Chapter 7 Summary and Vocabulary

- In Chapter 6, you saw many examples of constant-increase and constant-decrease patterns of change. They give rise to equations of the form $y = mx + b$. The change is called **linear** because the graph is a line. Now in Chapter 7, we turned our attention to patterns of change called **exponential growth** and **exponential decay.** They give rise to equations of the form $y = b \cdot g^x$.

- Graphs of exponential functions are curves. The change is called **exponential** because the independent variable is in the exponent. In exponential change, the number g is the growth factor. If $g > 1$, the situation is **exponential growth.** Among the common applications of exponential growth are compound interest and population growth. In the long run, exponential growth will always overtake a situation of linear increase. If $0 < g < 1$, the situation is **exponential decay.**

- These and other patterns can be described using the mathematical idea of a function. A **function** is a set of ordered pairs in which each first coordinate appears with exactly one second coordinate. Thus, functions exist whenever the value of one variable determines a unique value of another variable.

- A function may be described by a list of ordered pairs, a graph, an equation, or a written rule. If a function f contains the ordered pair (a, b), then we write $f(a) = b$. We say that b is the value of the function at a. If you know a formula for the function, you can obtain values and graphs of functions using calculators, spreadsheets, or paper and pencil.

- Constant-increase or constant-decrease situations are described by **linear functions.** Constant growth or decay situations are described by **exponential functions.** Repeatedly adding a quantity m to an initial value b gives rise to values of the linear function $f(x) = mx + b$. Repeatedly multiplying an initial value b by the growth factor g gives rise to values of the exponential function $f(x) = b \cdot g^x$. Spreadsheets are particularly useful for finding values of functions.

Theorems and Properties

Repeated Multiplication Property of Powers (p. 398)
Compound Interest Formula (p. 400)
Growth Model for Powering (p. 405)
Zero Exponent Property (p. 405)

Vocabulary

7-1
power, *n*th power
base
exponent
principal
interest
annual yield
compound interest

7-2
exponential growth
growth factor
exponential growth equation

7-3
exponential decay
half-life

7-4
exponential regression

7-5
function
input, output
value of the function
squaring function
independent variable
dependent variable
domain of a function
range of a function
relation

7-6
f(*x*) notation
function notation

Chapter 7 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. Evaluate $x^2 + x^0$ when $x = \frac{1}{5}$.
2. Write $8 \cdot 8 \cdot 8 \cdot 8 \cdot d \cdot d \cdot d \cdot d \cdot d \cdot d$ using exponents.
3. If $f(x) = 3x^0$, find $f(1{,}729)$.
4. If $g(y) = 3y - y^2$, find $g(-2)$.

In 5–7, Tyrone deposits $400 into a savings account that pays 4.4% interest per year.

5. Write and evaluate an expression for the amount of money Tyrone will have after 7 years, assuming he doesn't deposit or withdraw money from the account.
6. At the same time that Tyrone makes his deposit, his sister Oleta deposits $400 in a highly unusual savings account. The account pays exactly $22 interest each year. Who will have more money after 10 years?
7. Who will have more money after 25 years?

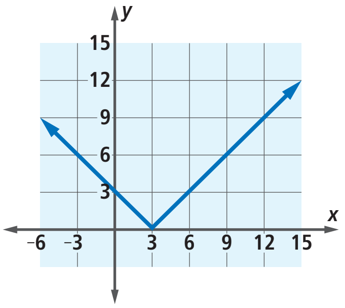
In 8–10, use the following information. A particular new 2006 car costs $34,975. Suppose its value depreciates 16% each year.

8. What is the growth factor of the value of the car?
9. Write a function $m(x)$ that approximates the car's value in x years. Specify the domain of your function.
10. Find $m(5)$, the approximate value of the car in 5 years.

For 11–13, $f(x) = 5 \cdot 0.74^x$. Calculate the value.

11. $f(1)$
12. $f(5)$
13. $f(7)$

In 14–16, use the absolute value function $f(x) = |x - 3|$ graphed below. Consider the domain as the set of all real numbers.



14. Determine $f(-12)$.
15. What is the range of the function f?
16. If the domain is restricted to $\{x: x \geq 5\}$, what is the range?

In 17–19, let $E(x) = 30(1.05)^x$ and $L(x) = 30 + 2x$.

17. Sketch a graph of these functions.
18. Which is greater, $L(9)$ or $E(9)$?
19. Give an example of a value of x when $L(x) < E(x)$.

In 20 and 21, write an equation describing the situation and graph the equation.

20. The population p of a country increases by 2.5% per year. In 1980, it had 76 million residents. Let k be the number of years since 1980.
21. The circulation c of a newspaper has decreased by 1% each month since January 2000, when it was 880,000. Let x be the number of months since January 2000.

In 22 and 23, graph the function on the given domain.

