Chapter 6 Summary and Vocabulary

- The simplest **quadratic equation** is of the form $x^2 = k$. If $k > 0$, there are two real solutions, $\sqrt{k}$ and $-\sqrt{k}$. When $k < 0$, the solutions are the imaginary numbers $i\sqrt{-k}$ and $-i\sqrt{-k}$, where, by definition, $\sqrt{-1} = i$. Any number of the form $a + bi$, where a and b are real numbers, is a **complex number**. Complex numbers are added, subtracted, and multiplied using the properties that apply to operations with real numbers and polynomials.

- Areas, paths of objects, and relations between the initial velocity of an object and its height over time lead to problems involving quadratic equations and functions. A projectile's height h above the ground on a planet with gravity g at time t after being launched with initial velocity v_0 from an initial height h_0 satisfies

$$h = \frac{1}{2}gt^2 + v_0t + h_0.$$

- When a, b, and c are real numbers and $a \neq 0$, the graph of the general quadratic equation $y = ax^2 + bx + c$ is a parabola. Using a process known as **completing the square,** this equation can be rewritten in **vertex form** $y - k = a(x - h)^2$. This parabola is a translation image of the parabola $y = ax^2$ you studied in Chapter 2. Its vertex is (h, k), its line of symmetry is $x = h$, and it opens up if $a > 0$ and opens down if $a < 0$. If data involving two variables are graphed in a scatterplot that appears to be part of a parabola, you can use three points on the graph to set up a system of equations that will allow you to find a, b, and c in the equation $y = ax^2 + bx + c$.

- The values of x for which $ax^2 + bx + c = 0$ can be found by using the **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

- The expression $b^2 - 4ac$ in the formula is the **discriminant** of the quadratic equation, and reveals the nature of its roots.

Vocabulary

6-1
quadratic expression
quadratic equation
quadratic function
standard form of a quadratic
binomial

6-2
absolute value
absolute-value function
square root
rational number
irrational number

6-3
corollary
vertex form
axis of symmetry
minimum, maximum

6-5
completing the square
perfect-square trinomial

6-6
quadratic regression

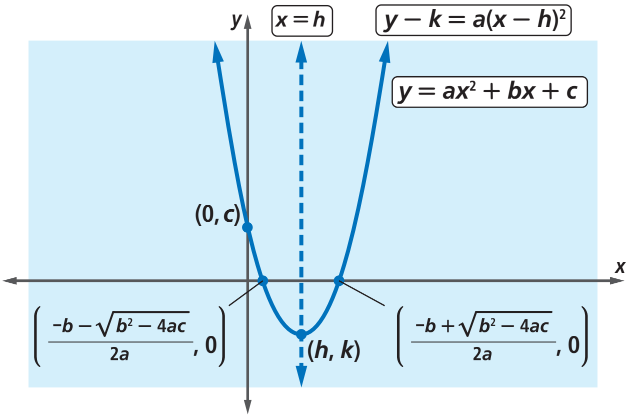
6-7
Quadratic Formula

6-8
imaginary number
*$\sqrt{k}$
*$\sqrt{-1}$, i
*pure imaginary number

6-9
*complex number
*real part, imaginary part
*equal complex numbers
*complex conjugate

6-10
*discriminant
*root of an equation
*zeros of a function

- If $b^2 - 4ac > 0$, there are two real solutions, as shown below.



- If $b^2 - 4ac = 0$, there is exactly one solution and the vertex of the parabola is on the x-axis. If a, b, and c are rational numbers and the discriminant is a perfect square, then the solutions are rational numbers.
- If $b^2 - 4ac < 0$, there are no real solutions and the parabola does not intersect the x-axis. The nonreal solutions are **complex conjugates**.

Postulates, Theorems, and Properties

Binomial Square Theorem (p. 376)
Absolute Value–Square Root Theorem (p. 381)
Graph-Translation Theorem (p. 387)
Parabola-Translation Theorem (Graph-Translation Corollary) (p. 388)
Parabola Congruence Theorem (p. 395)
Completing the Square Theorem (p. 402)
Quadratic Formula Theorem (p. 414)
Square Root of a Negative Number Theorem (p. 422)
Properties of Complex Numbers Postulate (p. 428)
Discriminant Theorem (p. 437)

Chapter 6 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

In 1–3, consider the parabola with equation $y = x^2 - 10x + 21$.

1. Rewrite the equation in vertex form.
2. What is the vertex of this parabola?
3. What are the x-intercepts of this parabola?

