Chapter 5 Summary and Vocabulary

- A **compound sentence** is the result of joining two or more sentences with the word *and* or *or*. The solution set for A *or* B is the **union** of the solution sets of A and B. The compound sentence A *and* B is called a **system.** The solution set to the system A *and* B is the **intersection** of the solution sets of A and B.
- Systems may contain any number of variables. If the system contains one variable, then its solutions may be graphed on a number line. If the system contains two variables, then its solutions may be graphed in the plane. The graph often tells you the number of solutions the system has, but may not yield the exact solutions.
- Systems of equations can be solved by hand or with technology. Some systems can be solved with tables and graphs. Algebraic methods use **linear combinations, substitution,** and matrices. The matrix method converts a system of n equations in n unknowns to a single matrix equation. To find the solution to a system in **matrix form,** multiply both sides of the equation by the **inverse** of the **coefficient matrix.**
- The graph of a linear inequality in two variables is a **half-plane** or a half-plane with its **boundary.** For a system of two linear inequalities, if the boundary lines intersect, then the **feasible region** is the interior of an angle plus perhaps one or both of its sides.
- Systems with two variables but more than two inequalities arise in **linear-programming problems.** In such a problem, you look for a solution to the system that maximizes or minimizes the value of a particular expression or formula.
- The Linear-Programming Theorem states that the feasible region of every linear-programming problem is always convex and that the solution that will maximize or minimize the pertinent expression must be a vertex of the feasible region.

Theorems and Properties

Addition Property of Inequality (p. 303)
Multiplication Property of Inequality (p. 303)
Substitution Property of Equality (p. 314)
Inverse Matrix Theorem (p. 329)
System-Determinant Theorem (p. 338)
Linear-Programming Theorem (p. 356)

Vocabulary

Lesson 5-1
compound sentence
double inequality
*union of two sets
*intersection of two sets

Lesson 5-2
system
*solution set of a system

Lesson 5-3
substitution method
consistent system
inconsistent system

Lesson 5-4
linear-combination method

Lesson 5-5
*inverse matrices
square matrix
singular matrix
*determinant

Lesson 5-6
matrix form of a system
coefficient matrix
constant matrix

Lesson 5-7
half-planes
boundary of the half-planes
lattice point

Lesson 5-8
feasible set, feasible region

Lesson 5-9
linear-programming problem

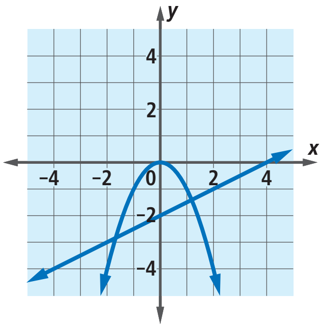
Chapter 5 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. Solve $-4n + 18 < 30$ and graph the solution set on a number line.

2. On a number line, graph $\{x | x \leq -3 \text{ or } x > 4\}$.

3. A graph of the system $\begin{cases} y = 0.5x - 2 \\ y = -x^2 \end{cases}$ is shown below. Approximate the solutions to the system to the nearest tenth.



4. Consider the system $\begin{cases} x + 4y = 16 \\ 2x + 8y = 30 \end{cases}$.

Is this system consistent or inconsistent? How do you know?

In 5 and 6, solve each system using an algebraic method and show how to check your answer.

5. $\begin{cases} p = 3q \\ r = q + 4 \\ 3r - 2p = 3 \end{cases}$

6. $\begin{cases} 7x + 5y = 12 \\ 2x + 9y = 14 \end{cases}$

7. The Natural Nut Company sells organic nuts. How many pounds of organic hazelnuts priced at \$7.83 per pound should be mixed with organic pecans priced at \$11.50 per pound to obtain 20 pounds of mixed nuts priced at \$10 per pound?

a. Write a system of equations representing this situation. Tell what each variable stands for in this problem.

b. Solve the system and answer the question.

8. Consider the system $\begin{cases} 3x + 2y = 24 \\ -2x + 7y = 39 \end{cases}$.

a. What is the coefficient matrix?

b. Find the inverse of the coefficient matrix.

