

## Lesson

## 13-2

## The Converse of an If-Then Statement

► **BIG IDEA** The converse of the statement, “If A, then B.” is the statement, “If B, then A.”

You might have heard of the question, “Which came first, the chicken or the egg?” This question is difficult because: If there is a chicken, then there must have been an egg. If there is an egg, then there must have been a chicken.

These two if-then statements are *converses*. The *converse* of the statement, “If  $p$ , then  $q$ ” is the statement, “If  $q$ , then  $p$ .” Another way of stating this is: the **converse** of an if-then statement is found by switching the antecedent and the consequent of the statement.

**STOP** QY

### Converses of True Statements

Often the converse of a true statement is not true. Here is an example.

**Statement** If  $x = 8$ , then  $x^2 = 64$ . (true)

**Converse** If  $x^2 = 64$ , then  $x = 8$ . (False;  $x$  could also be  $-8$ .)

#### GUIDED

##### Example 1

The given statement is true. Write its converse and explain why the converse is not true.

- If a quadrilateral has 4 sides of the same length and its diagonals have the same length, then the quadrilateral is a rectangle.
- If you live in the state of North Dakota, then you live in the United States.

##### Solutions

- Converse: If a quadrilateral is \_\_\_\_\_, then \_\_\_\_\_. This statement is false because \_\_\_\_\_.
- Converse: If \_\_\_\_\_, then \_\_\_\_\_. The statement is false because \_\_\_\_\_.

### Vocabulary

converse

equivalent statements

if and only if

### Mental Math

Evaluate when  $a = 2$  and  $b = -2$ .

- $(a + b)^2$
- $a^2 + b^2$
- $a^2 + 2ab + b^2$

#### ► QY

Give the converse of the statement: If  $\sqrt{2x} = 10$ , then  $x = 50$ .

Sometimes the converse of a true statement is true. When a statement and its converse are both true, then the antecedent and consequent are **equivalent statements**. Equivalent statements can be connected by the phrase *if and only if*. Here is an example.

**Statement** If  $x = 6$ , then  $2^x = 64$ . (true)

**Converse** If  $2^x = 64$ , then  $x = 6$ . (true)

Because the statement and its converse are both true, you can write:  
 $x = 6$  **if and only if**  $2^x = 64$ .

## GUIDED

### Example 2

The given statement is true and so is its converse. Write the converse and then combine the statement and its converse into one if-and-only-if statement.

- If a quadrilateral has 4 sides of the same length and its diagonals have the same length, then the quadrilateral is a square.
- If you live in the largest country in South America, then you live in Brazil.

### Solution

- Converse: If a quadrilateral \_\_\_\_\_?, then \_\_\_\_\_?. You can write: A quadrilateral has 4 sides of the same length and diagonals of the same length *if and only if* \_\_\_\_\_?.
- Converse: If you live in Brazil, then you live in the largest country in South America. You can write: \_\_\_\_\_? *if and only if* \_\_\_\_\_?.



Brazil is the fourth most populous country in the world with about 190 million people.

Source: *The World Factbook*

When you see the phrase “if and only if,” then you can separate the sentence into two if-then statements.

In Hardnox High School, a student is on the honor roll if and only if his or her grade point average is at least 3.75.

means

In Hardnox High School, if a student’s grade point average is at least 3.75, then the student is on the honor roll.

and

If a student in Hardnox High School is on the honor roll, then his or her grade point average is at least 3.75.

## Questions

### COVERING THE IDEAS

- State the converse of the statement: If there is smoke, then there is fire.

2. **Multiple Choice** If a statement is true, then its converse  
**A** must be true.      **B** may be true.      **C** must be false.

In 3–5, a statement is given.

- Is it true?
  - State its converse.
  - Is its converse true?
  - If either the statement or its converse is not true, correct them so that they are both true.
- If an integer is divisible by 3 and by 4, then it is divisible by 24.
  - If both the units and tens digits of an integer written in base 10 equal 0, then the integer is divisible by 100.
  - If  $7u < 56$ , then  $u > 8$ .

