Chapter 7 Summary and Vocabulary

- When $x > 0$, the expression x^m is defined for any real number m. This chapter covers the meanings and properties of x^m when m is a positive or negative rational number.
- In previous courses, you have learned basic properties of **powers**. For any nonnegative **bases** and nonzero real **exponents**, or any nonzero bases and integer exponents:

Product of Powers Postulate	$x^m \cdot x^n = x^{m+n}$
Power of a Power Postulate	$(x^m)^n = x^{mn}$
Power of a Product Postulate	$(xy)^n = x^n y^n$

- The following theorems can be deduced from these postulates. For any positive bases and real number exponents and any nonzero bases and integer exponents:

Quotient of Powers Theorem	$\frac{x^m}{x^n} = x^{m-n}$
Power of a Quotient Theorem	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$
Zero Exponent Theorem	$x^0 = 1$
Negative Exponent Theorem	$x^{-m} = \frac{1}{x^m}$
Exponent Theorem	$x^{\frac{1}{n}}$ is the positive solution to $b^n = x$.
Rational Exponent Theorem	$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \left(x^{\frac{1}{n}}\right)^m$

- From these properties, we see that $x^{\frac{1}{n}}$ is the positive root of x, and $x^{\frac{m}{n}}$ is both the mth power of the positive ***n*th root** of x, and the positive nth root of the mth power of x. These properties are not always true when $x < 0$, so we do not define x^m when $x < 0$ and m is not an integer. You can use these properties to simplify expressions and to solve equations of the form $x^n = b$. To solve such an equation, raise each side of the equation to the $\frac{1}{n}$ power.

Vocabulary

Lesson 7-1
powering, exponentiation
*base
*exponent
*power
**n*th-power function
identity function
squaring function
cubing function

Lesson 7-4
annual compound interest
principal
semi-annually
compounding daily
annual percentage yield, APY

Lesson 7-5
*geometric sequence, exponential sequence
constant multiplier
constant ratio

Lesson 7-6
cube root, *n*th root

- Equations involving powers are found in many fields, including investments, science, and music. In the **General Compound Interest Formula** $A = P\left(1 + \frac{r}{n}\right)^{nt}$, A is the value of an investment of P dollars earning interest at a rate r compounded n times per year for t years. When P, r, n, and t are given, you can solve for A; when A, r, n, and t are known, you can solve for P. Because of the multitude of ways to calculate interest, federal law requires institutions to advertise the **annual percentage yield (APY)** of their accounts.
- A **geometric sequence** is a sequence in which the ratios of consecutive terms are constant; each term is a constant multiple r of the preceding term. In symbols, given g_1, then for all $n \geq 2$, $g_n = rg_{n-1}$. The nth term of a geometric sequence can be found explicitly using the formula $g_n = g_1 r^{n-1}$.

Postulates and Theorems

Probability of Repeated Independent Events (p. 454)
Product of Powers Postulate (p. 459)
Quotient of Powers Theorem (p. 460)
Power of a Product Postulate (p. 460)
Power of a Quotient Theorem (p. 461)
Zero Exponent Theorem (p. 461)
Power of a Power Postulate (p. 462)
Negative Exponent Theorem (p. 467)
Annual Compound Interest Formula (p. 473)
General Compound Interest Formula (p. 473)
Recursive Formula for a Geometric Sequence (p. 479)
Explicit Formula for a Geometric Sequence (p. 481)
Number of Real Roots Theorem (p. 488)
$\frac{1}{n}$ Exponent Theorem (p. 489)
Rational Exponent Theorem (p. 493)

Chapter 7 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. A student writes $(3 + 4)^{\frac{1}{2}} = 3^{\frac{1}{2}} + 4^{\frac{1}{2}}$. Explain why this sentence is not true.

In 2–4, write as a whole number or simple fraction.

2. 7^{-2} **3.** $(214{,}358{,}881)^{\frac{1}{8}}$ **4.** $\left(\frac{343}{27}\right)^{-\frac{4}{3}}$

5. Write without an exponent: $\frac{7.3 \cdot 10^3}{10^{-4}}$.

In 6 and 7, simplify. Assume $x > 0$ and $y > 0$.