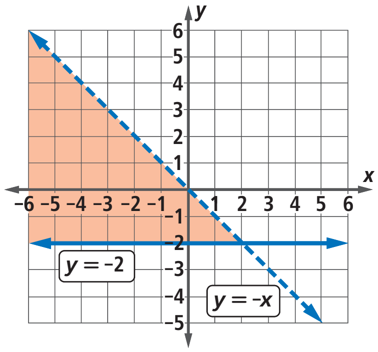
c. Use a matrix equation to solve the system.

9. a. Give an example of a 2×2 matrix that does not have an inverse.

b. How can you tell that the inverse does not exist?

10. Graph the solution set of $y > -\frac{5}{2}x - 2$.

11. The graph below shows the feasible region for what system?



12. Tell whether the following points are solutions to the system from Question 11.

a. (2.5, 3.2)

b. (–4, 0.5)

c. (–2, 2)

13. Surehold Shelving Company produces two types of decorative shelves. The Olde English style takes 20 minutes to assemble and 10 minutes to finish. The Cool Contemporary style takes 10 minutes to assemble and 20 minutes to finish. Each day there are at most 48 worker-hours available in the assembly department and at most 64 worker-hours available in the finishing department. If Surehold Shelving makes a \$15 profit on each Olde English shelf and a \$12 profit on each Cool Contemporary shelf, how many of each shelf should the company make to maximize its profit?

a. Let e = number of Olde English shelves and c = number of Cool Contemporary shelves the company makes. Translate the constraints into a system of linear inequalities.

b. Graph the system of inequalities and find the vertices of the feasible region.

c. Apply the Linear-Programming Theorem and interpret the results.

Chapter 5

Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Solve 2 × 2 and 3 × 3 systems using the linear-combination method or substitution. (Lessons 5-3, 5-4)

1. **Multiple Choice** After which of the following operations does the system $\begin{cases} -2x + 3y = 13 \\ 6x + y = 5 \end{cases}$ yield $20x = 2$?

A Multiply the first equation by 3 and add.

B Multiply the second equation by 3 and add.

C Multiply the first equation by -3 and add the second equation.

D Multiply the second equation by -3 and add the first equation.

In 2–7, solve and check.

2. $\begin{cases} 3z - 6w = 15 \\ 0.5z - w = 22 \end{cases}$

3. $\begin{cases} 2x + 10y = 16 \\ x = -3y \end{cases}$

4. $\begin{cases} r = s - 5 \\ 2r - s = -3 \end{cases}$

5. $\begin{cases} -7 = 2x + 6y \\ 0 = -4x - 8y + 22 \end{cases}$

6. $\begin{cases} 3r + 12t = 6 \\ r = 2s \\ t = \frac{7}{12}s \end{cases}$

7. $\begin{cases} a = 2b - 4 \\ b = 2c + 2 \\ c = 4a + 6 \end{cases}$

8. Consider the system $\begin{cases} y = -3x \\ -5x + 3y = -28 \end{cases}$.

a. Which method do you prefer to use to solve this system, substitution or the linear-combination method?

b. Solve and check the system using your method from Part a.

OBJECTIVE B Find the determinant and inverse of a square matrix. (Lesson 5-5)

In 9–12, a matrix is given.

a. Calculate its determinant.

b. Find the inverse, if it exists.

9. $\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

10. $\begin{bmatrix} 5 & 2 \\ -3 & 2 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix}$

12. $\begin{bmatrix} w & x \\ y & z \end{bmatrix}$

13. Suppose $M = \begin{bmatrix} 1 & 2 \\ -5 & 3 \end{bmatrix}$. Find M^{-1}.

14. If the inverse of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ does not exist, what must be true about its determinant?

15. Explain why the matrix $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ does not have an inverse.

16. a. Find $\begin{bmatrix} -9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}^{-1}$ using a calculator.

b. Check your answer using matrix multiplication.

OBJECTIVE C Use matrices to solve systems of two or three linear equations. (Lesson 5-6)

In 17–20, solve each system using matrices.

