

Lesson

5-7

Graphing Inequalities in the Coordinate Plane

Vocabulary

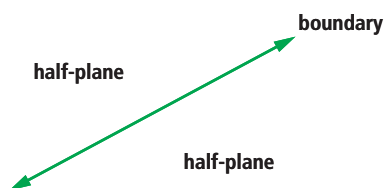
half-plane

boundary of a half-plane

lattice point

► **BIG IDEA** The graph of a linear inequality in two variables consists of the points on one side of a line, either with or without the line.

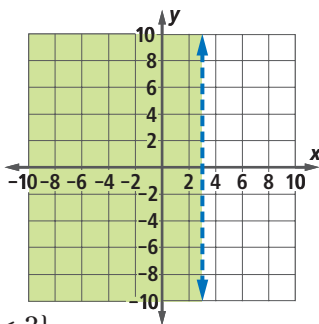
As you learned in Lesson 5-1, solutions to compound sentences with inequalities involving only one variable can be graphed on a number line. In this lesson, we review the graphs of inequalities in two variables.



When a line is drawn in a plane, the line separates the plane into three distinct sets of points: two regions called **half-planes** and the line itself. The line is called the **boundary of the half-planes**. The boundary does *not* belong to either half-plane.

Inequalities with Horizontal or Vertical Boundaries

Inequalities with one variable can be graphed on either a number line or in the coordinate plane. In the plane, think of $x < 3$ as $x + 0 \cdot y < 3$. The solution set is the set of all points (x, y) with x -coordinates less than 3. Graph $x = 3$ with a dashed line because the points on this line are *not* part of the solution set. Shade the half-plane to the left of the line $x = 3$. Shaded is $\{(x, y) \mid x < 3\}$.



STOP QY1

Half-Planes with Oblique Boundaries

The line $y = mx + b$ is oblique when $m \neq 0$. The half-planes of this line are described by the inequalities $y > mx + b$ (the half-plane above the line) and $y < mx + b$ (the half-plane below the line). Read $y > mx + b$ as “the set of all points where the y -coordinate is greater than $mx + b$.”

Mental Math

Kelley's hair is 8 inches long and grows at a rate of 0.5 inch per month.

- How long will Kelley's hair be m months from now?
- How long will Kelley's hair be 3 months from now?
- How long was Kelley's hair 3 months ago?
- Four months ago, Kelley got a hair cut. She cut 3 inches off her hair. How long was her hair before her hair cut?

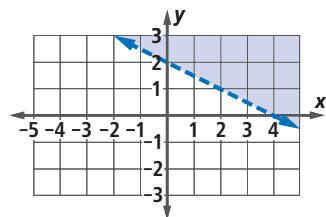
► QY1

Describe the graph of $\{(x, y) \mid y \geq -100\}$.

Example 1

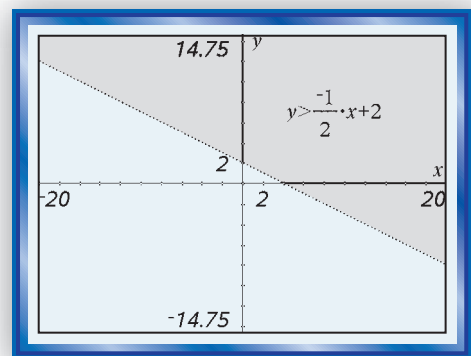
Graph the linear inequality $y > -\frac{1}{2}x + 2$.

Solution 1 First, graph the boundary $y = -\frac{1}{2}x + 2$. Use a dashed line because the boundary points do not satisfy the inequality. For the points on the boundary, the y -coordinates are equal to $-\frac{1}{2}x + 2$. The y -coordinates are greater than $-\frac{1}{2}x + 2$ for all points above the line. So the solution is the set of all points above the line. Shade the half-plane above the line to show this.



Solution 2 Use a graphing utility to graph the inequality. On the grapher shown, you can enter the inequality symbol directly. On other graphers, you may have to choose “ $y >$ ” from a menu, then enter the rest of the inequality.

Graphers may also differ in the appearance of the boundary line itself. Although this inequality calls for a dotted line, many graphing utilities only show a solid line. The display at the right shows a dotted line, but it is difficult to see.



Check Pick a point in the shaded region. We pick $(4, 6)$. Do the coordinates satisfy $y > -\frac{1}{2}x + 2$? Is $6 > -\frac{1}{2}(4) + 2$? Yes, it checks.

STOP QY2

If the equation for a line is not in slope-intercept form, you can still describe its half-planes rather easily. The half-planes of the line with equation $Ax + By = C$ are described by

$$Ax + By < C \text{ and } Ax + By > C.$$

To decide which inequality describes which side of the line, pick a point not on the line and test it in the inequality.