22. $h(k) = 1 - 3.5k$, $-10 \leq k \leq 8$

23. $c(x) = 10 \cdot 2^x$, $0 \leq x \leq 5$

24. **Matching** Decide which of the situations the function with the given equation describes.

i. constant increase

ii. constant decrease

iii. exponential growth

iv. exponential decay

a. $f(x) = -4x + 18$

b. $g(x) = 0.4(5)^x$

c. $h(x) = 5(0.4)^x$

d. $m(x) = \frac{2}{3}x - 7$

25. It is estimated that a house purchased in 1990 for \$100,000 has increased in value about 4% a year since that time. Suppose you want to use a spreadsheet to display the estimated value of the house from 1990 to 2010.

◇	A	B
1	Year	Value of House
2	1990	\$100,000
3		
4		
5		

a. What formula could you enter in cell A3 to get the appropriate value using cell A2?

b. Explain the process by which you would obtain appropriate amounts in cells B4 to B22.

Chapter 7 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Evaluate functions. (Lesson 7-6)

In 1–4, suppose $f(x) = 10 - 3x$. Evaluate the function.

1. $f(2)$
2. $f(-4)$
3. $f(1) + f(0)$
4. $f(3 + 6)$
5. If $g(x) = \left(\frac{11}{6}\right)^x$, give the value of $g(2)$.
6. If $h(x) = 2x^3$, calculate $h(4)$.
7. If $f(t) = -8t$ and $g(t) = 6t$, give the value of $f(-1) + g(-2)$.
8. If $E(m) = 6^m$ and $L(m) = m + 5$, find a value for m for which $E(m) < L(m)$.

OBJECTIVE B Calculate function values in spreadsheets. (Lesson 7-7)

In 9 and 10, use the spreadsheet below.

◇	A	B	C
1	x	1000(1.05)^x	1000+50x
2	0	1000	1000
3	1		
4	2		
5	3		

9. Rani wants to put values of the function with equation $y = 1{,}000(1.05)^x$ in column B of the spreadsheet.
   a. What formula can she enter in cell B3?
   b. What number will appear in cell B3?
   c. What should she do to get values of y in column B when $x = 2, 3, 4, 5, \ldots, 10$?

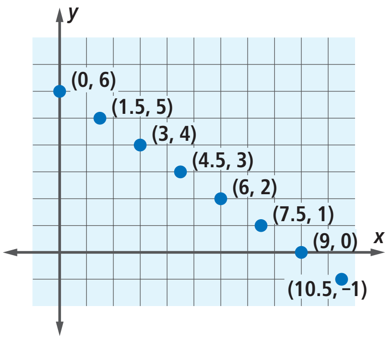
10. Olivia wants to put values of the function with equation $f(x) = 1{,}000 + 50x$ in column C of the spreadsheet.
    a. What formula can she enter in cell C3?
    b. What number will appear in cell C3?
    c. What should she do to get values of $f(x)$ in column C when $x = 2, 3, 4, 5, \ldots, 10$?

PROPERTIES The principles behind the mathematics

OBJECTIVE C Use the language of functions. (Lessons 7-5, 7-6)

11. Suppose $y = f(x)$.
    a. What letter names the independent variable?
    b. What letter names the function?

12. A linear function L is graphed below.



    a. What is $L(3)$?
    b. What is the domain of L?
    c. What is the range of L?
    d. Find a formula for $L(x)$ in terms of x.

13. Suppose a function f consists of only the ordered pairs (2, 200), (4, 400), (5, 500), and (10, 1,000).
    a. What is the domain of f?
    b. What is the range of f?
    c. Give a formula for $f(x)$ in terms of x.

USES Applications of mathematics in real-world situations

OBJECTIVE D **Calculate compound interest. (Lesson 7-1)**

14. An advertisement indicated that a 3-year certificate of deposit would yield 4.53% per year. If $2,000 is invested in this certificate, what will it be worth at the end of 3 years?

15. When Brie was born, she received a gift of $500 from her grandparents. Her parents put it into an account at an annual yield of 5.2%. Brie is now 12 years old. How much is this gift worth now?

16. In 2004, the endowment of Harvard University (the value of the university's assets) was reported to be about $22.14 billion. Suppose the trustees of the university feel they can grow this endowment by 6% a year. What would be the value of the endowment in 2010?

17. Which investment yields more money: (a) x dollars for 4 years at an annual yield of 8% or (b) the same amount of money at an annual yield of 4% for 8 years? Explain your reasoning.

OBJECTIVE E **Solve problems involving exponential growth and decay. (Lessons 7-2, 7-3, 7-4)**

18. In 1990, there were 4.4 million cell phone subscribers in the United States; by 2006, there were 219.4 million subscribers. The table below shows the number of cell phone subscribers for each year from 1990 to 2006.

