

Summary and Vocabulary

- ▶ Generalizations in mathematics include **assumptions** (assumed properties), **definitions** (meanings of terms or phrases), and **theorems** (statements deduced from assumptions, definitions, or other theorems). These generalizations are often presented as **if-then statements**. For example, one assumed property of real numbers is the Distributive Property of Multiplication over Addition. It can be written in if-then form as: If a , b , and c are real numbers, then $a(b + c) = ab + ac$.
- ▶ The **converse** of the statement, “If a , then b ” is the statement, “If b , then a .” The converse of a true statement is not necessarily true. When the converse is true, then the statement “ a if and only if b ” is true. Definitions are **if-and-only-if statements**. For example, x is an even number if and only if x can be written as $2n$, where n is an integer.
- ▶ By putting together if-then statements of assumptions and definitions, a **mathematical proof** can be created. From the definition of even number, you can prove that if the square of an integer is even, then the integer is even. You can also prove that $\sqrt{2}$ and square roots of other nonzero integers that are not perfect squares are **irrational numbers**. Using the definition of divisibility by any number and what it means for a number to be in base 10, you can prove divisibility tests and other interesting properties of numbers.
- ▶ Every equation or inequality that you solve showing steps and justifications can be thought of as a **proof**. Suppose you solve $8x + 50 = 2$ and obtain $x = -6$. If you can justify the steps that you used in your solution, you have proved: “If $8x + 50 = 2$, then $x = -6$.” The check is the converse: “If $x = -6$, then $8x + 50 = 2$.”
- ▶ Mathematical knowledge grows by deducing statements from those that are assumed to be true or have been proved earlier to be true. Among the oldest and most important theorems in all of mathematics are the **Quadratic Formula** and the **Pythagorean Theorem**. Proofs of the Quadratic Formula use the properties that are most associated with solving equations. The proofs of the Pythagorean Theorem that we show in this chapter use area formulas for triangles, squares, and trapezoids.

Vocabulary

13-1

if-then statement
antecedent
consequent
generalization

13-2

converse
equivalent statements
if and only if

13-3

justifications
proof argument
deduction

13-5

closed under an operation
even integer, even number
odd integer, odd number
semiperimeter

13-7

irrational number

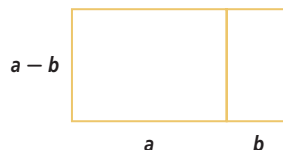
Theorems and Properties

Irrationality of \sqrt{n}
Theorem (p. 819)

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

- State conclusions and justifications to prove that if $8(2y - 1) = y + 37$, then $y = 3$.
- Determine the antecedent and consequent of the statement proved in Question 1.
- Amalia says that xy equals 0 if and only if both x and y equal 0.
 - Write the two if-then statements that are equivalent to Amalia's if-and-only-if statement.
 - Is Amalia correct? Explain your answer.
- Marcus is measuring the diagonal across a piece of paper. The paper is 7 in. by 8 in.
 - What is the exact length of a diagonal of the paper?
 - Explain why Marcus' ruler will not give him an exact measurement of the diagonal.
- Consider the following statement: All algebra students can solve quadratic equations.
 - Write the statement in if-then form.
 - Identify the antecedent and consequent for Part a.
 - Write the converse of the statement you wrote in Part a.
 - Decide whether the statement you wrote in Part c is true. Explain your answer.
- Prove or find a counterexample to the statement: If the tens digit of a 4-digit number is 4 and the units digit is 8, then the number is divisible by 4.

- What algebraic relationship is pictured by the rectangles, given that $b < a$?



- The product of two numbers is 717, and their sum is -242 . What are the numbers?
- Find the value of x in the diagram at the right.
 - Is x rational or irrational?
- True or False** Determine whether each of the following is true or false. Explain your answers.
 - If a triangle is formed by cutting a square in half along one of its diagonals, then the triangle is isosceles.
 - If a triangle is formed by cutting a square in half along one of its diagonals, then the triangle is equilateral.
 - True or False** Determine whether the following statement is true or false and explain your answer: A person can be President of the United States if and only if he or she was born in the United States.
 - If a number is divisible by 3 and another number is divisible by 4, then their product is divisible by 12. Illustrate this statement with a picture and explain why your picture shows that the statement is true.

Chapter 13

Chapter Review

SKILLS Procedures used to get answers

OBJECTIVE A Show and justify the steps in solving an equation. (Lesson 13-3)

In 1 and 2, fill in the table for the proof.

1.

Conclusions	What Was Done	Justifications
$4x + 5 = 17$?	?
$4x + 5 + -5 = 17 + -5$?	?
$4x + 0 = 12$?	?
$4x = 12$?	?
$\frac{1}{4} \cdot 4x = 12 \cdot \frac{1}{4}$?	?
$1 \cdot x = 3$?	?
$x = 3$?	?

2.

Conclusions	What Was Done	Justifications
$2n + 5 = 4n + 3$?	?
$2n + 2 = 4n$?	?
$2 = 2n$?	?
$1 = n$?	?

3. Prove: If $3t - 15 = 4t + 2$, then $t = -17$.

4. Prove: $\sqrt{16y - 16} = 2y$ if and only if $y = 2$.

OBJECTIVE B Find two numbers given their sum and product. (Lesson 13-4)

5. There are 26 students in a dancing class. If you know there are 165 possible boy-girl couples from this group, how many boys and how many girls are in the class?