QY2

How do you enter $y > -\frac{1}{2}x + 2$ on your grapher? Does the boundary line show as dotted or solid?

GUIDED**Example 2**

Is the set of points satisfying $3x - 2y > 6$ above or below the line with equation $3x - 2y = 6$?

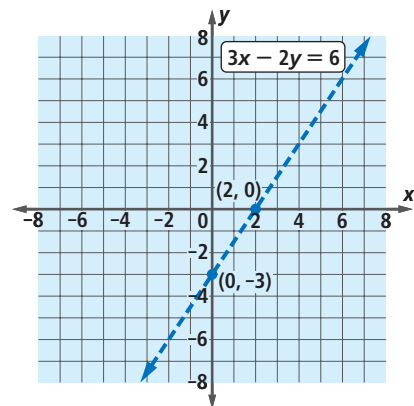
Solution Refer to the graph of $3x - 2y = 6$ at the right. Pick a point not on the line:

(?, ?) is a point ? (above/below) the line.

Test your point.

Is $3 \cdot \underline{\quad} - 2 \cdot \underline{\quad} > 6$?

(continued on next page)



This point _____? (does/does not) satisfy the inequality.

Based on your answer, the points satisfying $3x - 2y > 6$ are _____? (above/below) the line with equation $3x - 2y = 6$.

Lattice Points

In the examples you have seen, the domain of all variables is the set of real numbers or a subset of the real numbers. In these cases, the graph of an inequality consists of all points in a region, indicated by shading. However, in some situations the domain of each variable is a discrete set, such as the set of integers. In these cases, the solution set consists of points whose coordinates are both integers. These points are called **lattice points**. If there are not too many lattice-point solutions, you should indicate each with a dot on the plane, rather than shading an entire region.

READING MATH

A *lattice* is an open framework of interwoven strips forming regular patterned spaces. The grid lines on an x - y graph form a lattice. You may see a lattice on a window or gate, or as the top crust of a pie.

Example 3

Aleta wants to buy p pencils at 12 cents each and r erasers at 15 cents each and spend no more than \$1. What combinations (p, r) are possible?

Solution The domains of the variables p and r are nonnegative integers, so every solution is a lattice point, $p \geq 0$, and $r \geq 0$.

Write an inequality to represent the situation.

The cost of the pencils is $12p$ cents.

The cost of the erasers is $15r$ cents.

So $12p + 15r \leq 100$.

Now graph the corresponding equation $12p + 15r = 100$.

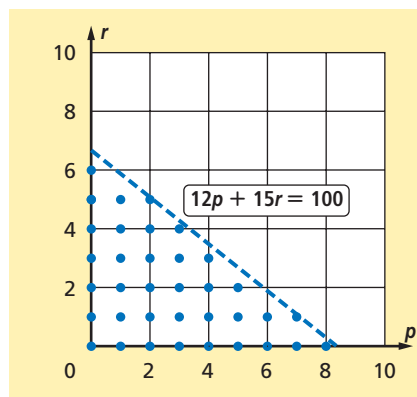
The p -intercept is $\frac{100}{12} = 8\frac{1}{3}$ and the r -intercept is $\frac{100}{15} = 6\frac{2}{3}$.

A graph is shown at the right. Notice that the line is dashed.

This is because the restricted domains of p and r mean that a point on the boundary is included in the solution set only if both coordinates of the point are integers.

Identify all lattice points satisfying the three inequalities $12p + 15r \leq 100$, $p \geq 0$, and $r \geq 0$ and mark them on the graph. There are 36 possible points.

When there are too many lattice points to graph them all, you should shade the region they are in and make a note that only the appropriate discrete values are solutions.



Questions

COVERING THE IDEAS

- Fill in the Blanks** A line separates a plane into two distinct regions called ____?. The line itself is called the ____? of these regions.
- Graph the solutions to $x \leq 4.93$
 - on a number line.
 - in the coordinate plane.
- Does the graph in the coordinate plane of all solutions to $y < 3$ consist of the points above or below the line with equation $y = 3$?
- To graph the inequality $y > 5 - 3x$, should you shade above or below the line with equation $y = 5 - 3x$?
- Matching** Tell which sentence each point satisfies.

a. $(-4, 4)$	i. $y > -\frac{1}{2}x + 2$
b. $(11, 2)$	ii. $y = -\frac{1}{2}x + 2$
c. $(0, 0)$	iii. $y < -\frac{1}{2}x + 2$

In 6 and 7, graph the inequality.