17. $\begin{cases} 3x - 4y = 12 \\ 4x - 6y = 48 \end{cases}$

18. $\begin{cases} 4a - 2b = -3 \\ 3a + 5b = 15 \end{cases}$

19. $\begin{cases} 2m = 5n + 4 \\ 3m = 6n - 3 \end{cases}$

20. $\begin{cases} 24 = 3x - 4y + 2z \\ 6 = x + 9y \\ 12 = 2x - 3y + 6z \end{cases}$

PROPERTIES Principles behind the mathematics

OBJECTIVE D Recognize properties of systems of equations. (Lessons 5-2, 5-3, 5-4, 5-6)

21. Are the systems $\begin{cases} 7x - 3y = 7 \\ 3x + 2y = 19 \end{cases}$ and $\begin{cases} y = 5 \\ x + y = 8 \end{cases}$ equivalent? Why or why not?

22. Give the simplest system equivalent to $4x = 8$ and $x + y = 6$.

23. What is a system with no solutions called?

In 24–27, a system is given.

a. Identify the system as inconsistent or consistent.

b. Determine the number of solutions.

24. $\begin{cases} 2x + 7y = 14 \\ 2x + 7y = 28 \end{cases}$

25. $\begin{cases} 6m - 2n = 7 \\ -3m = -2n - \frac{7}{2} \end{cases}$

26. $\begin{cases} 4a - 5b = 20 \\ 2a + 3b = -6 \end{cases}$

27. $\begin{cases} r = -t^2 \\ r = t - 3 \end{cases}$

28. For what value of k does $\begin{cases} 3x + ky = 6 \\ 15x + 5y = 30 \end{cases}$ have infinitely many solutions?

29. Find a value of t for which $\begin{cases} 2x + 8y = t \\ 3x + 12y = 7 \end{cases}$ has no solutions.

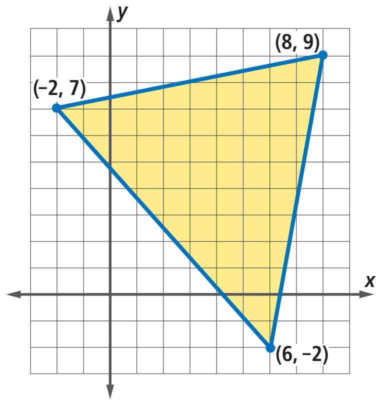
30. Suppose the determinant of the coefficient matrix of a system of equations is not zero. What can you conclude about the system?

OBJECTIVE E Recognize properties of systems of inequalities. (Lessons 5-8, 5-9)

31. **True or False** The boundaries are included in the graph of the solution set of $\begin{cases} y > 5 \\ y < 6 - x \end{cases}$.

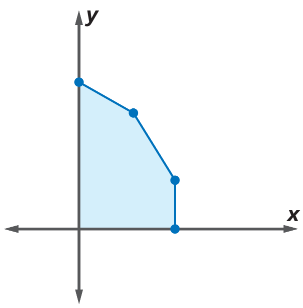
32. A system of inequalities is graphed below. Tell whether the point is a solution to the system. Justify your answer.

    a. (2, 4)   b. (8, 2)

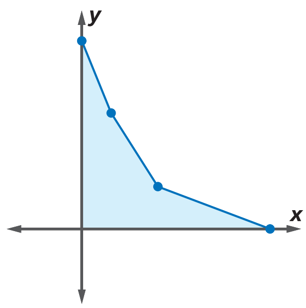


33. Tell whether the shaded region could be a feasible set in a linear-programming situation. Justify your answer.

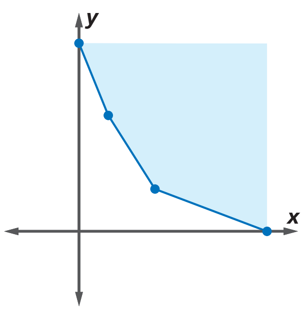
a.



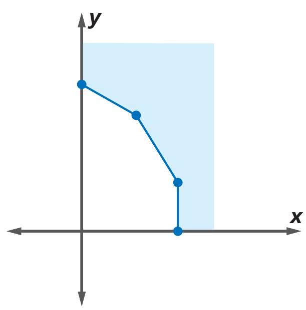
b.



c.

