Lesson

5-2 Solving Systems Using Tables, Graphs, or a CAS

Vocabulary

system

solution set of a system

BIG IDEA The solution(s) to a system of equations in two variables can be estimated by examining a table or graph and often solved exactly by using a CAS.

Minnie Strikes and Noah Spares are in a bowling league in which players are ranked using a handicap system. The maximum handicap is 50 pins per game and depends on the player's average. Minnie has a high average, so she has a low 5-pin handicap. Noah has a low average; his handicap is 45 pins. Noah practices, and his handicap decreases by 5 pins each month. Minnie does not practice, and her handicap increases by 3 pins per month. So after x months; Noah's handicap $N(x)$ is $45 - 5x$ and Minnie's handicap $M(x)$ is $5 + 3x$.

If the situation continues, then at some point Minnie's and Noah's handicaps will be the same. To determine when this will happen, you can solve a *system of equations*. Remember that a **system** is a set of conditions joined by the word *and*. Thus, if we call the handicaps y, the following compound sentence models the situation where the handicaps are equal: $y = 45 - 5x$ *and* $y = 5 + 3x$.

A system is often denoted by a brace: $\begin{cases} y = 45 - 5x \\ y = 5 + 3x \end{cases}$.

The **solution set of a system** is the intersection of the solution sets of the individual sentences in the system.

Mental Math

Suppose that the amount of hair a cat sheds varies directly as the square of the cat's height.

a. Fraidy is 3 times as tall as she was when she was a kitten. How much hair does she shed now compared to when she was a kitten?

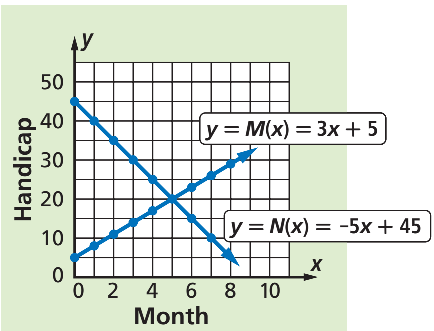
b. Mittens is 1.2 times as tall as Fluffy. How much hair does Mittens shed compared to Fluffy?

c. Suppose that a Burmese sheds four times as much as a Siamese. How many times as tall as the Siamese is the Burmese?

Finding Solutions Using Tables and Graphs

To solve a system, you can create a table or a graph. To speed up graphing, use the equations in slope-intercept form.

Noah's handicap $N(x) = -5x + 45$
Minnie's handicap $M(x) = 3x + 5$



Month	$N(x)$	$M(x)$
0	45	5
1	40	8
2	35	11
3	30	14
4	25	17
5	20	20
6	15	23
7	10	26

The table and graph show that the handicaps $M(x)$ and $N(x)$ both equal 20 when $x = 5$. They also show that when $x < 5$, Minnie has a lower handicap than Noah. When $x > 5$, Noah's handicap is lower.

Solutions to systems of equations can be written in several ways. For example, the solution to Minnie and Noah's system can be expressed by

(1) listing the solution: (5, 20).

(2) writing the solution set: {(5, 20)}.

(3) writing a simplified equivalent system: $\begin{cases} x = 5 \\ y = 20 \end{cases}$.

STOP QY1

QY1

If Minnie's handicap increased by 5 pins each month, when would her handicap be equal to Noah's?

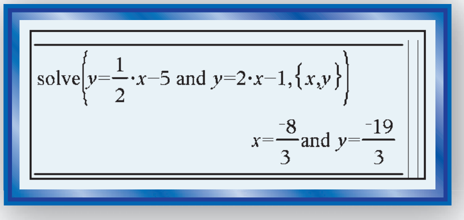
Solving Systems with a CAS

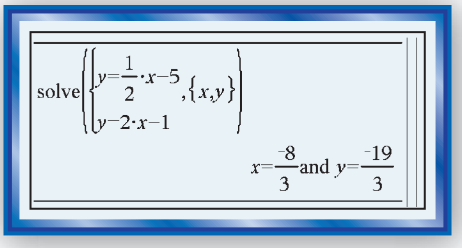
You can also use a CAS to solve a system of equations. This is a good method if solutions are not easily found in tables or on graphs.

Example 1

Solve the system $\begin{cases} y = \frac{1}{2}x - 5 \\ y = 2x - 1 \end{cases}$.

Solution Most CAS have more than one way to enter and solve systems. Two different approaches on one machine are shown at the right. The first approach enters the system as a compound sentence. The second enters the system using a brace. You should find out how to do this on your CAS.





So the solution is $\left(-\frac{8}{3}, -\frac{19}{3}\right)$.

STOP QY2

QY2

Verify the solution to Example 1 on your CAS.

Solving Nonlinear Systems

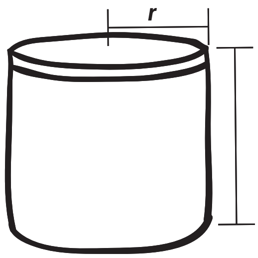
Systems can involve nonlinear equations.

