



**Example 1**

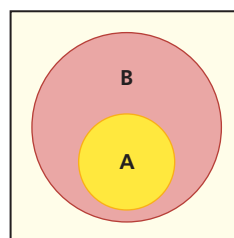
**True or False** If  $B$  is an insect, (then)  $B$  has six legs.

**Solution** The statement is true because every insect has six legs. (Having six legs is one of the defining characteristics of insects.)

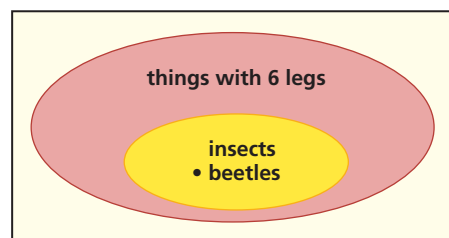
An if-then statement with a variable describes a pattern. As in any pattern, a value of the variable for which both the antecedent and consequent of an if-then statement is true is an instance of the statement. A beetle is an insect and a beetle has six legs. So a beetle is an instance of Statement 1.

Situations in which the antecedent is false do not affect whether an if-then statement is true. A dining room table might have six legs, but a table is not an insect. So a dining room table is not an instance of Statement 1.

A true if-then statement can be represented with a Venn diagram. The set of insects is placed inside the set of things with six legs.



If A, then B.



Beetles are the largest group in the animal kingdom, representing one-fourth of all animals.

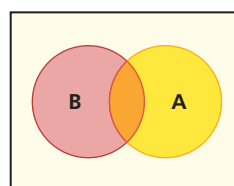
Source: San Diego Zoo

**Example 2**

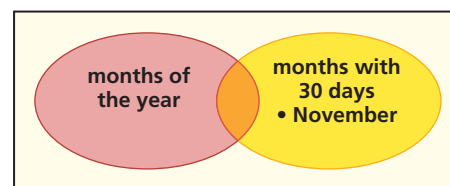
**True or False** If  $x$  is a month of the year, then  $x$  has 31 days.

**Solution** This statement is not true because there are some months that have fewer than 31 days.

If there is a value of the variable for which the antecedent of an if-then statement is true and the consequent is not true, then the if-then statement is *false*. This value is a *counterexample* to the statement. The month of November has only 30 days, so it is a counterexample to the if-then statement. One counterexample is enough to cause an if-then statement to be false.



If A, then sometimes (but not always) B.

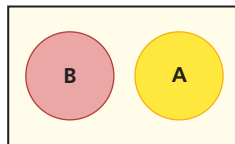


Even though there are values of the variable for which the statement of Example 2 is true, because there is a counterexample, the generalization is false.

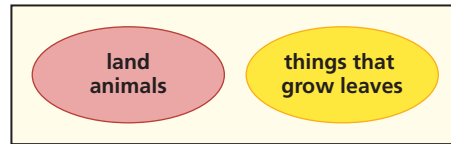
**Example 3**

Draw a Venn diagram for the statement: If  $G$  is a land animal,  $G$  grows leaves.

**Solution** A tiger is a land animal and a tiger does not grow leaves, so the statement is false. In fact, no land animals grow leaves. So if  $G$  is a land animal,  $G$  never grows leaves. The Venn diagram has two circles that do not overlap.



If A, then not B.

**If-Then Statements in Mathematics**

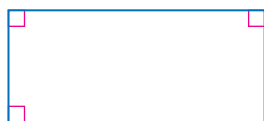
If-then statements occur throughout mathematics because they clarify what is given information and what are conclusions.

**GUIDED****Example 4**

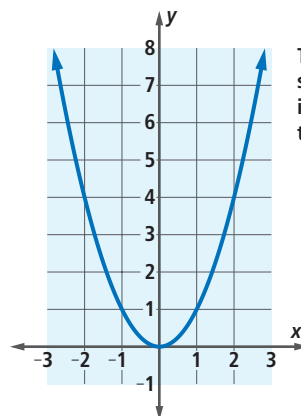
Fill in the blanks.

If-Then Statement	True or False	Explanation
a. If $3n + 5 = 65$ , then $n = 20$ .	true	If $3n + 5 = 65$ , then $3n = \underline{\quad?}$ . If $3n = \underline{\quad?}$ , then $n = 20$ .
b. If a figure is a square, then it is a rectangle.	?	Definition of square: a square is $\underline{\quad?}$ .
c. If $x$ is a real number, then $x^2 > 0$ .	?	0 is a real number and $0^2$ is not greater than 0.
d. If a quadrilateral has 3 right angles, then it is a square.	false	?

Counterexamples to the two false statements in Guided Example 4 can be pictured. For Statement c, graph  $y = x^2$ . Notice that the graph is not always above the  $x$ -axis, so it is not the case that  $x^2 > 0$  for every value of  $x$ . For Statement d, show a quadrilateral that has 3 right angles but is not a square.



This quadrilateral has 3 right angles and is not a square.