In 4–7, perform the operations and put the answer in $a + bi$ form.

4. $5i \cdot i$
5. $\sqrt{-12} \cdot \sqrt{-3}$
6. $\frac{6 + \sqrt{-64}}{3}$
7. $(1 + 5i)(-4 + 3i)$
8. If $z = 6 - 5i$ and $w = 2 + 5i$, write $z - w$ in $a + bi$ form.
9. **Multiple Choice** How does the graph of $y - 3 = -(x + 4)^2$ compare to the graph of $y = -x^2$? It is translated:
   - A 4 units to the right and 3 units down.
   - B 4 units to the right and 3 units up.
   - C 4 units to the left and 3 units up.
   - D 4 units to the left and 3 units down.
10. Graph $y - 3 = -(x + 4)^2$.
11. If $f(x) = -(x + 4)^2 + 3$, find the domain and range of f.

In 12–15, solve the equation. Show your work.

12. $|t + 12| = 21$
13. $\sqrt{(6s + 5)^2} = 9$
14. $17 = (y + 18)^2$
15. $3x^2 + 18x = 4x + 5$
16. This statement is false: $\sqrt{x} = |x|$ for all real values of x. Correct it.
17. Expand: $(2a - 5)^2 + (2a + 5)^2$.
18. The discriminant of an equation $ax^2 + bx + c = 0$ is negative. What does this indicate about the solutions of the equation?
19. Consider the graph of $y = ax^2 + bx + c$. How many x-intercepts does the graph have if its discriminant is
    a. -2? b. 0? c. 17?

In 20 and 21, suppose the height h in feet of a ball at time t seconds is given by $h = -16t^2 + 28t + 6$.

20. How high is the ball after 1.5 seconds?
21. When does the ball hit the ground?
22. A 12-foot by 16-foot rectangular garden has an x-foot wide walkway that surrounds it on all sides. Write an expression for the area of the walkway.
23. **Multiple Choice** The graph of each equation is a parabola. Which parabola is not congruent to the others?
    - A $y = (x + 4)^2$
    - B $y = 4x^2$
    - C $y + 4 = x^2$
    - D $y + 2 = x^2$
24. If circles are drawn so that every circle intersects every other circle, there is a pattern to the maximum number of intersection points.
    a. Draw a scatterplot of these data.
    b. Fit a quadratic model to these data.
    c. Predict in how many points 17 circles will intersect if every circle intersects every other circle.

Number of Circles n	Maximum Number of Intersections $f(n)$
1	0
2	2
3	6
4	12
5	20

Chapter 6 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Expand products and squares of binomials. (Lesson 6-1)

In 1–8, expand.

1. $(7p + 2)(p - 3)$
2. $(r + 2q)(r - q)$
3. $6(3w - 2)(2w + 1)$
4. $(3x + \sqrt{2})(3x - \sqrt{2})$
5. $(a + b)^2$
6. $(2x + 1)^2$
7. $13(y - 3)^2$
8. $7(s + t)^2 - 3(s - t)^2$

OBJECTIVE B Convert quadratic equations from vertex to standard form, and vice versa. (Lessons 6-4, 6-5)

In 9 and 10, rewrite in standard form.

9. $y = 5(x + 1)^2$
10. $y + 3 = 0.25(x - 4)^2$

In 11 and 12, write each equation in vertex form.

11. $y = x^2 + 9x - 5$
12. $3y = 7x^2 - 6x + 5$
13. **Multiple Choice** Which equation is equivalent to $y - 1 = 2(x - 1)^2$?

A $y = 2x^2 + 4x - 3$
B $y = 2x^2 + 4x + 3$
C $y = 2x^2 - 4x + 3$
D $y = 2x^2 - 4x - 3$

OBJECTIVE C Solve quadratic equations. (Lessons 6-2, 6-7, 6-8, 6-10)

In 14–22, solve.

14. $(x - 3)^2 = 0$
15. $r^2 - 26 = 0$
16. $d^2 = -24$
17. $-25 = y^2$
18. $2x^2 + x - 1 = 0$
19. $3 - s^2 - 5s = 6$
20. $z^2 + 2z - 8 = 7$
21. $k^2 = 6k - 9$
22. $2x(x - 2) = -1 + x^2 - 2x$

OBJECTIVE D Solve absolute-value equations. (Lesson 6-2)

In 23–28, solve.

