Lesson 7-1

Compound Interest

Vocabulary

power, *n*th power
base
exponent
principal
interest
annual yield
compound interest

BIG IDEA Compound interest is the way most banks and other savings institutions pay savers who put their money into their accounts.

Powers and Repeated Multiplication

A number having the form x^n is called a **power.** When n is a positive integer, x^n describes repeated multiplication. For example, $10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$ and $3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$. These are examples of the following property.

Repeated Multiplication Property of Powers

When n is a positive integer, $x^n = \underbrace{x \cdot x \cdot \ldots \cdot x}_{n \text{ factors}}$.

The number x^n is called the ***n*th power** of x and is read "x to the nth power" or just "x to the n." In the expression x^n, x is the **base** and n is the **exponent.** Thus, 3^5 is read "3 to the 5th power," or "3 to the 5th," where 3 is the base and 5 is the exponent. In the expression $100{,}000(1.02)^x$ found on page 397, 1.02 is the base and x is the exponent. The number 100,000 is the coefficient of the power 1.02^x.

Mental Math

Write as a fraction in lowest terms.

a. 0.02

b. 1.75

c. –3.57

How Is Interest Calculated?

An important application of exponents and powers occurs with savings accounts. When you save money, you can choose where to put it. Of course, you can keep it at home, but banks, savings and loan associations, and credit unions will pay you to let them hold your money for you. The amount you give them at the start is called the **principal.** The amount they pay you is called **interest.**

Interest is always a percent of the principal. The percent that the money earns per year is called the **annual yield.**

Example 1

Suppose you deposit *P* dollars in a savings account upon which the bank pays an annual yield of 4%. If the account is left alone, how much money will be in it at the end of a year?

Solution

Total = principal + interest (4% of principal)

$= P + 0.04P$

$= (1 + 0.04)P = 1.04P$

You will have 1.04P, or 104% of the principal.

STOP **QY1**

> **QY1**
>
> If you deposited $1,000 in a savings account with an annual yield of 4%, what would you have at the end of a year?

Compound Interest and How It Is Calculated

When the year is up, the account will have extra money in it because of the interest it earned. If that money is left in the account, then at the end of second year, the bank will pay interest on all the money that is now in the account (the original principal and the first year's interest). This leads to **compound interest,** which means that the interest earns interest.

Example 2

Suppose you deposit $100 in a savings account upon which the bank pays an annual yield of 4%. Assume the account is left alone in Parts a and b.

a. How much money will be in the account at the end of 4 years?

b. How much interest would you earn in the 4 years?

Solution

a. Refer to Example 1. Each year the amount in the bank is multiplied by $1 + 0.04 = 1.04$.

End of first year: $100(1.04) = 100(1.04)^1 = 104.00$

End of second year: $100(1.04)(1.04) = 100(1.04)^2 = 108.16$

End of third year:

$100(1.04)(1.04)(1.04) = 100(1.04)^3 = 112.4864 \approx 112.48$

End of fourth year:

$100(1.04)(1.04)(1.04)(1.04) = 100(1.04)^4 \approx 116.9858 \approx 116.98$

At the end of 4 years there will be $116.98 in the account.

b. Because you started with $100, you earned $116.98 − $100 = $16.98 in the 4 years.

Examine the pattern in the solution to Example 2. At the end of t years there will be $100(1.04)^t$ dollars in the account. By replacing 100 by P for principal, and 0.04 by r for the *annual yield,* we obtain a general formula for compound interest.

Compound Interest Formula

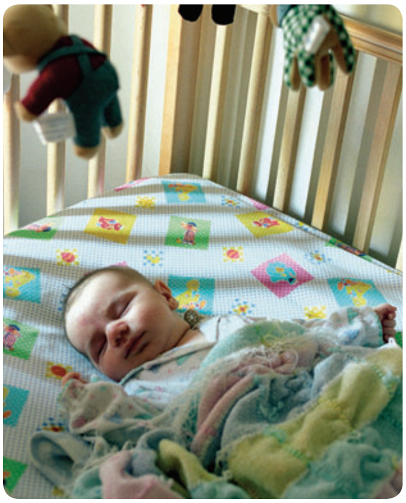
If a principal P earns an annual yield of r, then after t years there will be a total amount A, where $A = P(1 + r)^t$.