- $100x - 80y > 200$
- $y - \frac{x}{3} \leq 6$
- What name is given to a point with integer coordinates?

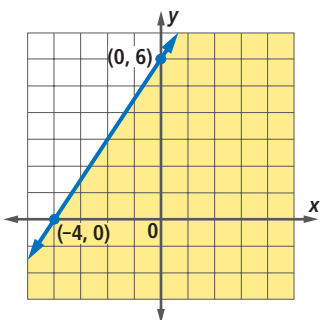
In 9–11, refer to Example 3.

- What is the greatest number of pencils Aleta can purchase?
- What is the greatest number of erasers Aleta can purchase?
- If Aleta wants an equal number of pencils and erasers, what is the greatest number of each she can purchase?
- Norma Lee Lucid wants to buy d DVDs at \$19.95 each and m music CDs at \$14.95 each. She wants to spend less than \$100.00.
 - Write an inequality in d and m describing this situation.
 - Graph all solutions.
 - What pairs (d, m) are possible?

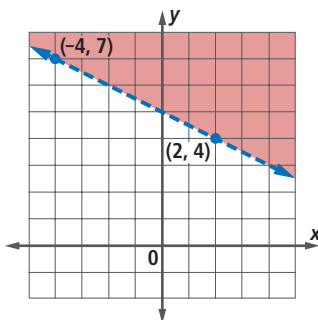
APPLYING THE MATHEMATICS

In 13 and 14, write an inequality that describes the graph.

13.



14.



In 15 and 16, a group of 4 adults and 8 children are planning to attend a family reunion at an amusement park. Children's tickets cost \$24.95 each and adult tickets cost \$32.50 each. The group budgeted \$300.00 for the trip, which is not quite enough money for everyone to attend. Some children and adults will have to stay home.

15. State whether each of these combinations will be able to attend.
- 4 children and 4 adults
 - 6 children and 3 adults
 - 7 children and 4 adults
16. a. Graph all pairs of numbers of children and adults who can attend.
b. How many pairs are there?
17. Can the graph of a linear inequality in two variables also be the graph of a function? Explain why or why not.

REVIEW

18. a. Write the matrix form of the system $\begin{cases} 3x + 6y = 18 \\ -4x - 5y = 20 \end{cases}$.
b. Solve the system. (Lesson 5-6)

19. Use the System-Determinant Theorem to determine whether

$$\text{the system } \begin{cases} A + 3B - 2C = 5.5 \\ 7A - 5B + C = 1 \\ B - C = -1 \end{cases} \text{ has exactly one solution.}$$

(Use a calculator to find the determinant.) (Lesson 5-6)

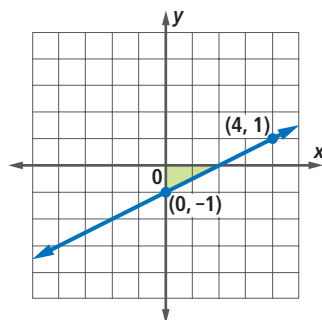
20. Antonio claims to have a shortcut for determining if the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an inverse whenever c and d are nonzero. If the fractions $\frac{a}{c}$ and $\frac{b}{d}$ are equal, Antonio argues, then the matrix has no inverse. Otherwise, it does. Does Antonio's method work? Explain your answer. (Lesson 5-5)
21. Solve $37w + 3 > 77$ and $8 - 4w \geq -10$. Write the solution in set-builder notation and graph it on a number line. (Lesson 5-1)
22. Meteor Crater in Arizona is approximately in the shape of a cylinder 1500 meters in diameter and 180 meters deep. Scientists estimate that the meteor that created it weighed about 300,000 tons at entry. Assuming the volume of the crater varies directly with the weight of the meteor, what volume of crater would you expect for a meteor weighing 500,000 tons? (Lesson 2-1)



When the crater was first discovered, it was surrounded by about 30 tons of meteoritic iron chunks, scattered over an area 12 to 15 kilometers in diameter.

EXPLORATION

23. This problem was made up by the Indian mathematician Mahavira and dates from about 850 CE. "The price of nine citrons and seven fragrant wood apples is 107; again, the mixed price of seven citrons and nine fragrant wood apples is 101. Oh you arithmetician, tell me quickly the price of a citron and a wood apple here, having distinctly separated these prices well." At this time algebra had not been developed yet. How could this question be answered by someone without using algebra?
24. Write a system of three inequalities that describes the shaded region below.



QY ANSWERS

- It is the half plane together with its boundary consisting of all points whose y -coordinate is greater than or equal to -100 .
- Answers vary.