23. $|11 - y| = 15$
24. $|s + 3| = 5$
25. $-\sqrt{(3x + 2)^2} = -5$
26. $\sqrt{(x - 2)^2} = 3$
27. $|4t - 20| = 0$
28. $|96 - A| = -4$

OBJECTIVE E Perform operations with complex numbers. (Lesson 6-8, 6-9)

In 29–34, simplify.

29. $-i^2$
30. $\sqrt{-49}$
31. $\sqrt{-9} \cdot \sqrt{-25}$
32. $\sqrt{-7} \cdot \sqrt{7}$
33. $2i \cdot 3i$
34. $3\sqrt{-4} + \sqrt{-9}$

In 35 and 36, write the conjugate.

35. $8 - 3i$
36. $-7i$

In 37–42, suppose $r = 7 - i$ and $s = 3i + 2$. Evaluate and simplify.

37. rs

38. s^2

39. $2r - s$

40. $ir + 3s - i$

41. $\frac{r}{s}$

42. $\frac{is}{r}$

PROPERTIES Principles behind the mathematics

OBJECTIVE F **Apply the definition of absolute value and the Absolute Value-Square Root Theorem.** (Lesson 6-2)

43. For which real numbers x is $|x| - x > 0$?

44. For which real numbers x is $|x| = -\pi$?

In 45–47, use the Absolute Value-Square Root Theorem to simplify.

45. $\sqrt{(11 + 5)^2}$

46. $-\sqrt{t^2}$

47. $-\sqrt{(-3)^2} + \sqrt{5^2}$

OBJECTIVE G **Use the Graph-Translation Theorem to interpret equations and graphs.** (Lessons 6-3, 6-4)

48. The preimage graph of $y = x^2$ is translated 50 units to the right and 300 units up. What is an equation for its image?

49. Describe how the graphs of $y = |x|$ and $y = |x + 2|$ are related.

50. Multiple Choice Which of the following is not true for the graph of the equation $y - 3 = -\frac{1}{2}(x + 1)^2$?

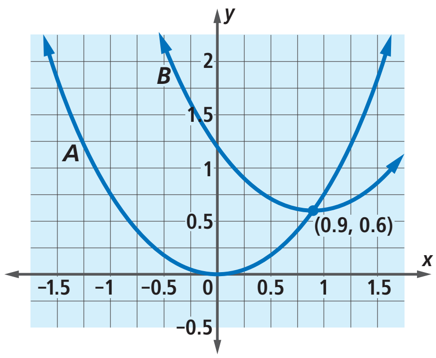
A The vertex is (-1, 3).

B The maximum point is (-1, 3).

C The equation of the axis of symmetry is $x = -1$.

D The graph opens up.

51. Assume parabola A is congruent to parabola B in the graph below.



a. What translation maps parabola A onto parabola B?

b. What is an equation for parabola B if parabola A has equation $y = \frac{3}{4}x^2$?

52. Compare the solutions to $4 = (x - 1)^2$ with the solutions to $x^2 = 4$.

53. Compare the solutions to $(k + 2)^2 = 25$ with the solutions to $k^2 = 25$.

54. The graph of $y = (5x + 3)^2$ is congruent to the graph of $y = ax^2$. What is a?

OBJECTIVE H **Use the discriminant of a quadratic equation to determine the nature of the solutions to the equation.** (Lesson 6-10)

In 55–57, an equation is given.

a. Evaluate its discriminant.

b. Give the number of real solutions.

c. Tell whether the solutions are rational, irrational, or nonreal.

55. $6x^2 + 9x + 1 = 0$

56. $z^2 = 81z + 81$

57. $5 + k = k^2 + 9$

USES Applications of mathematics in real-world situations

OBJECTIVE I Use quadratic equations to solve area problems or problems dealing with distance, velocity, and acceleration. (Lessons 6-1, 6-4, 6-5, 6-7)

58. A framed mirror is 28 inches by 34 inches, including the frame. The frame is w inches wide.
    a. Write an expression for the area of the mirror without the frame.
    b. If the area of the mirror is 85% of the total area, how wide is the frame?

In 59 and 60, use the equation, $h = -16t^2 + v_0t + h_0$ for the height h in feet of an object after t seconds. Ignore wind resistance.

59. A ball is thrown directly upward from an initial height of 5 feet at an initial velocity of 37 feet per second.
    a. Estimate to the nearest hundredth the highest point reached by the ball.
    b. To the nearest hundredth of a second, when does the ball reach its highest point?
    c. How far from the ground will the ball be after 2 seconds?