The compound interest formula is read "A equals P times the quantity 1 plus r, that quantity to the tth power."

GUIDED

Example 3

When Jewel was born, her parents put $2,000 into an account for college. What will be the total amount of money in the account after 18 years at an annual yield of 5.4%?



Tuition fees at public four-year colleges increased 35% between 2001 and 2006.

Source: The College Board

Solution Here $P = \$2{,}000$, $r = 5.4\%$, and $t = 18$. Substitute the values into the Compound Interest Formula. Use $5.4\% = 0.054$.

$$A = P(1 + r)^t$$
$$= \underline{\ ?\ }(1 + \underline{\ ?\ })^{\underline{\ ?\ }}$$

To evaluate this expression, use a calculator key sequence such as the following.

__?__ [×] __?__ [^] __?__ [ENTER]

Your display shows __?__, which rounded down to the nearest cent is __?__.

In 18 years, at an annual yield of 5.4%, $2,000 will increase to __?__.

STOP QY2

QY2

Suppose you invest $6,240 in an account at 6.3% annual yield for 10 years. How much will be in the account at the end of the 10 years?

Eighteen years may seem like a long time, but it is not an unusually long amount of time for money to be in college accounts or retirement accounts.

Why Do You Receive Interest on Savings?

Banks and other savings institutions pay you interest because they want money to lend to other people. The bank earns money by charging a higher rate of interest on the money they lend than the rate they pay customers who deposit money.

Thus, if the bank could loan the \$1,000 you deposited at 4% (perhaps to someone buying a car) at 12% a year, the bank would receive 0.12(\$1,000), or \$120 from that person. So the bank would earn \$120 – \$40 = \$80 in that year on your money. Part of that \$80 goes for salaries to the people who work at the bank, part for other bank costs, and part for profit to the owners of the bank.

Questions

COVERING THE IDEAS

1. How is the expression 4^{10} read?

2. Consider the expression $10x^9$. Name each of the following.
   a. base b. power c. exponent d. coefficient

3. a. Calculate 7^3 without a calculator.
   b. Calculate 7^3 with a calculator. Show your key sequence.

In 4–6, rewrite the following expressions using exponents.

4. $\underbrace{\frac{5}{9} \cdot \frac{5}{9} \cdot \ldots \cdot \frac{5}{9}}_{t \text{ times}}$

5. $18 \cdot -3 \cdot -3 \cdot -3 \cdot -3$

6. $21 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$

7. On page 397, three possibilities are offered for the growth of a population. What is the predicted population in 50 years using the indicated possibility?
   a. Possibility 1
   b. Possibility 2
   c. Possibility 3

8. **Matching** Match each term with its description.
   a. money you deposit i. annual yield
   b. interest paid on interest ii. compound interest
   c. yearly percentage paid iii. principal

In 9 and 10, write an expression for the amount in the bank after 1 year if *P* dollars are in an account with the annual yield given.

9. 2%

10. 3.25%

11. a. Write the Compound Interest Formula.
    b. What does A represent?
    c. What does P stand for?
    d. What is r?
    e. What does t represent?

In 12–14, assume the interest is compounded annually.

12. Suppose you deposit \$300 in a new savings account paying an annual yield of 2.5%. If no deposits or withdrawals are made, how much money will be in the account at the end of 5 years?

13. A bank advertises an annual yield of 4.81% on a 5-year CD (certificate of deposit). If the CD's original amount was \$2,000, how much will it be worth after 5 years?