The point  $(0, 0)$  shows that if  $x = 0$ , then  $x^2 = 0$ .

## Putting Statements into If-Then Form

Statements with the words “all,” “every,” and “no” can be rewritten in if-then form without changing their meaning.

### GUIDED

#### Example 5

Rewrite each statement in if-then form.

Statement	If-then Form with Variables
a. Every whole number is a real number.	If $x$ is a whole number, then ____? ____.
b. All people born in the United States are U.S. citizens.	If ____? ____, then $P$ is a U.S. citizen.
c. No power of a positive number is negative.	If $p$ is a positive number and $g$ is any real number, then ____? ____.

## Questions

### COVERING THE IDEAS

In 1 and 2, identify the antecedent and the consequent of the if-then statement.

- If the sun shines this afternoon, I will be happy.
- The world would be a better place if people did not litter.

In 3 and 4, rewrite the statement as an if-then statement with a variable in it. Underline the antecedent once and the consequent twice.

- Every integer greater than 1 is either a prime number or a product of prime numbers.
- When a person drives over 35 miles per hour on that street, that person is speeding.
- Explain why this if-then statement is true.  
If  $3x + 16 > 10$ , then  $x > -2$ .
- Explain why this if-then statement is false.  
If  $3x + 16 > 10$ , then  $x > 0$ .

In 7–9, an antecedent is given. Complete the statement with two consequents that make it true.

- If  $L$  and  $W$  are the length and width of a rectangle, then \_\_\_\_? \_\_\_\_.
- If  $x^m \cdot x^n = x^{m+n}$ , then \_\_\_\_? \_\_\_\_.
- If  $n$  is divisible by 10, then \_\_\_\_? \_\_\_\_.



Interstate highways nationwide usually have posted speed limits between 55 and 75 mi/h.

Source: Federal Highway Administration

In 10–13, an if-then situation is given.

- a. Tell whether the statement is true or false.
  - b. Draw a Venn diagram picturing the situation.
10. If a figure is a rectangle, then the figure is a square.
  11. If you are in the United States and in Chattanooga, then you are in Tennessee.
  12. If a number is an even number, then it is a prime number.
  13. If  $x > 0$ , then  $-x > 0$ .

### APPLYING THE MATHEMATICS

In 14–17, a false statement is given.

- a. Find a counterexample that shows it is false.
  - b. Find the largest domain of the variable for which the statement is true.
14. If  $w$  is a real number, then  $w^3$  is positive.
  15. If  $t$  is a real number, then  $t^4$  is positive.
  16. If  $r$  is a real number, then  $2r \geq r$ .
  17. If  $x$  is a real number and  $y = x^2 + x$ , then  $y \geq 0$ .

In 18 and 19, draw a counterexample to the statement.

18. For all real numbers  $a$ , the graph of  $y = ax^2$  is a parabola.
19. A triangle cannot have two angles each with measure over  $75^\circ$ .
20. Draw the following four true statements in one Venn diagram.
  - a. Every rhombus is a parallelogram.
  - b. Every rectangle is a parallelogram.
  - c. If a figure is both a rhombus and a rectangle, then it is a square.
  - d. Every square is both a rhombus and a rectangle.

In 21 and 22, rewrite the statement as an if-then statement and indicate why each statement is false.

21. All sentences have a subject, verb, and object.
22. Every president of the United States has served fewer than three terms.

### REVIEW

23. **Skill Sequence** Factor each expression. (Lesson 12-4)
  - a.  $a^2 - 36$
  - b.  $n^2 - 5n - 36$
  - c.  $x^2 - 5xy - 36y^2$

24. **Fill in the Blank** Do this problem in your head. Because one thousand times one thousand equals one million, then  $1,005 \cdot 995 = \underline{\quad?}$ . (Lesson 11-6)
25. Tickets to a hockey game cost \$22 for adults and \$16 for children. The total attendance at one game was 3,150 and the total revenue from ticket sales for the game was \$66,258. How many of each kind of ticket were sold for the game? (Lesson 10-5)
26. Write  $2^{-3} + 4^{-3}$  as a simple fraction. (Lesson 8-4)
27. Let  $g(x) = \frac{3}{4}x - 7$ . (Lesson 7-6)
- Calculate  $g(80)$ .
  - Calculate  $g(-80)$ .
  - Describe the graph of  $g$ .
28. **True or False** The slope of the line through  $(x_1, y_1)$  and  $(x_2, y_2)$  is the opposite of the slope of the line through  $(x_2, y_2)$  and  $(x_1, y_1)$ . (Lesson 6-6)



The Carolina Hurricanes won hockey's 2006 Stanley Cup.

### EXPLORATION

29. Consider this statement: No four points in a plane can all be the same distance from each other.
- Write the statement in if-then form.
  - Is the statement true or false?
  - If “in a plane” is deleted from the statement, show that the resulting statement is false.

### QY ANSWER

If  $A$  is a land animal, then  
 $A$  grows leaves.