6. $\left(1728x^9y^{27}\right)^{\frac{1}{3}}$ **7.** $\frac{84x^{21}y^5}{6x^3y^7}$

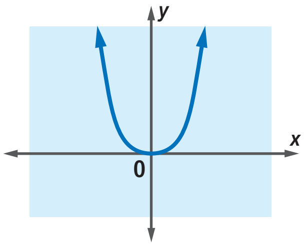
8. True or False Justify your answer.

a. $-5^{\frac{1}{3}} < 5^{-\frac{1}{3}}$ **b.** $3^{-6.4} < 3^{-6.5}$

9. Solve $x^{\frac{3}{2}} = 0.8$ for x. Round your answer to the nearest thousandth.

10. Approximate $4.26^{\frac{1}{6}}$ to the nearest hundredth.

11. A graph of a power function, $y = x^n$, is shown at the right. Is n even or odd? Justify your answer.



12. Consider this sequence, which gives the maximum height of a bouncing ball after the nth bounce:

$$\begin{cases} h_1 = 5 \\ h_n = 0.85h_{n-1}, \text{ for integers } n \geq 2. \end{cases}$$

a. Find the height of the ball after the 1st, 2nd, 3rd, and 4th bounces.

b. Write an explicit formula for this sequence.

13. True or False The range of all power functions is the set of real numbers. Justify your answer.

In 14 and 15, the first few terms of a geometric sequence are given.

a. Find the next two terms.

b. Write a recursive formula for the sequence.

c. Write an explicit formula for the nth term of the sequence.

14. 3, 0.6, 0.12, 0.024, …

15. –4, –12, –36, –108, …

16. A quiz has 10 multiple choice questions. Each question has three choices. What is the probability of guessing all 10 questions correctly?

17. The formula $m = 1.23 \cdot (1.2)^b$ gives the approximate number of minutes m that b bacon slices will take to cook in a typical microwave. How long will it take 2 slices of bacon to cook? Give your answer in minutes and seconds.

18. Identify all real 10th roots of 1024.

19. True or False $625^{\frac{1}{4}} = -5$. Justify your answer.

20. A bank account pays 4.25% annual interest. Suppose you deposit \$500 in the account and then do not deposit or withdraw any money for 6 years.

a. How much will you have in the account after 6 years if the interest is compounded annually?

b. What is the APY if the interest is compounded quarterly?

21. A zero-coupon bond paying 6.8% interest compounded monthly for 7 years has matured, giving the investor \$7854. How much did the investor pay for the bond 7 years ago?

Chapter 7

Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Evaluate b^n when $b > 0$ and n is a rational number. (Lessons 7-2, 7-3, 7-6, 7-7, 7-8)

In 1–6, write as a simple fraction or whole number. Do not use a calculator.

1. 17^0
2. 2^{-3}
3. $\left(\frac{5}{3}\right)^{-1}$
4. $64^{-\frac{1}{2}}$
5. $8^{\frac{4}{3}}$
6. $1{,}000{,}000^{\frac{1}{3}}$

In 7–9, approximate to the nearest hundredth.

7. $123^{\frac{2}{3}}$
8. $5 \cdot 8^{\frac{1}{4}}$
9. $7^{1.5}$

OBJECTIVE B Simplify expressions or solve equations using properties of exponents. (Lessons 7-2, 7-3, 7-7, 7-8)

In 10–12, solve.

10. $13^4 \cdot 13^{12} = 13^x$
11. $\frac{2^6}{2^{-3}} = 2^y$
12. $(7^z)^3 = 7^9$

In 13–18, simplify. Assume all variables represent positive numbers.

13. $(-2x^3)^2$
14. $(-2x^2)^3$
15. $\left(\frac{p}{q}\right)^7\left(\frac{5q}{3p}\right)^3$
16. $\frac{21b}{(7b^{-5})(6b^5)}$
17. $\frac{-16x^8y^{\frac{5}{2}}}{4x^2y^{\frac{1}{2}}}$
18. $\frac{(r^3s^2)^{\frac{1}{2}}}{r^2s^3}$

OBJECTIVE C Describe geometric sequences explicitly and recursively. (Lesson 7-5)

In 19 and 20, the first few terms of a geometric sequence are given.

a. Find an explicit formula for the nth term.
b. Find a recursive formula for the sequence.
c. Find the 16th term.

