Lesson 7-4 Modeling Exponential Growth and Decay

Vocabulary

exponential regression

BIG IDEA Situations of exponential growth and decay can be modeled by equations of the form $y = bg^x$.

As you saw with the population of California in Lesson 7-2, sometimes a scatterplot shows a data trend that can be approximated by an exponential equation. Similar to linear regression, your calculator can use a method called **exponential regression** to determine an equation of the form $y = b \cdot g^x$ to model a set of ordered pairs.

Mental Math

Each product is an integer. Write the integer.

a. $\frac{1}{4} \cdot 360$

b. $\frac{2}{3} \cdot 45$

c. $\frac{4}{5} \cdot 135$

d. $\frac{5}{12} \cdot 228$

Modeling Exponential Decay

In the Activity below, exponential regression models how high a ball bounces.

Activity

Each group of 5 or 6 students needs at least 3 different types of balls (kickball, softball, and so on), a ruler, markers or chalk, and large paper with at least 25 parallel lines that are 3 inches apart.

Step 1 Tape the paper to a wall or door so the horizontal lines can be used to measure height above the floor. To make measuring easier, number every fourth line (12 in., 24 in., 36 in., and so on).

Step 2 One student will drop each ball from the highest horizontal line and the other students will act as spotters to see how high the ball bounces. The 1st spotter will mark the height to which the ball rebounds after the 1st bounce. The 2nd spotter will mark the rebound height after the 2nd bounce, and so on, until the ball is too low to mark.

Step 3 Make a table similar to the one at the right and record the rebound heights after each bounce.

Bounce	Ball Height (in.)
0 (drop height)	?
1	?
2	?
3	?
?	?

Step 4 For each ball, enter the data in a list and create a scatterplot on your calculator.

Step 5a. Use the linear regression capability of your calculator to find the line of best fit for the data. Graph this line on the same screen as your scatterplot. Sketch a copy of the graph.

(*continued on next page*)

b. Find the deviation between the actual and predicted height of the ball after the 3rd bounce.

c. Use your linear regression model to predict the height of the ball after the 8th bounce.

Step 6a. Use the exponential regression capability of your calculator to find an exponential curve to fit the data. Graph this equation on the same screen as your scatterplot. Sketch a copy of the scatterplot and curve.

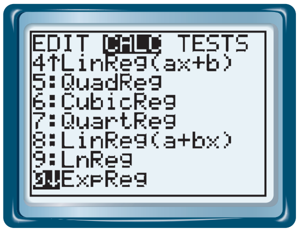
b. Find the deviation between the actual and predicted height of the ball after the 3rd bounce.

c. Use your exponential regression model to predict the height of the ball after the 8th bounce.

Step 7 Which seems to be the better model of the data the linear equation or the exponential equation? Explain how you made your decision.

Step 8 Repeat Step 6 to find exponential regression equations that fit the bounces of the other balls.

Step 9 Write a paragraph comparing the "bounciness" of the balls you tested.



STOP QY

Modeling Exponential Growth

Advances in technology change rapidly. Some people say that if you purchase a computer today it will be out of date by tomorrow. When computers were first introduced to the public, they ran much more slowly. As computers have advanced over the years, the speed has increased greatly. On the next page is an example of data that a person collected to show the advancement in computer technology. The processing speed of a computer is measured in megahertz (MHz).

GUIDED

Example

The table and graph on the next page show the average speed of a computer and the year it was made.

a. Write an equation to model the data.

b. Find the deviation between the actual speed for the year 2000 and the predicted speed.

c. Use the model to predict the processing speed of a computer made in 2020.

QY

A student dropped a ball from a height of 0.912 meter and used a motion detector to get the data below.

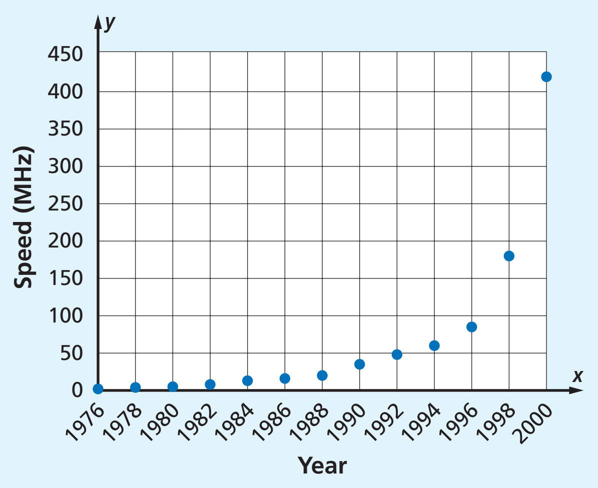
Bounce	Rebound Height (m)
0	0.912
1	0.759
2	0.603
3	0.496
4	0.411
5	0.328
6	0.271

a. Write an exponential equation to fit the data.

b. After the 8th bounce, how high will the ball rebound?