Year	Cell Phone Subscribers (in thousands)
1990	4,369
1991	3,380
1992	8,893
1993	13,067
1994	19,283
1995	28,154
1996	38,195
1997	48,706
1998	60,831
1999	76,285
2000	97,036
2001	118,398
2002	134,561
2003	148,066
2004	169,467
2005	194,479
2006	219,420

    a. Create a scatterplot with y = cell phone subscribers and x = the year since 1990. Why is the exponential model a better model for these data than a linear model?
    b. Use exponential regression to find an equation to fit the data.
    c. Use your equation from Part b to predict the number of cell phone subscribers for the year 2013.

19. Twelve fish were introduced into a large lake. In 3 years, the population had multiplied by a factor of 20.

a. In 15 years, at this growth rate, by how much would the population be multiplied?

b. **Multiple Choice** If P is the number of fish t years after introduction, which formula relates P and t?

A $P = 12(20)^t$ B $P = 12(20)^{3t}$
C $P = 12(20)^{\frac{t}{3}}$

20. Suppose a car depreciates 20% in value each year, and its purchase price was $22,000.

a. What is the growth factor in the situation?

b. What is the car's value 1 year after purchase?

c. What is its value n years after purchase?

OBJECTIVE F Determine whether a situation is constant increase, constant decrease, exponential growth, exponential decay, or a nonconstant change. (Lesson 7-4)

In 21–24, does the equation describe a situation of constant increase, constant decrease, exponential growth, or exponential decay?

21. $y = \frac{1}{5}x - 10$

22. $m = -3n + 4$

23. $p = \frac{2}{3}(3)^r$

24. $y = 3\left(\frac{2}{3}\right)^x$

25. A store is going out of business. It advertises that it is reducing prices 1% on day 1, then 2% more on day two, then 3% more on day 3, and so on for 100 days. Is this a situation of constant decrease, exponential decay, or neither of these?

26. Is the sequence: $\frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \ldots$ one of constant increase, constant decrease, exponential growth, exponential decay, or a different kind of increase or decrease?

OBJECTIVE G Compare linear increase with exponential growth. (Lesson 7-7)

27. Country A has 10 million people and its population is growing by 2% each year. Country B has 20 million people and its population is growing by 1 million people per year.

a. Give an equation for the population $A(n)$ of country A, n years from now.

b. Give an equation for the population $B(n)$ of country B, n years from now.

c. In 30 years, if these trends continue, which country would have the greater population? Explain why.

d. In 100 years, if these trends continue, which country would have the greater population? Explain why.

28. Explain why exponential growth always overtakes linear increase if the time frame is long enough.

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE H Graph exponential relationships. (Lessons 7-2, 7-3)

In 29–32, graph the equation and describe a situation that it might represent.

29. $A = 1{,}000(1.02)^x$

30. $C = 0.07 + 0.03t$

31. $y = 50 - 3x$

32. $V = 10{,}000(0.90)^t$

33. Graph the function of Question 20 for integer values of the domain from 1 to 10.

OBJECTIVE I **Graph functions.**
(Lessons 7-5, 7-6)

In 34–37, graph the function on the domain $-5 \leq x \leq 5$.

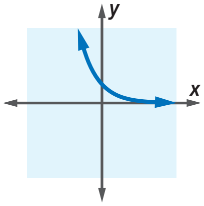
34. $f(x) = 30 - 2x$

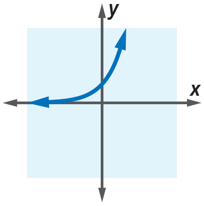
35. $g(x) = x^2$

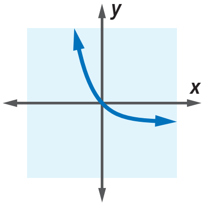
36. $h(x) = 3 \cdot 2^x$

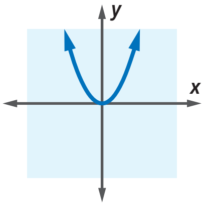
37. $m(x) = x^3 - x$

38. Multiple Choice Which is the graph of the function A when $A(x) = \left(\frac{1}{3}\right)^x$?

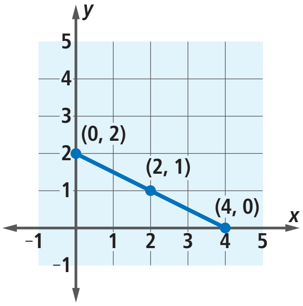
A


B


C


D


39. Refer to the graph of a linear function L below.



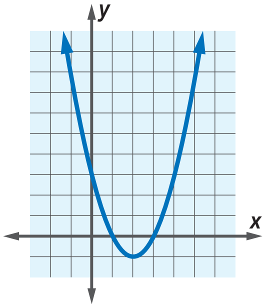
a. What is the value of $L(2)$?

b. What is the value of $L(0)$?

c. What is the domain of L?

d. What is the range of L?

40. On the grid below, each tick mark is one unit.



a. What is $f(0)$?

b. What is $f(3)$?

c. What is the domain of f?

d. What is the range of f?