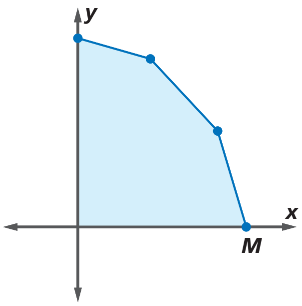


d.



34. Where in a feasible set are the possible solutions to a linear-programming problem?

35. Does the point M in the region below represent a possible solution to a linear programming problem? Why or why not?



USES Applications of mathematics in real-world situations

OBJECTIVE F **Use systems of two or three linear equations to solve real-world problems.** (Lessons 5-2, 5-3, 5-4, 5-6)

36. A painter is creating a shade of pink by mixing 5 parts white for every 2 parts red. If 3 gallons of this shade are needed, how much white and how much red is needed?

37. To make longer dives, scuba divers breathe oxygen-enriched air. A diver wants to create a tank of air that contains 36% oxygen by combining two sources. The first source is standard compressed air that contains 20% oxygen. The second source contains 60% oxygen. What percent of each source will the diver need to put into the tank?

38. After three tests, Rachelle's average in math was 76. After four tests, her average was 81. If the teacher drops Rachelle's lowest test score, Rachelle's average will be 85. What was Rachelle's lowest test score?

39. A nutritionist plans a dinner menu that provides 5 grams of protein and 6 grams of carbohydrates per serving. The meat she is serving contains 0.6 gram of protein and 0.4 gram of carbohydrates per ounce. The vegetables have 0.2 gram of protein and 0.8 gram of carbohydrates per ounce. How many ounces of meat and how many ounces of vegetables will be in each serving?

40. Molly Millions makes $6000 per month, while Mike Myzer makes $2800 per month. The following table shows average daily expenses for Molly and Mike. Both are paid on the first day of the month.

	Molly	Mike
Housing	$127	$28
Transportation	$22	$3
Food	$21	$6
Other	$15	$2

a. Who has more money left from their paycheck at the end of the month?

b. On which day of the month do Mike and Molly have about the same amount left from their pay?

OBJECTIVE G **Solve problems using linear programming.** (Lesson 5-9)

41. A real estate developer is planning a building that will combine commercial space (offices and stores) with residential space. The total development will have 40,000 square feet. Construction costs are \$225 per square foot for residential space and \$185 per square foot for commercial space, and the developer has a construction budget of \$8,500,000. Zoning laws dictate that there be no more than 30,000 square feet of commercial space in the development. The developer plans to sell the property and anticipates a profit of \$70 per square foot for residential space and \$60 per square foot for commercial space.

a. Let c = number of square feet of commercial space and r = number of square feet of residential space. Translate the constraints into a system of linear inequalities.

b. Graph the system of inequalities and find the vertices of the feasible region.

c. Find the solution (c, r) that maximizes profit.

42. Some parents shopping for their family want to know how much hamburger and how many potatoes to buy. From a nutrition table, they find that one ounce of hamburger has 0.7 mg of iron, 0 IU of vitamin A, and 7.4 grams of protein. One medium potato has 1.9 mg of iron, 17.3 IU of vitamin A, and 4.3 grams of protein. For this meal the parents want each member of the family to have at least 5 mg of iron, 15 IU of vitamin A, and 35 grams of protein. One potato costs \$0.10 and 1 ounce of hamburger costs \$0.15. The parents want to minimize their costs, yet meet daily requirements. What quantities of hamburger and potatoes should they buy for the family?

a. Identify the variables for this problem.

b. Translate the constraints of the problem into a system of inequalities. (You should have five inequalities; a table may help.)

c. Graph the system of inequalities from Part b and find the vertices of the feasible region.

d. Write an expression for the cost to be minimized.

e. Apply the Linear-Programming Theorem to determine which vertex minimizes the cost expression of Part d.

f. Interpret your answer to Part e. What is the best buy for this family?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

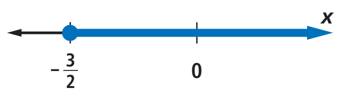
OBJECTIVE H **Solve and graph linear inequalities in one variable.** (Lesson 5-1)

43. Other weather conditions aside, the Space Shuttle will not delay or cancel a launch as long as the temperature is greater than 48°F and no more than 99°F.

a. Graph the allowable launch temperatures.

b. Write the set of allowable launch temperatures in set-builder notation.