Example 2

Kaila is a packaging engineer at Healthy Soup Company. The marketing department has requested that she design a new soup can. The new can must hold 360 mL of soup and be in the shape of a cylinder. An eye-catching new label must be large enough to show the product name and all the nutrition content data. Marketing wants the area of the label to be about 225 cm^2. What are the dimensions of this new can?

Solution Define variables. Let h = the height of the can and r = the radius of the can.

Draw a picture, as shown at the right.



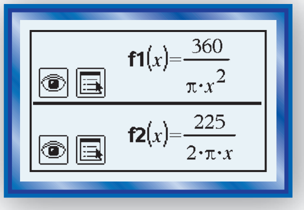
Write a system of equations. The volume of the can is given by the formula $V = \pi r^2 h$. The can must hold 360 cm^3 of soup (1 mL = 1 cm^3). So, $\pi r^2 h = 360$.

The area of the rectangular label that wraps around the can is the product of the can's circumference $2\pi r$ and its height h. Marketing wants the area to be about 225 cm^2. So, $2\pi rh = 225$.

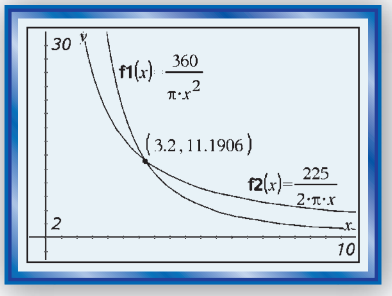
So, a system describing the situation is $\begin{cases} \pi r^2 h = 360 \\ 2\pi rh = 225 \end{cases}$.

Solve the system. One way to solve this system is by graphing. First, solve each equation for h so that you can enter the functions into a graphing utility.

$$\begin{cases} h = \dfrac{360}{\pi r^2} \\ h = \dfrac{225}{2\pi r} \end{cases}$$

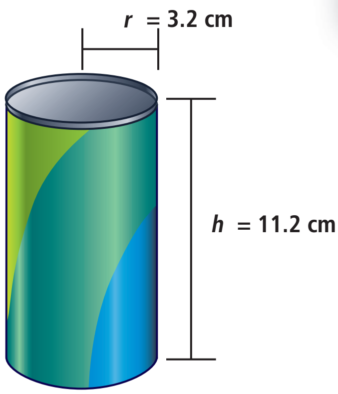


Think of r as the independent variable and h as the dependent variable. Enter the equations into a graphing utility as shown at the right. r becomes x and h becomes $f(x)$.



Both r and h must be positive, so only consider the branches of these inverse variation and inverse square variation graphs in the first quadrant. These branches intersect at only one point. Use the `intersect` command on your graphing utility to find the coordinates of the point of intersection.

The display shows the point of intersection is about (3.2, 11.2), so the radius of the can is approximately 3.2 cm and the height is approximately 11.2 cm.



This is the only solution that meets the criteria of a 360 mL volume and a 225 cm^2 label area.

When a system is nonlinear, it is possible for it to have more than one solution, as in Example 3.

GUIDED

Example 3

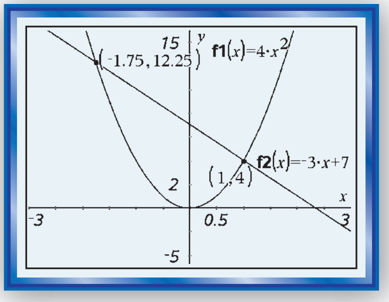
Solve the system $\begin{cases} y = 4x^2 \\ 2y + 6x = 14 \end{cases}$.

Solution 1 First, solve both equations for y.

The first equation is already solved for y. The second equation is equivalent to y = __?__ + 7.

Graph both equations in a window that shows all the points of intersection.

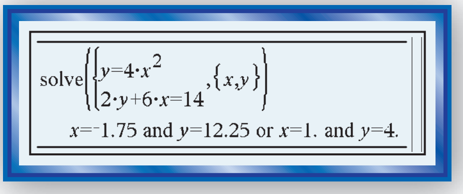
Window dimensions: __?__ ≤ x ≤ __?__ and __?__ ≤ y ≤ __?__.



Use the intersection feature of the graphing utility to identify both intersection points.

The graphs intersect at (__?__, __?__) and (__?__, __?__).

Solution 2 Solve on a CAS.



So the two possible solutions are (__?__, __?__) and (__?__, __?__).

You will learn an algebraic method of solution later in this book.

Questions

COVERING THE IDEAS

1. What is a system?

In 2 and 3, refer to Minnie and Noah's situation.

2. After how many months is Noah's handicap less than Minnie's?
3. Suppose Noah's handicap has decreased to 40 pins, while Minnie's handicap has increased to 20 pins. If Noah's handicap continues decreasing by 3 pins each month and Minnie's declines by 1 pin each month, when will their handicaps be equal?
   a. Solve this new system of equations by using the `solve` command.
   b. Solve the system by graphing.
   c. Which method do you prefer and why?