SKILLS
PROPERTIES
USES
REPRESENTATIONS

In 6–9 find the two numbers that satisfy the given conditions.

6. $n + m = 10, nm = 24$

7. $xy = 2.3, x + y = 5.6$

8. $uv = 35, u + v = 12$

9. $p + q = -46, pq = 529$

10. Mrs. Violet doesn't know the dimensions of her rectangular garden, but she knows it has an area of 23.52 square meters. She also remembers that she needs 19.6 meters of fencing for her garden. Find the dimensions of Mrs. Violet's garden.

PROPERTIES The principles behind the mathematics

OBJECTIVE C Identify the antecedent and consequent of an if-then statement not necessarily given in if-then form. (Lesson 13-1)

In 11–14 identify the antecedent and the consequent.

11. If an animal has feathers then it is a bird.

12. It is spring if the trees are blooming.

13. No irrational number can be represented as the ratio of two integers.

14. James doesn't listen to music when he studies.

OBJECTIVE D Determine whether if-then and if-and-only-if statements in algebra or geometry are true or false. (Lessons 13-1, 13-2)

In 15–18, is the statement true or false?

15. A number is divisible by 3 if it is divisible by 9.
16. If $x = 7$ or $x = 3$, then $x^2 + 10x + 21 = 0$.
17. A triangle is equilateral if and only if two of its sides are equal and it has one 60° angle.
18. If only two outcomes are possible and they are equally likely, then the probability of each is 50%.

OBJECTIVE E Prove divisibility properties of integers. (Lessons 13-5, 13-6)

19. Prove that a 3-digit number abc is divisible by 7 only if the number $2a + 3b + c$ is divisible by 7.
20. Show that if n is even then n^3 is divisible by 8.
21. Show that all 6-digit integers of the form $xyzxyz$ are divisible by 13.
22. Show that if the 4-digit number $abcd$ written in base 10 is divisible by 11, then $b + d - (a + c)$ is divisible by 11.

OBJECTIVE F Apply the definitions and properties of rational and irrational numbers. (Lesson 13-7)

In 23–26, tell whether the number is rational or irrational.

23. $\sqrt{6}$
24. $0.\overline{142857}$
25. $\sqrt{169}$
26. $2\pi - 3$

27. Is it possible for two irrational numbers to have a product that is rational? Explain why or why not.
28. Is it possible for two rational numbers to have a product that is irrational? Explain why or why not.

USES Applications of mathematics in real-world situations

OBJECTIVE G Determine whether if-then and if-and-only-if statements in real-world contexts are true or false. (Lessons 13-1, 13-2)

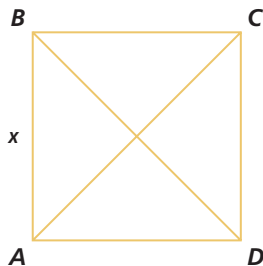
In 29–32, a statement is given.

- a. Is the statement true?
 - b. Is the converse true?
 - c. If either the statement or the converse is not true, change the statement so that both are true. Rewrite the new statement in if-and-only-if form.
29. A year with 366 days is a leap year.
 30. If you live in France, you live within 10 kilometers of the Eiffel Tower.
 31. If you are an eleventh grader, you are in high school.
 32. All horses are four-legged animals.

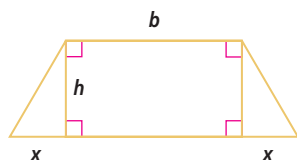
REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE H Display or prove properties involving multiplication using areas of polygons or squares. (Lesson 13-8)

33. Picture the property that for all positive numbers a and b , $(a + b)^2 = a^2 + b^2 + 2ab$.
34. Square $ABCD$ is pictured below. Show that the area of $ABCD$ is equal to the sum of the areas of the four small triangles.

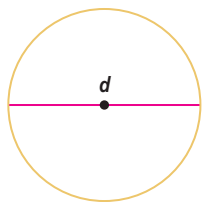


35. Draw a rectangle with dimensions a and b , and draw a diagonal from one corner to the other, making two triangles. Prove that the diagonal cuts the area of the rectangle in half.
36. Use the isosceles trapezoid below to show that $\frac{1}{2}(2b + 2x)h = xh + bh$.



OBJECTIVE I Determine whether lengths of geometric figures are rational or irrational. (Lesson 13-7)

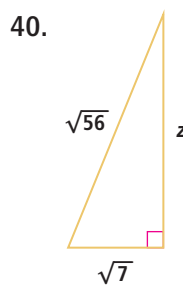
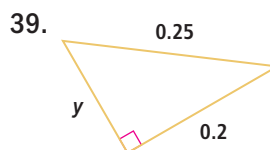
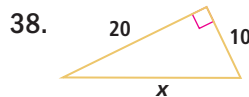
37. Consider the circle below. Its circumference is 32 inches.



- What is the exact radius of the circle?
- Is this number rational or irrational?

In 38–40,

- determine the missing length, and
- determine whether your answer to Part a is rational or irrational.



- Draw a segment whose length is $1 + \sqrt{8}$ centimeters.
- Is that length rational or irrational?