Lesson

7-6 Function Notation

Vocabulary

$f(x)$ notation

function notation

▶ **BIG IDEA** When a function f contains the ordered pair (x, y), then y is the value of the function at x, and we may write $y = f(x)$.

Refer to the graph on page 397. Recall that each equation represents a different prediction of the population of a town x years in the future. Each model describes a function. The functions are of three types.

Possibility 1 $P = 100{,}000$ describes a *constant function* where the population does not change.

Possibility 2 $P = 100{,}000 + 3{,}000x$ describes a *linear function* with 3,000 new people per year.

Possibility 3 $P = 100{,}000(1.02)^x$ describes an *exponential function* with a growth rate of 2% per year.

It is important to see these functions on the same axes because we want to compare them. But in talking about three functions, we might easily get confused. If we say the letter P, which P are we talking about? It would be nice to be able to name a function in a simple and useful manner.

Mental Math

Give the coordinates of a solution to the inequality.

a. $y \leq -22x + 6$

b. $5m - 4n > 3$

c. $-b + 3 < a - 4.5$

$f(x)$ Notation

Conveniently, mathematics does have another way to name functions. With this method, Possibility 3 can be written $E(x) = 100{,}000(1.02)^x$.

We chose the letter E as a name for the function as a reminder of exponential growth. The symbol $E(x)$ shows that x is the input variable. It is read "E of x." What is the purpose of using this new symbol? It allows us to show the correspondence between specific pairs of values for the input (number of years x) and output (population predicted by the exponential growth model). For example, when $x = 3$, the output is $E(3) = 100{,}000(1.02)^3 = 106{,}120.8$. So $E(3) = 106{,}120.8 \approx 106{,}121$ people. When $x = 20$, the output is $E(20) = 100{,}000(1.02)^{20} \approx 148{,}595$ people.

The other population models can be written in function notation as well.

Possibility 2 could be written $L(x) = 100{,}000 + 3{,}000x$.
Possibility 1 could be written $C(x) = 100{,}000$.
Now each model has a different name.

It is important to know that $E(x)$ *does not* denote the multiplication of E and x. The parentheses indicate the input of a function.

Example 1

Given a function with equation $f(x) = 5x - 19$, find $f(2)$.

Solution $f(x) = 5x - 19$ is a general formula that tells how to find the output for any input. The symbol $f(2)$ stands for "the output of function f when the input is 2." So substitute 2 for x and evaluate the expression on the right side.

$f(2) = 5(2) - 19$

$= 10 - 19 = -9$

So $f(2) = -9$.

In Example 1, we say that –9 is the *value of the function* when $x = 2$.

GUIDED

Example 2

Use the three functions given earlier for population models to find $E(10)$, $L(10)$, and $C(10)$. Explain what the results mean in the context of the population situation.

Solution

$E(x) = 100{,}000(1.02)^x$

$E(10) = 100{,}000(1.02)^{\underline{\ ?\ }}$

After 10 years, the population based on the exponential model is predicted to be about _?_ people.

$L(x) = 100{,}000 + 3{,}000x$

$L(10) = \underline{\ ?\ } + \underline{\ ?\ }(\underline{\ ?\ }) = \underline{\ ?\ }$

After 10 years, the population based on the linear model is predicted to be _?_ people.

$C(x) = 100{,}000$

$C(10) = 100{,}000$

After _?_ years, the population based on the _?_ model is predicted to be _?_ people.

In working with functions, questions arise in which you are given the value of one variable and are asked to find the value of the other variable. In Example 1, you were given the *input* value. You substituted to find the output value. In the next two examples, you are given the value of the *output*. This results in an equation to solve. When you have a formula for a function, symbolic methods may be used to solve the equation to find the input value.

Example 3

Possibility 2 used the linear function $L(x) = 100{,}000 + 3{,}000x$ to model the population of a town x years in the future. According to this model, in how many years will the population reach 150,000?

Solution In $L(x) = 100{,}000 + 3{,}000x$, replace $L(x)$ with 150,000. Then solve for x.

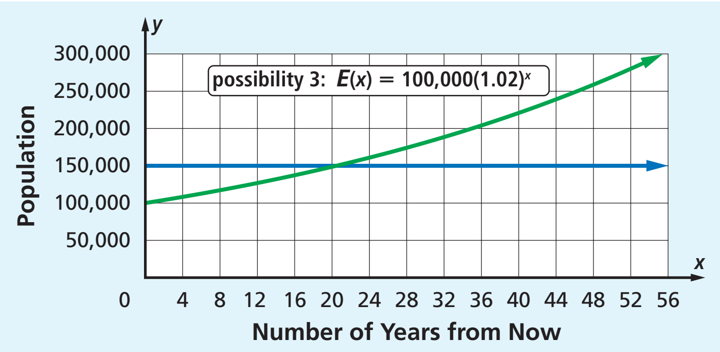
$$150{,}000 = 100{,}000 + 3{,}000x$$
$$50{,}000 = 3{,}000x$$
$$16.67 \approx x$$

The linear model predicts that in about 17 years the population will be 150,000. So $L(x) = 150{,}000$ when $x \approx 17$.