19. 4, –12, 36, –108, 324, …
20. 2, 0.5, 0.125, 0.03125, …
21. Find the 25th term of a geometric sequence whose first term is 4 and whose constant multiplier is 1.075. Express your answer
    a. exactly.
    b. to the nearest thousandth.
22. **Multiple Choice** Which of the following could be the first three terms of a geometric sequence?
    A 9, 3, –6, …
    B $6\frac{2}{3}, 66\frac{2}{3}, 666\frac{2}{3}, \ldots$
    C $\frac{4}{7}, \frac{11}{7}, \frac{18}{7}, \ldots$
    D 0.4, 0.16, 0.64, …

In 23–26, give the first four terms of the geometric sequence described.

23. constant ratio of 3, first term 7
24. first term is $\frac{2}{3}$, fourth term is $\frac{16}{81}$
25. $\begin{cases} t_1 = 6 \\ t_n = \frac{2}{3}t_{n-1}, \text{ for integers } n \geq 2 \end{cases}$
26. $\begin{cases} h_1 = -4 \\ h_{n+1} = -1.5h_n, \text{ for integers } n \geq 1 \end{cases}$

OBJECTIVE D **Find all real solutions to equations of the form $x^n = b$, where $x \geq 0$ and n is a rational number.** (Lessons 7-6, 7-7, 7-8)

In 27–35, find all real solutions.

27. $12x^2 = 432$
28. $33 = a^3$
29. $y^4 = 16$
30. $p^{-2} = 16$
31. $6 = y^{\frac{1}{3}}$
32. $7q^{-\frac{3}{5}} = 9$
33. $x^{\frac{4}{9}} = 12$
34. $r^{-\frac{3}{2}} = \frac{1}{8}$
35. $1.55 = m^{\frac{1}{6}}$

PROPERTIES Principles behind the mathematics

OBJECTIVE E **Recognize properties of nth powers and nth roots.** (Lessons 7-2, 7-6, 7-7, 7-8)

In 36–38, True or False. Justify your answer.

36. $-5 = (390{,}625)^{\frac{1}{8}}$
37. $\pi^{-5.3} < \pi^{-5.4}$
38. $t^{-\frac{3}{4}} = \frac{1}{(t^3)^{\frac{1}{4}}}$ $(t > 0)$
39. a. Identify all the square roots of 121.
    b. Simplify $121^{\frac{1}{2}}$.
40. **True or False** $3i$ is a 4th root of 81.
41. **True or False** For all $x > 1$, $x^{\frac{1}{4}} < x$.
42. Suppose $0 < v < 1$. Arrange from least to greatest: v, $v^{\frac{3}{2}}$, v^{-3}, $v^{\frac{1}{3}}$, $v^{-\frac{5}{3}}$.

In 43–46, use properties A–D below. Assume $R > 0$, $m \neq 0$, and $n \neq 0$. Identify the property or properties that justify the equality.

A $R^0 = 1$

B $R^n = \frac{1}{R^n}$

C $R^{\frac{1}{n}}$ is the positive solution to $x^n = R$.

D $R^{\frac{m}{n}} = (R^m)^{\frac{1}{n}} = \left(R^{\frac{1}{n}}\right)^m$

43. $(3.456)^{6-6} = 1$
44. $\left(q^{\frac{1}{4}}\right)^4 = q$
45. $(81)^{-\frac{1}{4}} = \frac{1}{3}$
46. $\left(\frac{1}{b}\right)^{-\frac{7}{8}} = (b^7)^{\frac{1}{8}}$
47. a. For what integer values of n does the equation $x^n = 19$ have exactly one real solution?
    b. How many solutions does it have for other nonzero integer values of n?
48. Explain why rational exponents are not defined for negative bases, using $(-125)^{\frac{1}{3}}$ and $(-125)^{\frac{2}{6}}$ as examples.
49. **Fill in the Blank** The positive nth root of a positive number equals the __?__ power of that number.

USES Real-world applications of mathematics

OBJECTIVE F **Solve real-world problems that can be modeled by expressions with nth powers or nth roots.** (Lessons 7-1, 7-6, 7-7, 7-8)

50. To qualify for a quiz show, a person must answer all questions correctly in three categories: literature, science, and current events. Suppose a person estimates that the probability of getting one question correct in literature is ℓ, in science is s, and in current events is c. If the person is asked 3 literature, 3 science, and 4 current events questions, what is the probability that the person gets all the questions right?

