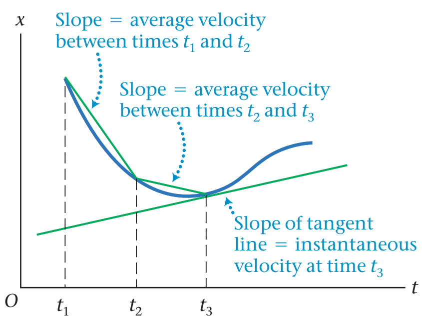
Average Velocity

Average velocity is *displacement* divided by elapsed time. Average velocity is positive for motion in the positive direction, negative for motion in the negative direction.

Graphical Interpretation

In an x-versus-t plot, the average velocity is the slope of a line connecting two points.

2-3 INSTANTANEOUS VELOCITY



The velocity at an instant of time, v, is the average velocity over shorter and shorter time intervals: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$.

Graphical Interpretation

In an x-versus-t plot, the instantaneous velocity is the slope of a tangent line at a given instant of time.

2-4 ACCELERATION

Average Acceleration

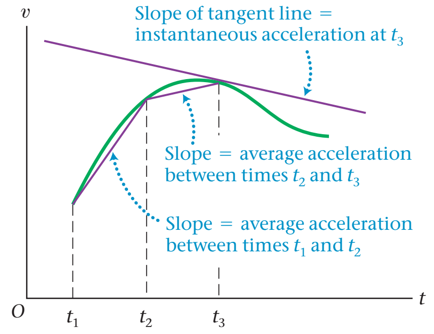
Average acceleration is the change in velocity divided by the elapsed time.

Instantaneous Acceleration

The acceleration at an instant of time, a, is the limit of the average acceleration over shorter and shorter time intervals: $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$.

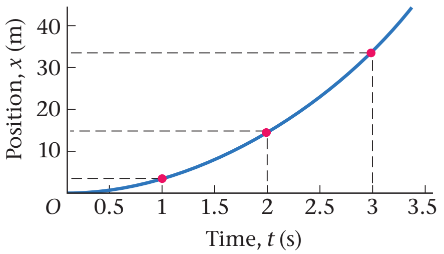
Graphical Interpretation

In a v-versus-t plot, the instantaneous acceleration is the slope of a tangent line at a given instant of time.



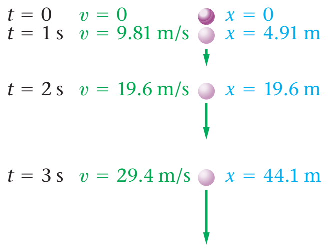
2-5 MOTION WITH CONSTANT ACCELERATION

Several different equations are used to describe the motion of objects moving with constant acceleration. These "equations of motion" are listed in Table 2-5. The ones that are used most often are (1) velocity as a function of time ($v = v_0 + at$), (2) position as a function of time $\left(x = x_0 + v_0 t + \frac{1}{2}at^2\right)$, and (3) velocity as a function of position ($v^2 = v_0^2 + 2a\Delta x$).



2-7 FREELY FALLING OBJECTS

Objects in free fall move under the influence of gravity alone. An object is in free fall as soon as it is released, whether it is thrown upward, thrown downward, or released from rest. As a result, objects in free fall move with a constant downward acceleration of magnitude $g = 9.81\ \text{m/s}^2$.



ANSWERS TO ENHANCE YOUR UNDERSTANDING QUESTIONS

1. **(a)** No. Distance is always increasing on a trip, whereas the magnitude of displacement can increase or decrease. **(b)** Yes. For example, in a 5-mi round trip the distance is 5 mi but the magnitude of the displacement is zero.
2. **(a)** Positive. **(b)** Zero. **(c)** Negative.
3. C < B < D < A.
4. **(a)** The speed is decreasing because velocity and acceleration have opposite signs. **(b)** The velocity is increasing (becoming less negative) because the acceleration is positive. **(c)** The speed is decreasing because velocity and acceleration have opposite signs. **(d)** The velocity is decreasing (becoming less positive) because the acceleration is negative.
5. **(a)** $x = 6$ m. **(b)** $v = -5$ m/s. **(c)** $a = 8\ \text{m/s}^2$.
6. The alligator encounters duck 1 first.
7. **(c)** 3 m.

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com. MP

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)
(The effects of air resistance are to be ignored in this chapter.)

1. You take your dog on a walk to a nearby park. On the way, your dog takes many short side trips to chase squirrels, examine fire hydrants, and so on. When you arrive at the park, do you and your dog have the same displacement from home? Have you and your dog traveled the same distance? Explain.
2. Does an odometer in a car measure distance or displacement? Explain.
3. An astronaut orbits Earth in the space shuttle. In one complete orbit, is the magnitude of the displacement the same as the distance traveled? Explain.
4. After a tennis match the players dash to the net to congratulate one another. If they both run with a speed of 3 m/s, are their velocities equal? Explain.
5. Does a speedometer measure speed or velocity? Explain.
6. Is it possible for a car to circle a racetrack with constant velocity? Can it do so with constant speed? Explain.
7. For what kinds of motion are the instantaneous and average velocities equal?
8. Assume that the brakes in your car create a constant deceleration, regardless of how fast you are going. If you double your driving speed, how does this affect **(a)** the time required to come to a stop, and **(b)** the distance needed to stop?
9. The velocity of an object is zero at a given instant of time. **(a)** Is it possible for the object's acceleration to be zero at this time? Explain. **(b)** Is it possible for the object's acceleration to be nonzero at this time? Explain.
10. If the velocity of an object is nonzero, can its acceleration be zero? Give an example if your answer is yes; explain why not if your answer is no.