14. How much interest will be earned in 7 years on a principal of \$1,000 at an annual yield of 5.125%?

APPLYING THE MATHEMATICS

In 15 and 16, assume the interest is compounded annually.

15. Susana invests \$250 at an annual yield of 4%. Jake invests \$250 at an annual yield of 8%. They leave the money in the bank for 2 years.

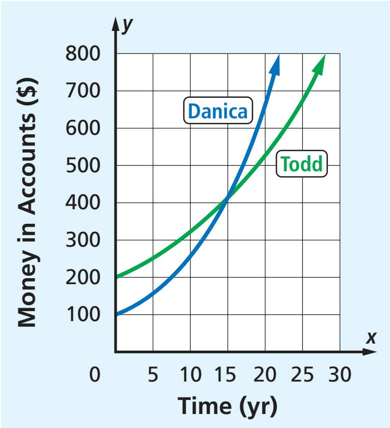
a. How much interest does each person earn?

b. Jake's interest rate is twice Susana's. Does Jake earn twice the interest that Susana does? Why or why not?

16. Which yields more money, (a) an amount invested for 6 years at an annual yield of 5%, or (b) the same amount invested for 3 years at an annual yield of 10%? Explain your answer.

In 17–19 on the next page, use the following: Danica invested \$100 in an account that earns an annual yield of 10%. On the same day, Todd deposited \$200 in an account earning 5% annually. Below are a graph and a spreadsheet that compare the amount in Danica's and Todd's accounts.

◇	A	B	C
1	Time since Investment (yr)	Danica's Account	Todd's Account
2	0	\$100.00	\$200.00
3	2	\$121.00	\$220.50
4	4	\$146.41	\$243.10
5	6	\$177.15	\$268.01
6	8	\$214.37	\$295.49
7	10	\$259.37	\$325.78
8	12	\$313.84	\$359.17
9	14		
10	16		
11	18		
12	20		



17. a. What formula will show the amount in Danica's account after t years?
    b. Complete the column indicating the amounts in Danica's account.

18. Repeat Question 17 for Todd's account.

19. In what year will Danica and Todd have the same amount in their accounts?

20. Use your calculator to make a table. If a principal of \$1,000 is saved at an annual yield of 5% compounded annually and nothing is withdrawn from the account, in how many years will it double in value?

REVIEW

21. In World Cup Soccer, a team gets 3 points for a win and 1 point for a tie. Let W be the number of wins and T the number of ties. **(Lessons 6-9, 3-7)**
    a. If a team has more than 3 points, what inequality must W and T satisfy?
    b. Graph all possible pairs (W, T) for a team that has played 3 games and has more than 3 points.

Ecuador's Ulises De La Cruz, left, and England's Joe Cole battle for the ball during a 2006 World Cup soccer match between England and Ecuador.

22. Miho puts \$6.00 into her piggy bank. Each week thereafter she puts in \$2.50. (The piggy bank pays no interest.)
    a. Write an equation showing the total amount of dollars Y after X weeks.
    b. Graph the equation. **(Lesson 6-2)**

23. Find the probability of getting a number that is a factor of 12 in one toss of a fair die. **(Lesson 5-6)**

In 24 and 25, solve the sentence. (Lessons 4-5, 4-4)

24. $38c - 14 = 6(c - 3) + 4$

25. $8(2 + \frac{1}{8}u) > 2u + 1 - u$

26. **Multiple Choice** Which formula describes the numbers in the table at the right? **(Lesson 1-2)**
    A $y = x + (x + 1)$
    B $y = 2x$
    C $y = 2^x$
    D $y = x^2$

x	0	2	4	6	8
y	1	4	16	64	256

27. Find t if $2^t = 32$. **(Previous Course)**

EXPLORATION

28. Find out the yield for a savings account in a bank or other savings institution near where you live. (Often these yields are in newspaper ads.)

QY ANSWERS

1. \$1,040

2. \$11,495.20