51. The Merchandise Mart in Chicago has about $4 \cdot 10^6$ ft^2 of floor space. This area is what percent of the $6.6 \cdot 10^6$ ft^2 of floor space in the Pentagon in Washington DC?

52. The intensity I of light varies inversely with the square of the distance d from the light source. Write a formula for I as a function of d using

a. a positive exponent.

b. a negative exponent.

In 53 and 54, use this information. Kepler's third law states that the ratio of the squares of the periods of any two planets equals the ratio of the cubes of their mean distances from the Sun. If the periods of the planets are t and T and their mean distances from the Sun are d and D, respectively, then $\frac{T^2}{t^2} = \frac{D^3}{d^3}$.

53. Find the ratio $\frac{D}{d}$ of the distance.

54. Kepler used his third law to determine how far planets were from the Sun. He knew that for Earth, $t \approx 365$ days and $d \approx 150{,}000{,}000$ km. He also knew that for Mars, $T \approx 687$ days. Use this information to find D, the mean distance from Mars to the Sun.

In 55 and 56, use this information about similar figures. If A_1 and A_2 are the surface areas of two similar figures and V_1 and V_2 are their volumes, then $\frac{A_1}{A_2} = \left(\frac{V_1}{V_2}\right)^{\frac{2}{3}}$.

55. Two similar figures have volumes 36 cm^3 and 48 cm^3. What is the ratio of the amounts of paint needed to cover their surfaces?

56. Solve the formula for $\frac{V_1}{V_2}$.

OBJECTIVE G **Apply the compound interest formulas. (Lesson 7-4)**

57. Deion wants to invest now in a bond that will give him $5000 in 7 years. The bond pays 5.25% interest compounding monthly. How much must Deion invest?

In 58 and 59, Camila put $10,000 in a 6-year 4.825% savings certificate in which interest is compounded daily 365 days per year.

58. What is the APY of Camila's savings certificate?

59. a. How much interest will she earn during the entire 6-year period?

b. How much interest will she earn during the sixth year?

In 60 and 61, Brooklyn now has $7231 in an account earning interest at a rate of 3.125% compounded quarterly.

60. Assuming she made no deposits or withdrawals in the past four years, how much money was in the account 4 years ago?

61. How much interest did she earn during the past two years?

OBJECTIVE H **Solve real-world problems involving geometric sequences. (Lesson 7-5)**

62. Suppose a ball bounces up to 77% of its previous height after each bounce, and the ball is dropped from a height of 8 m.

a. Find an explicit formula for the height after the nth bounce.

b. Find the height of the ball after the tenth bounce, to the nearest decimeter.

63. Raul set a copy machine to reduce images to 82% of their original size.

a. If the original image was 20 cm by 25 cm, what are the dimensions of the copy?

b. If each time Raul made a copy he used the copy as the preimage for the next copy, what are the dimensions of the fifth copy?

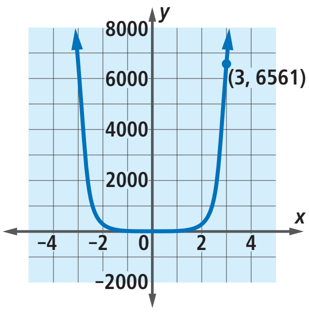
64. A weight on a pendulum moves 120 cm on its first swing. On each succeeding swing back or forth it moves 95% of the distance of the previous swing. Write the first four terms of the sequence of swing lengths.

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

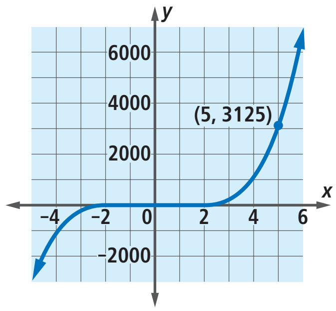
OBJECTIVE I Graph *n*th power functions. (Lesson 7-1)

In 65 and 66, an *n*th power graph is drawn. Write an equation for each function.

65.



66.



In 67 and 68, a function is given.

a. Graph the function.

b. Identify its domain and range.

c. Describe any symmetries of the graph.

67. $y = x^5$

68. $y = x^4$

69. Use a graph to explain why the equation $x^6 = -5$ has no real solutions.