60. A ball is dropped from the top of a building that is 1000 feet tall. To the nearest tenth of a second, how long after it is dropped will the ball reach the ground?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE J Fit a quadratic model to data. (Lesson 6-6)

61. A researcher marked four evenly-spaced concentric circles on a field. She then released 1000 frogs and returned an hour later to count how many frogs were in each circle. She found 200 frogs: 11 frogs in the inner circle, 38 frogs in the first ring, 60 frogs in the second ring, and 91 frogs in the third ring. The researcher then created another evenly spaced concentric circle and repeated the experiment.
    a. Fit a model to the data that can predict the number of frogs in the fourth ring.
    b. How many frogs should the researcher expect to find in the fourth ring?
    c. Why will the model overestimate the number of frogs in the 20th concentric circle?

62. Consider the sequence {1, 5, 12, 22, 35,…}, where differences between consecutive terms increase by 3.
    a. Use quadratic regression to write an explicit formula for the terms of this sequence.
    b. Find the 23rd term in this sequence.

63. In 1980, Omaha, Nebraska, had a population of 314,255. By 1990, the population had increased to 344,463. The population continued to increase and reached 390,007 by 2000.
    a. Write a quadratic equation that models Omaha's population growth starting in 1980.
    b. Use your equation to predict Omaha's population in 2010.
    c. Based on your model, in which year will Omaha's population first exceed 1 million?

OBJECTIVE K **Graph quadratic functions and absolute-value functions and interpret them.** (Lessons 6-2, 6-3, 6-4, 6-10)

In 64–67, sketch a graph of the function, and label the vertex and x-intercepts with their coordinates.

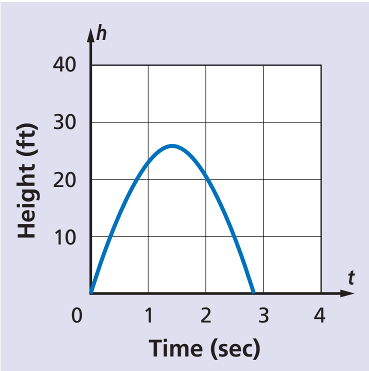
64. $y = 3x^2 + 18x$

65. $y - 6 = -\frac{1}{3}(x + 3)^2$

66. $y - 7 = |x - 2|$

67. $y + 1 = 2x^2$

68. The height of a ball thrown upward at time t is shown on the graph below.

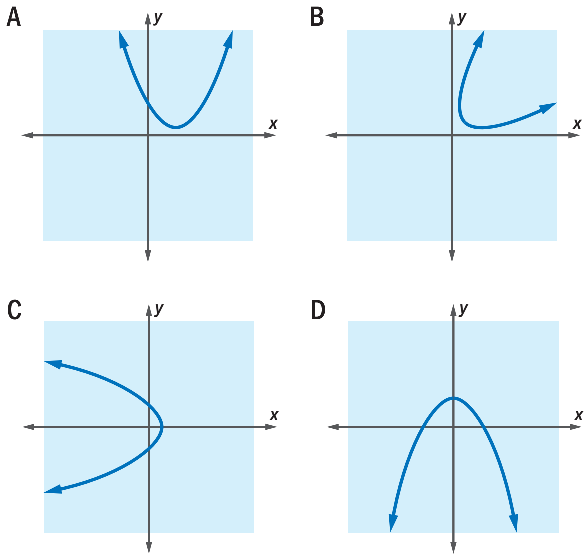


a. About when did the ball get to its maximum height?

b. About how high did the ball get?

c. About when was the ball 10 feet high?

In 69 and 70, refer to the parabolas shown below.



69. Which graph is of the form $y - k = a(x - h)^2$ with h positive?

70. Which graph is of the form $y - k = a(x - h)^2$ with a negative?

OBJECTIVE L **Use the discriminant of a quadratic equation to determine the number of x-intercepts of a graph of the associated quadratic function.** (Lesson 6-10)

In 71 and 72, give the number of x-intercepts of the graph of the parabola.

71. $y = 7x^2 + 5x - 13$

72. $y = \frac{1}{7}(x + 3)^2 - 6$

73. Does the parabola $y = 3x^2 - 9x$ ever intersect the line $y = -1$? Explain your reasoning.

74. How many x-intercepts does the graph of $y = -\frac{1}{5}(x - t)^2$ have when $t \neq 0$?