44. Multiple Choice Which inequality is graphed below?



A $x > -\frac{3}{2}$ **B** $x < -\frac{3}{2}$

C $x \geq -\frac{3}{2}$ **D** $x \leq -\frac{3}{2}$

In 45 and 46, solve the inequality and graph its solution set.

45. $-3x + 9 > 21$ **46.** $2n + 3(n - 12) \geq 9$

47. Write an inequality that describes the graph below.

In 48–51, graph on a number line.

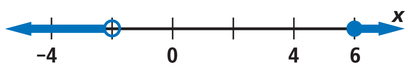
48. $\{x \mid x > 7 \text{ and } x < 11\}$

49. $\{t \mid -3 \le t < 5\} \cap \{t \mid t \ge 1\}$

50. $\{n \mid n > 9\} \cup \{n \mid n > 4\}$

51. $\{y \mid y \le 7 \text{ or } 8 \le y \le 10\}$

52. Write the compound sentence that is graphed below.



OBJECTIVE I **Estimate solutions to systems by graphing.** (Lesson 5-2)

In 53–55, estimate all solutions to the system by graphing.

53. $\begin{cases} y = 4x - 5 \\ y = 0.5x + 3 \end{cases}$

54. $\begin{cases} 3x - 2y = -6 \\ y = x^2 \end{cases}$

55. $\begin{cases} 3x + 5y = -19 \\ xy = 7 \end{cases}$

OBJECTIVE J **Graph linear inequalities in two variables.** (Lesson 5-7)

In 56–59, graph on a coordinate plane.

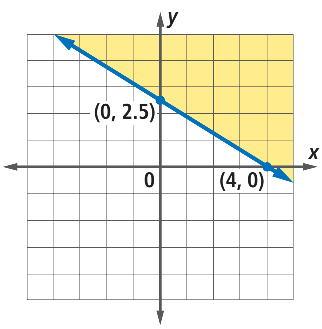
56. $x < -3$ or $y \ge 1$

57. $x \ge 7$ and $y \ge 13$

58. $y \ge -2x + 2$

59. $2x - 5y < 8$

60. Write an inequality to describe the shaded region below.



OBJECTIVE K **Solve systems of inequalities by graphing.** (Lesson 5-8)

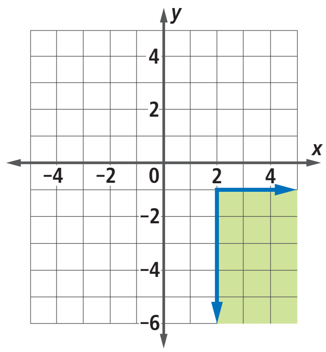
In 61–63, graph the solution set.

61. $\begin{cases} x \ge 3 \\ y \le -2 \end{cases}$

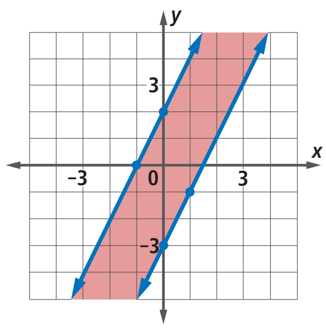
62. $\begin{cases} 5c + 7d < 35 \\ 5c - 7d > -1 \end{cases}$

63. $\begin{cases} 3x \ge -6 \\ 2(x + y) \le 6 \\ 6 > y - 2x \end{cases}$

64. Use a compound sentence to describe the shaded region below.



65. **Multiple Choice** Which of the following systems describes the shaded region below?



A $\begin{cases} y < \frac{1}{2}x + 2 \\ y < \frac{1}{2}x - 3 \end{cases}$

B $\begin{cases} y > 2x + 2 \\ y < x - 3 \end{cases}$

C $\begin{cases} y \le 2x + 2 \\ y \ge 2x - 3 \end{cases}$

D $\begin{cases} y \le x - 1 \\ y \ge 2x + 1.5 \end{cases}$