Year	Years since 1976	Speed (MHz)
1976	0	2
1978	2	4
1980	4	5
1982	6	8
1984	8	13
1986	10	16
1988	12	20
1990	14	35
1992	16	48
1994	18	60
1996	20	85
1998	22	180
2000	24	420

Source: *Microprocessor Quick Reference Guide*

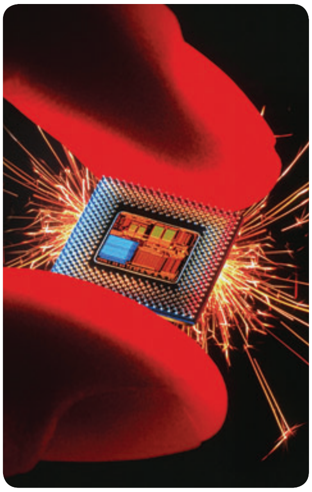


Solutions

a. First enter the data into your calculator lists. Instead of letting years be the x-values, let x = the years since 1976. So for 1976 itself, $x = 0$ and for 1978, $x = 2$. Next, use exponential regression on your calculator to find an exponential equation to fit the data. For $y = b \cdot g^x$, the calculator gives $b \approx 2.241$ and $g \approx 1.218$. (Your calculator may call this equation $y = ab^x$.)
The exponential equation that best fits the data is $y =$ __?__.

b. For 2000, $x = 24$ and the actual speed was 420 MHz. Substitute 24 into the equation to find the predicted value. The predicted speed is $y =$ __?__ MHz. The deviation is 420 − __?__ = __?__. The actual processing speed in 2000 was __?__ more than the predicted speed.

c. The year 2020 is 2020 − 1976, or 44 years after 1976, so substitute 44 for x in your exponential equation. $y =$ __?__ (__?__)$^{?}$, so the predicted processor speed for the year 2020 is __?__ MHz.



On April 25, 1961, the patent office awarded the first patent for an integrated circuit to Robert Noyce while Jack Kilby's application was still being analyzed. Today, both men are acknowledged as having independently conceived of the idea.

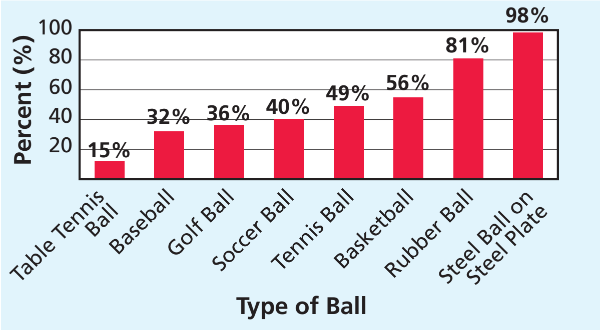
Source: PBS

Questions

COVERING THE IDEAS

1. Suppose a ball is dropped and it rebounds to a height of y feet after bouncing x times, where $y = 6(0.55)^x$. Use the equation to
   a. give the height from which the ball was dropped, and
   b. give the percent the ball rebounds in relation to its previous height.

In 2–4, use the graph to answer the questions. The percent written above each bar represents the percent of the previous height to which each type of ball will rebound.



Source: Exploratorium®

2. Which ball's rebound height could be modeled by the equation $y = 10(0.49)^x$?

3. If a basketball is dropped from a height of 15 feet above the ground, how high will it rebound after the 1st bounce? After the 5th bounce?

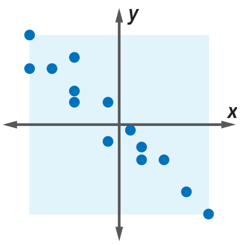
4. Find and compare the rebound percentages in the Activity on page 419 to those in the graph. Are they similar or different?

5. A computer's memory is measured in terms of megabytes (MB). The table at the right shows how much memory an average computer had, based on the number of years it was made after 1977. Use exponential regression to predict the amount of memory for a computer made in 2020.

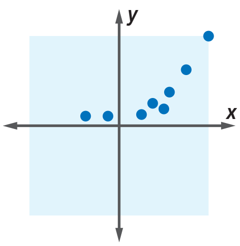
Years After 1977	Memory (MB)
0	0.0625
2	1.125
3	8
6	16
7	30
9	32
13	40
17	88
21	250
27	512

6. For each scatterplot, tell whether you would expect exponential regression to produce a good model for the data. Explain your reasoning.

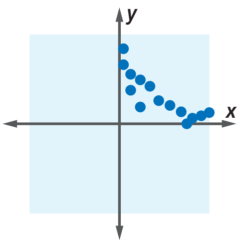
a.



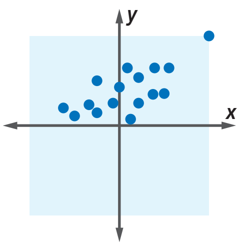
b.



c.



d.