In 6–8, write the two if-then statements that are meant by the if-and-only-if statement.

- You will receive full credit for this question if and only if you get both parts correct.
- A quadrilateral is a rectangle if and only if it has four right angles.
- $5x + 4y = 20$  if and only if  $y = -1.25x + 5$ .

In 9 and 10, rewrite the definition as an if-and-only-if statement.

- Every linear function has an equation of the form  $f(x) = ax + b$ .
- The reciprocal of a nonzero number  $x$  is the number  $y$  such that  $xy = 1$ .

In 11–14, are statements (1) and (2) equivalent? If not, why not?

- |                      |                     |
|----------------------|---------------------|
| 11. (1) $x = 3$      | 12. (1) $y = 15$    |
| (2) $2x = 6$         | (2) $y^4 = 50,625$  |
| 13. (1) $z = 8$      | 14. (1) $a + b = c$ |
| (2) $z^2 + 48 = 14z$ | (2) $c - b = a$     |

### APPLYING THE MATHEMATICS

In 15–18, a statement is given.

- Is it true?
  - State its converse.
  - Is its converse true?
  - If either the statement or its converse is not true, correct them so that they are both true.
- If  $(x - 5)(2x + 3) = 45$ , then  $2x^2 - 7x + 30 = 90$ .

16. If a polygon has 7 sides, then it is a hexagon.
17. If a parabola has an equation of the form  $y = (x - 5)(x + 3)$ , then its  $x$ -intercepts are  $-5$  and  $3$ .
18. If  $3x + 4y = 6$  and  $2x - 5y = 7$ , then  $x = 6$  and  $y = -3$ .

In 19–21, a statement is given.

- a. Tell whether the statement is true.
- b. If the statement is true, give an example. If it is not true, modify it so that it is true.
19. A line is a vertical line if and only if its slope is undefined.
20.  $x^a \cdot x^b = x^{a+b}$  if and only if  $a$  and  $b$  are positive integers.
21. A person can be a U.S. citizen if and only if the person was born in the United States.

### REVIEW

In 22 and 23, tell whether the given if-then statement is true or false. If it is false, provide a counterexample to the statement.

(Lessons 13-1, 9-1)

22. If a dining room table seats 8 people, then it seats 6 people.
23. If  $n$  is even, the graph of  $y = ax^n$  crosses the  $x$ -axis twice.
24. **Multiple Choice** Rectangles of width  $x$  are cut off from two adjacent sides of a 12 in.-by-12 in. sheet of wrapping paper, as shown at the right. What is the area, in square inches, of the square region that remains? (Lesson 11-6)
- A  $12 - x^2$                                       B  $144 + x^2$   
 C  $144 - x^2$                                       D  $144 - 24x + x^2$
25. If 8 pencils and 5 erasers cost \$4.69 and 3 pencils and 4 erasers cost \$2.80, find the cost of 2 pencils. (Lesson 10-5)

In 26–28, simplify the expression. (Lessons 8-6, 8-5)

26.  $5(3\sqrt{x})^2$                       27.  $(6 \cdot 2^{-2})^3$                       28.  $\frac{9 \pm \sqrt{18}}{3}$

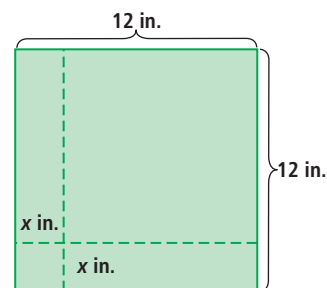
### EXPLORATION

29. Write two statements from outside of mathematics that are true but whose converses are false.



A valid U.S. passport is required for U.S. citizens to enter and leave most foreign countries.

Source: U.S. Department of State



### QY ANSWER

If  $x = 50$ , then  $\sqrt{2x} = 10$ .