Because functions can also be described with tables and graphs, tables and graphs are useful in solving problems in which you are given the output and need to find the input.

Example 4

For $E(x) = 100{,}000(1.02)^x$, use the graph to find when the population reaches 150,000.



Solution Graph $y = 100{,}000(1.02)^x$. To help you see the point on this graph whose y-coordinate is 150,000, also graph the horizontal line $y = 150{,}000$. Trace on the graph to find where these two graphs intersect. When x is between 20 and 21 years, $E(x)$ is approximately 150,000.

Check Substitute 20 and 21 for x in the equation $E(x) = 100{,}000(1.02)^x$.

Using $x = 20$, $E(x) = E(20) = 100{,}000(1.02)^{20} \approx 148{,}595$.

Using $x = 21$, $E(x) = E(21) = 100{,}000(1.02)^{21} \approx 151{,}567$.

$148{,}595 < 150{,}000 < 151{,}567$, so the answer is reasonable.

Unless the situation suggests a better letter, the most common letter used to name a function is f. That is, instead of writing $y = 3x + 5$, we might write $f(x) = 3x + 5$. Then f is the linear function with slope 3 and y-intercept 5. It is read "f of x equals 3 times x plus 5." This way of writing a function is called **$f(x)$ notation** or **function notation.** The symbol $f(x)$ is attributed to the great Swiss mathematician Leonhard Euler (1707–1783).

Activity

The CAS allows you to work with functions.

1. a. Use the DEFINE command to define $f(x) = 3x^2 + 2x + 10$ in your CAS.
   b. Find $f(2)$.
   c. Find $f(3) + f(-6)$.
   d. Find $f(2006) - f(2005)$.
   e. Find $f(a)$.
   f. Find $f(y)$.
   g. Find f(math). (Do not type in multiplication symbols between the letters. CAS sees *math* as just one big variable called a string variable.)
   h. Find f(your name).
2. a. Define $g(x) = \frac{x^3 + 999}{8x - 1}$.
   b. Find $g(7)$, $g(\pi)$, $g(t)$, and g(mom).
3. Explain the relationships between function notation and substitution.
4. Without using a CAS, find $h(4)$ and h(algebra) if $h(x) = 5x + 2x^2 + 1$.

Questions

COVERING THE IDEAS

1. How is the symbol $f(x)$ read?

In 2 and 3, let $E(x) = 100{,}000(1.02)^x$, $L(x) = 100{,}000 + 3{,}000x$, and $C(x) = 100{,}000$.

2. a. Without a calculator, find the values of $E(1)$, $L(1)$, and $C(1)$.
   b. What do these values mean in the population projection situation?
3. a. With a calculator if necessary, find the values of $E(25)$, $L(25)$, and $C(25)$.
   b. What do these values mean in the population projection situation?
4. If $f(x) = 4 \cdot 0.12^x$, find each value.
   a. $f(1)$ b. $f(3)$ c. $f(5)$

5. Let p be a function with $p(n) = 100 \cdot 2^n$. Find the value of $p(n)$ when $n = 7$.

6. Determine if the statement is *always true, sometimes but not always true,* or *never true*. If $g(x) = 4x^2$ and $g(x) > 0$, then $x > 0$.

APPLYING THE MATHEMATICS

7. The table below shows the amount of money earned by a person who worked x hours.

Hours Worked x	Total Amount Earned $f(x)$
5	\$45
10	\$90
15	\$135
20	\$180
25	\$225
30	\$270

a. What is the value of $f(5)$?

b. What is the value of $f(15)$?

c. Find x if $f(x) = \$180$.

8. Let $h(n) = 0.5n$.

a. Calculate $h(0)$, $h(1)$, and $h(2)$.

b. Calculate $h(-1) + h(-2)$.

c. Why do you think the letter h was chosen for this function?

9. A computer purchased for \$1,600 is estimated to depreciate at a rate of 25% per year. The computer's value after t years is given by $W(t) = 1{,}600(0.75)^t$.

a. Evaluate $W(4)$ and explain what the value means.

b. Graph the function W for values of t with $0 \le t \le 7$.

c. Use your graph to estimate the solutions to $W(t) < 1{,}000$ and explain what your answer means.

10. Suppose $L(x) = 12x - 18$.

a. Calculate $L(5)$.

b. Calculate $L(3)$.

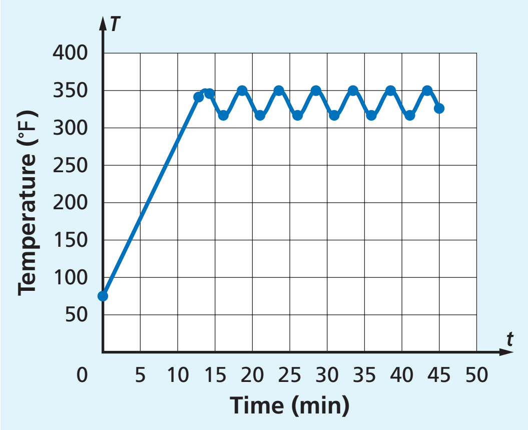
c. Evaluate $\frac{L(5) - L(3)}{5 - 3}$.

d. What is the meaning of your calculation in Part c?

In 2005, approximately 89% of U.S. public middle and junior high schools had the use of computers in the classroom.

Source: Quality Education Data, Inc.

11. Oven temperature T varies with the length of time t the oven has been on. An oven, whose initial temperature was 80°, was set for 325°. The actual temperature was measured and then graphed over a 45-minute interval. Below is the graph of the function f where $T = f(t)$.



a. Estimate $f(25)$.

b. What is the meaning of $f(25)$?

c. Estimate the solution to the equation $f(t) = 200$.

d. What is the meaning of the solution in Part c?

REVIEW

In 12 and 13, could the table of values represent a function with *x* as the independent variable and *y* as the dependent variable? Why or why not? (Lesson 7-5)

12.

x	–3	–2	–1	0	1	2	3
y	0	1	5	1	6	0	2

13.

x	0	1	5	1	6	0	2
y	–3	–2	–1	0	1	2	3

14. Consider the relation described by the equation $x^2 + 3y^2 = 31$. (Lesson 7-5)

a. Is (2, 3) a solution?

b. Is (2, –3) a solution?

c. Is $x^2 + 3y^2 = 31$ the equation of a function? Explain.

15. Rewrite $8y - 4x + 1 = 25 + 6x$ in each form. (Lessons 6-8, 6-4)

a. standard form

b. slope-intercept form

16. ***Skill Sequence*** **(Lessons 4-4, 2-2, 2-1)**

a. Simplify $3(5 - 2m)$.

b. Simplify $3(5 - 2m) - 2(7m + 1)$.

c. Solve $3(5 - 2m) - 2(7m + 1) = 43$.

EXPLORATION

17. Some functions involve more than one input variable. The chart below shows wind chill as a function of the wind speed and the temperature. For wind speed V and temperature T, let $W(V, T)$ = wind chill and let $F(V, T)$ = frostbite time.

Wind (mph)	Temperature (°F)																	
Calm	40	35	30	25	20	15	10	5	0	–5	–10	–15	–20	–25	–30	–35	–40	–45
5	36	31	25	19	13	7	1	–5	–11	–16	–22	–28	–34	–40	–46	–52	–57	–63
10	34	27	21	15	9	3	–4	–10	–16	–22	–28	–35	–41	–47	–53	–59	–66	–72
15	32	25	19	13	6	0	–7	–13	–19	–26	–32	–39	–45	–51	–58	–64	–71	–77
20	30	24	17	11	4	–2	–9	–15	–22	–29	–35	–42	–48	–55	–61	–68	–74	–81
25	29	23	16	9	3	–4	–11	–17	–24	–31	–37	–44	–51	–58	–64	–71	–78	–84
30	28	22	15	8	1	–5	–12	–19	–26	–33	–39	–46	–53	–60	–67	–73	–80	–87
35	28	21	14	7	0	–7	–14	–21	–27	–34	–41	–48	–55	–62	–69	–76	–82	–89
40	27	20	13	6	–1	–8	–15	–22	–29	–36	–43	–50	–57	–64	–71	–78	–84	–91
45	26	19	12	5	–2	–9	–16	–23	–30	–37	–44	–51	–58	–65	–72	–79	–86	–93
50	26	19	12	4	–3	–10	–17	–24	–31	–38	–45	–52	–60	–67	–74	–81	–88	–95
55	25	18	11	4	–3	–11	–18	–25	–32	–39	–46	–54	–61	–68	–75	–82	–89	–97
60	25	17	10	3	–4	–11	–19	–26	–33	–40	–48	–55	–62	–69	–76	–84	–91	–98

Frostbite Times: 30 minutes, 10 minutes, 5 minutes

Source: National Weather Service

a. Pick three (V, T) pairs and find $W(V, T)$ and $F(V, T)$.

b. Find two solutions for the equation $W(V, T) = -55$.

c. Find two solutions for $W(V, T) = -39$ that have different values for $F(V, T)$.

18. Let $m(x)$ = the mother of person x and $f(x)$ = the father of person x. Using yourself for x, find $m(f(x))$ and $f(f(x))$. (*Hint:* Start with the inner-most parentheses.) What are simpler descriptions for each of these two functions?