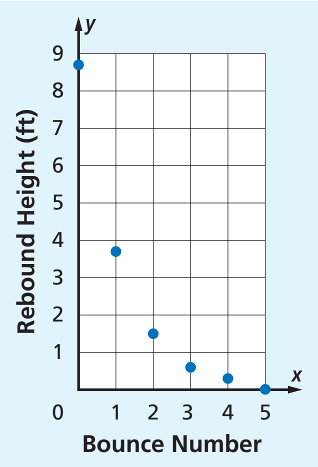


APPLYING THE MATHEMATICS

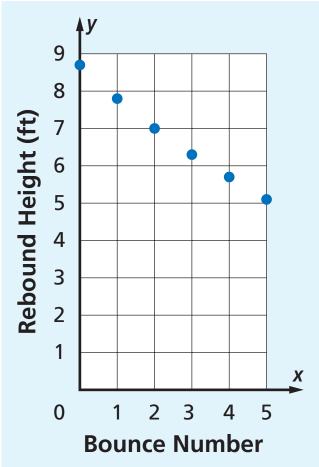
Matching In 7–9, the graphs relate the bounce height of a ball to the number of times that it has bounced. Match a graph to the equation.

a. $y = 5.1(0.90)^x$ **b.** $y = 8.7(0.90)^x$ **c.** $y = 8.7(0.42)^x$

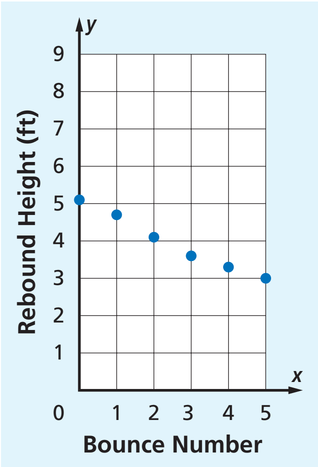
7.



8.



9.



10. The table at the right shows the number of weeks a movie had played in theaters, how it ranked, and how much money it grossed each weekend. (Note that $x = 0$ is the weekend the movie opened.)

a. Create a scatterplot with y = gross sales after x weeks in theaters. Why is the exponential model a better model for these data than a linear model?

b. Use exponential regression to find an equation to fit the data.

c. What gross sales are predicted for the weekend of the 20th week?

Weeks in Theaters	Rank	Weekend Gross ($)
0	1	114,844,116
1	1	71,417,527
2	2	45,036,912
3	2	28,508,104
4	3	14,317,411
5	5	10,311,062
6	7	7,515,984
7	11	4,555,932
8	13	3,130,214
9	18	2,204,636
10	22	890,372
11	25	403,186

11. Lydia and Raul started with 2 pennies in a cup, shook them out onto the table, and added a penny for each coin that showed a head. They continued to repeat this process and their data are recorded in the table at the right.

a. Create a scatterplot of their data.

b. Use exponential regression to derive an equation relating the trial number to the number of pennies they will have on the table.

Trial Number	Number of Pennies
0	2
1	2
2	3
3	5
4	8
5	13
6	17
7	25
8	38
9	60

For 12 and 13, create a real-world problem that could be modeled by the given equation.

12. $y = 72(1.08)^x$

13. $y = 14(0.65)^x$

REVIEW

14. The population of a city is 1,250,000. Write an expression for the population y years from now under each assumption. (Lessons 7-3, 7-2, 6-1)
    a. The population grows 2.5% per year.
    b. The population decreases 3% per year.
    c. The population decreases by 1,500 people per year.

15. Graph $y = 125\left(\frac{2}{5}\right)^x$ for integer values of x from 0 to 5. (Lesson 7-3)

16. Rewrite x^3y^4 using the Repeated Multiplication Property of Powers. (Lesson 7-1)

17. An art store buys a package of 40 bristle paintbrushes for \$80.00 and a package of 30 sable paintbrushes for \$150. If they plan to sell an art kit with 4 bristle paintbrushes and 3 sable paintbrushes, how much should they charge for the kit to break even on their costs? (Lesson 5-3)

a young artist at work

18. ***Skill Sequence*** Divide and simplify each expression. (Lesson 5-2)
    a. $\frac{4}{x} \div \frac{5}{x}$
    b. $\frac{4}{x} \div \frac{5}{2x}$
    c. $\frac{4}{x} \div \frac{5}{x^2}$

19. Recall that if an item is discounted x%, you pay $(100 - x)$% of the original price. Calculate in your head the amount you pay for a camera that originally cost \$300 and is discounted each indicated amount. (Lesson 4-1)
    a. 10%
    b. 25%
    c. $33\frac{1}{3}$%

20. Evaluate $(3a)^3(4b)^2$ when $a = -2$ and $b = 6$. (Lesson 1-1)

EXPLORATION

21. In the ball-drop activity on pages 419–420 and Questions 2–4 on page 422, you explored the rebound height of a ball as a percent of its previous height. Different types of balls have different percents. Does the height from which the ball is dropped affect the percent a ball will rebound? Explain your answer.

22. Do the activity described in Question 11 on page 423. How close is your exponential model to the one in that question?

QY ANSWERS

a. $y = 0.917(0.816)^x$

b. approximately 0.18 m