4. a. Use brace notation to write the system $y = 7x - 4$ and $y = 10$.

b. Solve the system in Part a using any of the four methods below. Check your solution with any of the other methods.

i. by hand

ii. using the `solve` command

iii. using a graphing utililty to find the intersection points

iv. using a table of values

c. Explain why you chose the methods that you used in Part b.

5. a. Solve the system $\begin{cases} y = \frac{1}{2}x - 1.5 \\ y = 2x + 3 \end{cases}$ by graphing.

b. What do you notice about the relationship between the two lines graphed in Part a?

c. Check your answer to Part a on a CAS using the `solve` command.

In 6 and 7, refer to Example 2.

6. We only looked for intersection points in the first quadrant. Why?

7. a. How can you tell that (3.2, 11.2) is an approximate and not an exact solution?

b. A CAS gives the solution (3.2, 11.190582). Is it an exact solution? Justify your answer.

8. Refer to Example 3. Would a window with $-2 \le x \le 4$ and $1 \le y \le 13$ show both intersections of the graphs? Why or why not?

APPLYING THE MATHEMATICS

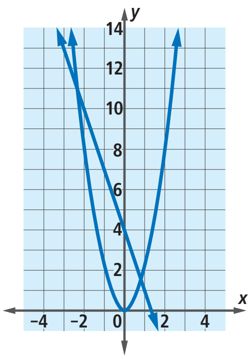
In 9–11, use the given systems and their graphs.

a. Tell how many solutions the system has.

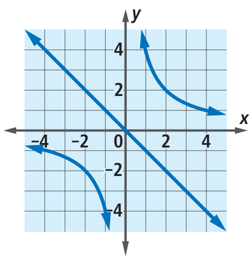
b. Estimate the solutions, if there are any, to the nearest hundredth.

c. Verify that your solutions satisfy all equations of the system.

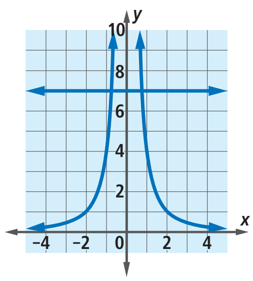
9. $\begin{cases} y = 2x^2 \\ y = -3x + 4 \end{cases}$



10. $\begin{cases} y = -x \\ xy = 4 \end{cases}$



11. $\begin{cases} y = 7 \\ x^2y = 4 \end{cases}$



In 12 and 13, a system is given.

a. **Graph the system.**

b. **Tell how many solutions the system has.**

c. **Estimate any solutions to the nearest tenth.**

12. $\begin{cases} y = -3x^2 \\ x - y = 1 \end{cases}$

13. $\begin{cases} y = \frac{-6}{t^2} \\ y = 12 \end{cases}$

14. The Aguilar family needs to hire a moving company. They received two quotes.
    Company 1: \$1000 plus \$0.10 per pound
    Company 2: \$750 plus \$0.20 per pound
    The Aguilars would like to find the value of the *break-even* point, where the cost is the same with both movers.
    a. Write a system describing this situation.
    b. Find the break-even point on a CAS by making a table of values starting at 0 and with a step size of 500.
    c. What is an appropriate domain for the functions of this system? Explain.

15. Use a graph to find out if there is a pair of numbers x and y whose product is 1740 and whose sum is 89.

16. Tomi is planting a vegetable garden that will be rectangular in shape. He has purchased 72 linear feet of fencing material to enclose the garden. He has enough fertilizer to take care of 320 square feet of garden.
    a. Write a system of equations relating the length L and width W of the rectangle to represent this situation.
    b. Assume that L is the independent variable. Rewrite each equation of the system to give W in terms of L.
    c. Solve the system using any method.

REVIEW

17. Solve $2 - 4x < 5$ and graph your solution on a number line. **(Lesson 5-1)**

18. Alkas is throwing a party. She tells Felix to bring a friend, but no more than 5 friends. Write a double inequality describing the number F of friends Felix can bring to the party without upsetting Alkas. **(Lesson 5-1)**

19. Find the 2×2 matrix for a transformation that maps the point (1, 0) onto (–4, 1) and the point (0, 1) onto (–1, –4). **(Lesson 4-7)**

20. True or False If $Ax + By$ is a linear combination of x and y, and $Cx + Dy$ is another linear combination of x and y, then the sum of these two expressions is a linear combination of x and y. **(Lesson 3-2)**

21. In basketball, a player's effective shooting percentage e is given by the formula $e = \frac{f + 1.5t}{a}$, where f is the total number of two-point shots made, t is the number of three-point shots made, and a is the total number of shots attempted. Write a formula for the number t of three-point shots a player has made in terms of e, f, and a. **(Lesson 1-7)**

EXPLORATION

22. The break-even point is used often when making business decisions. Do some research about how the break-even point is applied in different types of financial analyses. Write a summary of your findings.

QY ANSWERS

1. after 4 months

2. Answers vary. Sample: