Lesson 7-5 The Language of Functions

Vocabulary

function
input
output
value of the function
squaring function
independent variable
dependent variable
domain of a function
range of a function
relation

▶ **BIG IDEA** A function is a relationship between two variables in which the value of the first variable is associated with, or determines, a unique value of the second variable.

In this course, you have seen many situations that involve two variables. In an investment situation, the length of time that money has been invested determines the value of the investment. In temperatures, the Fahrenheit temperature determines the Celsius temperature or vice versa. In a sequence of dot patterns, the term number determines the number of dots. When the value of a first variable determines the value of a second variable, we call the relationship between the variables a *function*.

Mental Math

To the nearest mile per gallon, estimate the mpg of a car that went

a. 300 miles on 11 gallons of gas.

b. 250 miles on 9 gallons of gas.

c. 400 miles on 14 gallons of gas.

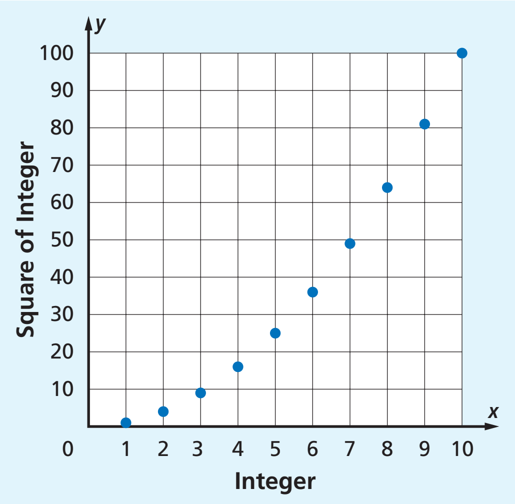
A Squaring Function

Consider the squares of the integers from 1 to 10. There are two variables. The first is the integer. The second is its square. We can describe the relationship between these integers and their squares in many ways.

Table or List

Integer	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

Graph



Equation $y = x^2$, where x is an integer from 1 to 10

Words The square of an integer from 1 to 10 is the result of multiplying the integer by itself.

In general, you can think of functions either as special kinds of correspondences or as special sets of ordered pairs. A **function** is a correspondence in which each value of the first variable (the **input**) corresponds to *exactly one* value of the second variable (the **output**), which is called a **value of the function.** We think of the first variable as determining the value of the second variable. The table, graph, equation, and words on page 425 describe a **squaring function.** The value of a number determines the value of its square. For example, when $x = 3$, the value of the squaring function is 9.

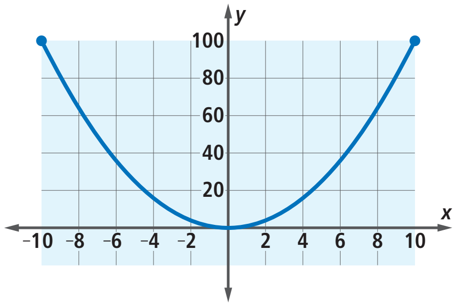
STOP **QY1**

> **QY1**
> What is the value of the squaring function when $x = 7$?

The graph on page 425 shows the squaring function as a set of ordered pairs. A function is a set of ordered pairs in which each first coordinate appears with *exactly one* second coordinate. That is, once you know the value of the first variable (often called x), then there is only one value for the second variable (often called y). For this reason, the first variable is called the **independent variable** and the second variable is called the **dependent variable.**

The Domain and Range of a Function

Suppose in the squaring function that x can be any real number from –10 to 10. You cannot list all the ordered pairs of the function, but the function still can be described by the equation $y = x^2$, where x is a real number from –10 to 10. The function can still be described in words: The square of any real number from –10 to 10 is the result of multiplying the number by itself. And the function can still be described by a graph, as shown at the right.



The difference between this squaring function and the one on page 425 is in the *domain of the function*. The **domain of a function** is the set of allowable inputs in the function, that is, the set of possible values of the first (independent) variable. In the squaring function on page 425, the domain is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. In the squaring function on this page, the domain is the set of real numbers from –10 to 10.

If a function has a graph, you can read its domain from the graph. The domain is the set of x-coordinates of the points of the graph.

Corresponding to the domain of a function is its **range,** the set of possible values of the second (dependent) variable. The range is the set of possible values of the function. In the squaring function on page 425, the range is {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}. In the squaring function on this page, the range is the set of real numbers from 0 to 100.

STOP QY2

When a set of numbers is not specifically given for the domain of a function, you should assume that the domain is the set of all numbers possible in the situation.

Example 1

Consider the reciprocal function $y = \frac{1}{x}$, which pairs real numbers with their reciprocals.

a. Give the domain.

b. Find the value of the function when $x = 4$.

Solution

a. The domain is the set of all values that can replace x, the independent variable. Any number except 0 can be used. (Because $\frac{1}{0}$ is undefined, 0 has no reciprocal.) So the domain is all real numbers except 0.

b. Substitute 4 for x. The value of the function is $\frac{1}{4}$.

GUIDED

Example 2

What are the domain and range of the function described by the equation $y = 4x - 3$?

Solution The domain is the set of allowable values of x. Because no situation is given for x, you should assume that its domain is ___?___.

The range is the set of possible values of y. The graph of $y = 4x - 3$ is an oblique line. So any value of y is possible, and the range is ___?___.

In many places in this book, you have seen one function modeling another.

Example 3

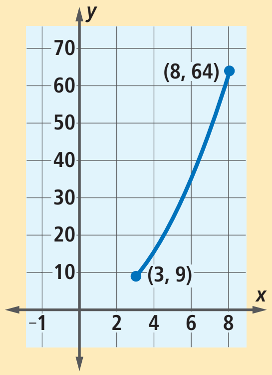
In Lesson 7-3, the equation $y = 21{,}000(0.85)^x$ describes a function that models a car's value y when it is x years old if it was purchased for \$21,000 and depreciates 15% a year. What is the range of this function?

Solution You can think of the car's value as decreasing constantly even though you have not yet studied values of powers when the exponent is not an integer. Refer to the graph in Lesson 7-3 on page 413. According to the model, the value of the car keeps decreasing but never reaches 0. So the range of the function is the set of real numbers y with $0 < y \leq 21{,}000$ which is written in set-builder notation as $\{y: 0 < y \leq 21{,}000\}$.

STOP QY3

▶ QY2

A third squaring function is graphed below. What is its domain? What is its range?



▶ QY3

Use a calculator to estimate the value of $21{,}000(0.85)^x$ to the nearest penny when $x = 25$, 25.3, and 26.

Relations That Are Not Functions

The word **relation** describes any set of ordered pairs. It is possible to have relations between variables that are not functions. This happens when the first variable x in a relation corresponds to more than one value of the second variable y. For example, the relation described by the equation $x = y^2$ does *not* describe a function. When $x = 4$, then $y = 2$ or $y = -2$. So the value $x = 4$ corresponds to two different values for y, 2 and –2. Because a value of x does not always determine exactly one value of y, the relation is not a function.

Everybody has a blood type. The most common blood-type classification system is the ABO system discovered by Karl Landsteiner in the early 1900s.

Source: University of Utah

Functions Whose Domains or Ranges Are Not Sets of Numbers

It is possible to have functions whose domains and ranges are not sets of real numbers, or even that have little to do with mathematics. For example, every person on Earth has a unique blood type. So there is a function whose domain is the set of all living people on Earth and whose range is the set of all blood types. Each ordered pair of this function is of the form (a person, that person's blood type). This function has no equation and cannot be graphed, but it is still a function with a domain (the set of all people) and a range (the set of all blood types).

Questions

COVERING THE IDEAS

1. Consider the function described by $y = x^2$ with domain the set of integers from 1 to 5.
   a. What is the value of this function when $x = 3$?
   b. The point (4, 16) is on the graph of this function. Which of these coordinates is the input and which is the output?
   c. Which variable is the independent variable and which is the dependent variable?
   d. What is the range of this function?
2. Consider a cubing function described by $y = x^3$ with domain the set of real numbers from –50 to 50. Find the value of this function when
   a. $x = 36$. b. $x = -36$. c. $x = 0$.
3. a. What is the value of the reciprocal function when $x = -1$?
   b. What is the value of the reciprocal function when $x = 3.5$?
4. Give the two definitions of *function* stated in this lesson.

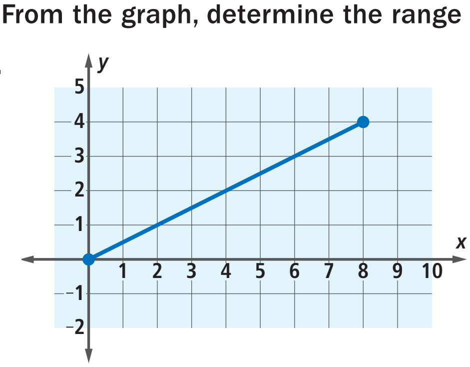
5. Explain why 0 is not in the range of the reciprocal function.

In 6 and 7, the graph of a function is given.

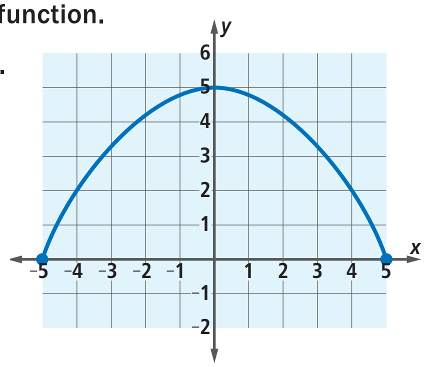
a. From the graph, determine the domain of the function.

b. From the graph, determine the range of the function.

6.



7.



8. Determine if the following statement is *always, sometimes but not always,* or *never* true. The graph of a function may contain both points (6, 5) and (6, 7).

9. **Multiple Choice** Which table does *not* describe a function?

A

x	y
1	6
2	53
3	8

B

x	y
1	6
2	6
3	6

C

x	y
6	1
6	2
6	3

D

x	y
6	6
6	6
6	6

APPLYING THE MATHEMATICS

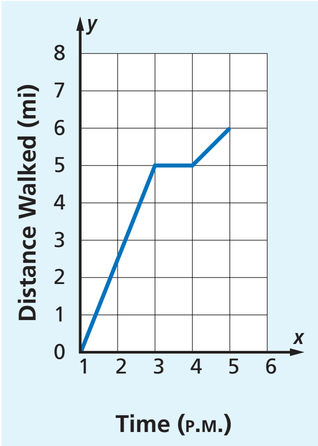
10. The graph at the right is of a function showing the distance walked by a hiker over time.

a. Find the value of the function when $x = 2$ P.M.

b. Find the value of the function when $x = 3{:}30$ P.M.

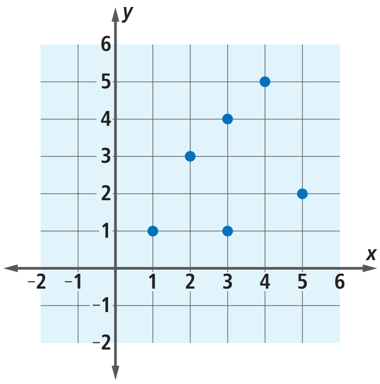
c. Find x for which the value of the function is 5.5 miles.

d. Use inequalities to describe the domain and range of this function.

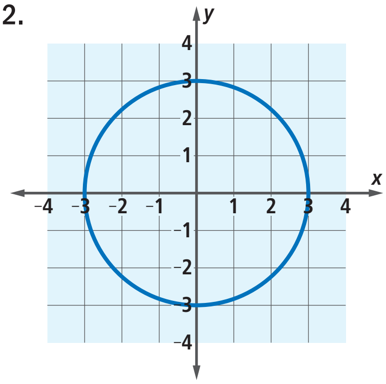


In 11 and 12, determine if the graph of ordered pairs (*x*, *y*) is that of a function. Justify your answer.

11.



12.



13. The function $y = 5 \cdot 0.8^x$ is graphed at the right.

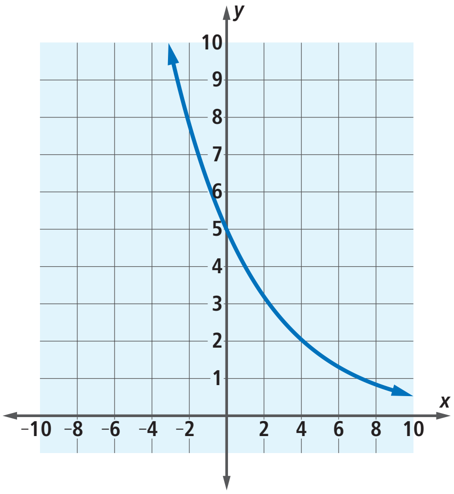
a. **True or False** Zero is in the range of this function.

b. What is the value of this function when $x = 0$?

c. What is the domain of the function that is graphed?

d. What is the range of the function that is graphed?

e. Suppose x can be any positive integer. What is the greatest possible value of y?



In 14 and 15, a situation is described in which one quantity can be used to predict values of the second quantity. Tell which quantity you wish to be the input and which should be the output in order to have a function. Sketch a reasonable graph. Do not mark numbers on the axes. Think only about the basic shape of the graph.

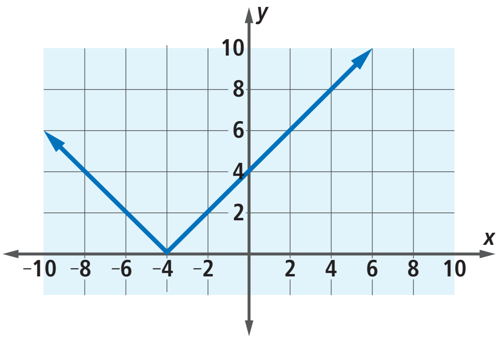
14. the amount of time since a cup of hot coffee was poured and its temperature

15. the height of a skydiver who has jumped from an airplane

People have been using parachutes for hundreds of years, even during the 1100s in China.

Source: United States Parachute Association

In 16–18, use the graph of an absolute value function below. Find the range for the part of the function whose domain is given.



16. domain = $\{x: x \geq 1\}$

17. domain = $\{x: -6 \leq x \leq -2\}$

18. domain = $\{x: x \leq 0\}$

In 19 and 20, an equation for a function is given. Determine the domain and the range of the function. You may use a graphing calculator to help you.

19. $y = \frac{1}{75}x - 3$

20. $y = 100 \cdot \left(\frac{1}{2}\right)^x$, when $x \geq 0$

21. Since 1980, world records in the men's marathon have been set many times.

Date	Record-Setter	Country	Time	Location of Race
Dec. 6, 1981	Robert de Castella	Australia	2 hr, 8 min, 18 sec	Fukuoka, Japan
Oct. 21, 1984	Steve Jones	Britain	2 hr, 8 min, 5 sec	Chicago, USA
Apr. 20, 1985	Carlos Lopes	Portugal	2 hr, 7 min, 12 sec	Rotterdam, Netherlands
Apr. 17, 1988	Belayneh Dinsamo	Ethiopia	2 hr, 6 min, 50 sec	Rotterdam, Netherlands
Sept. 20, 1998	Ronaldo de Costa	Brazil	2 hr, 6 min, 5 sec	Berlin, Germany
Oct. 24, 1999	Khalid Khannouchi	Morocco	2 hr, 5 min, 42 sec	Chicago, USA
Apr. 14, 2002	Khalid Khannouchi	USA	2 hr, 5 min, 38 sec	London, England
Sept. 28, 2003	Paul Tergat	Kenya	2 hr, 4 min, 55 sec	Berlin, Germany

Source: www.marathonguide.com

a. Consider the function using the pairs (date, time). This function is an example of a *decreasing function*. Why do you think it is called a decreasing function?

b. Consider the eight ordered pairs (record-setter, time). By examining the definition of function, explain why these eight ordered pairs do *not* make up a function.

REVIEW

22. In China, most families are allowed to have only one child. This policy was implemented to reduce the population, with a goal of reaching 700 million citizens by 2050. Suppose the 2005 population of 1.3 billion decreases by 1% each year. (**Lesson 7-3**)

a. Write an expression for the population of China x years after 2005.

b. Will the goal of having 700 million citizens or less in the year 2050 be met?

23. Find the slope of the line given by the equation $\frac{7}{20}(x + 18) = \frac{8}{3}(y - 11)$. (**Lessons 6-2, 4-4, 3-8**)

24. Triangle 1 has an area of 12 cm^2 and is similar to Triangle 2, which has an area of 108 cm^2. What is the ratio of similitude of Triangle 1 to Triangle 2? (**Lesson 5-10**)

25. The maximum number p of people allowed on a certain elevator times the average weight w of an adult should not exceed 1,500 pounds. Write an inequality describing the rule and solve for p. (**Lesson 3-6**)

EXPLORATION

26. Find equations for two different functions with the same domain that contain both the ordered pairs (1, 1) and (2, 6).

QY ANSWERS

1. 49

2. domain: $\{x: 3 \le x \le 8\}$; range: $\{y: 9 \le y \le 64\}$

3. \$361.15; \$343.97; \$306.98