

Using Algebra to Prove



▶ 13-1 If-Then Statements

13-2 The Converse of an If-Then Statement

13-3 Solving Equations as Proofs

13-4 A History and Proof of the Quadratic Formula

13-5 Proofs of Divisibility Properties

13-6 From Number Puzzles to Properties of Integers

13-7 Rational Numbers and Irrational Numbers

13-8 Proofs of the Pythagorean Theorem

In previous chapters, you looked at instances and made generalizations about patterns.

For example, the instances $3 \cdot 5 + 3 = 3 \cdot 6$, $7.4 \cdot 5 + 7.4 = 7.4 \cdot 6$, and $\left(-\frac{8}{99}\right) \cdot 5 + \left(-\frac{8}{99}\right) = \left(-\frac{8}{99}\right) \cdot 6$ can be described by the general pattern

$n \cdot 5 + n = n \cdot 6$.

In making this generalization, you have used *inductive reasoning*.

Inductive reasoning is the

process of arriving at a general conclusion (not necessarily true) from specific instances.

You use inductive reasoning quite often in

everyday situations. For example, if every

single family house you see on a block is

yellow, you may want to conclude that every

house in a city is also yellow. That conclusion

is wrong. There are houses in a neighborhood

or city that are not yellow. But the general



pattern $n \cdot 5 + n = n \cdot 6$ does happen to be true for all real numbers. We know this because we can *prove* it mathematically.

To prove a generalization, you must use *deductive reasoning*. Deductive reasoning starts from properties that are assumed to be true. For example, we assumed the Distributive Property of Multiplication over Addition to be true: If a , b , and c , are any real numbers, then $ab + ac = a(b + c)$.

If this is true for all real numbers, then it is true when $a = n$, $b = 5$, and $c = 1$. Substituting these values for a , b , and c , $n \cdot 5 + n \cdot 1 = n(5 + 1)$.

Using another assumed property, that if n is any real number, then $n \cdot 1 = n$. Adding 5 and 1 then gives $n \cdot 5 + n = n \cdot 6$.

This string of justified if-then statements has *proved* that for all real numbers n , $n \cdot 5 + n = n \cdot 6$.

You may not have written the words *if* and *then*, but in this course you have often strung if-then statements to follow each other.

A string of justified statements that follow from each other like these is a *proof*. In this chapter, you will see many examples of proofs that use the algebra that you have studied. These proofs involve the solving of equations, divisibility properties of arithmetic, and geometric figures. They comprise one of the most important uses of algebra: showing that a statement is true when there are infinitely many cases to consider.