Lesson

6-6 Fitting a Quadratic Model to Data

Vocabulary

quadratic regression

BIG IDEA *Quadratic regression* is like linear regression in that it finds the model with the least sum of squares of differences from the given data points to the values predicted by the model.

In Chapter 3, you learned how to find an equation for the line through two points, and how to find a linear model for data that lie approximately on a straight line. You can also fit a *quadratic model* to data that lie approximately on a parabola. **Quadratic regression** fits a model of the form $y = ax^2 + bx + c$ to data.

Mental Math

Find the slope of the image of $3x + 2y = 7$ under the transformation.

a. $T_{6,-1}$

b. R_{90}

c. R_{270}

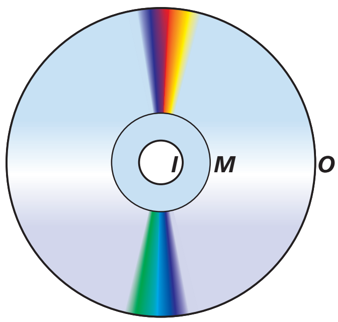
d. $r_{y=x}$

Activity

MATERIALS old CD that can get scratched up, two $8\frac{1}{2}$–inch by 11–inch pieces of paper, quarter, CAS or graphing calculator

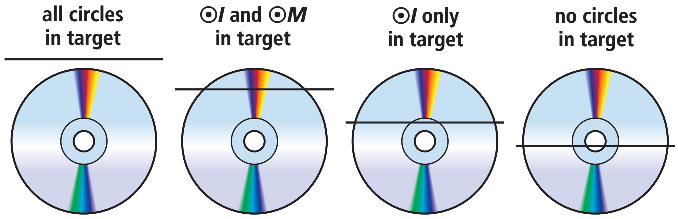
You are going to investigate how the radius *x* of a circular object affects the probability *y* of the object landing completely in a fixed region when dropped.

Step 1 A CD like the one pictured at the right has 3 circles on it. Circle *I* represents the hole in the CD. Circle *M* (for middle) represents the start of the silver writing surface. Circle *O* represents the outer edge of the CD. Draw a line across the middle of your $8\frac{1}{2}$–by–11 piece of paper, dividing it into two $5\frac{1}{2}$–by–$8\frac{1}{2}$ rectangular targets.



Step 2 On a separate sheet of paper, make four columns labeled Drop Number, *I*, *M*, and *O*. In the Drop Number column, write the integers 1 through 25. Place your divided sheet of $8\frac{1}{2}$–inch–by–11–inch paper on the floor and stand above it holding the CD waist high, parallel to the floor. Drop the CD.

If no part of the CD is touching the paper, drop it again. If any part of the CD is touching the paper, determine the score for each circle, *I*, *M*, and *O* as follows: If a circle on the CD lands completely inside one of the two rectangular targets, give it 1 point for that drop. Pictured below are the four possible situations and scores after a drop. The horizontal line represents either the edge of the paper or the line you drew on the paper.



circle	*I*	*M*	*O*	*I*	*M*	*O*	*I*	*M*	*O*	*I*	*M*	*O*
point	1	1	1	1	1	0	1	0	0	0	0	0

Drop the CD 25 times and fill in your table.

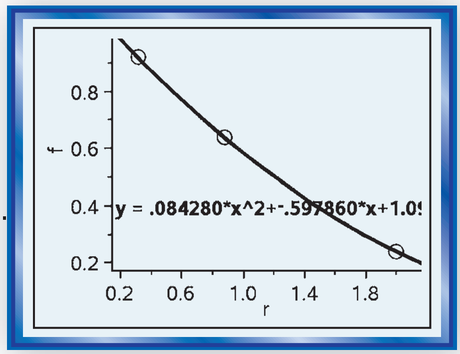
Step 3 Calculate the relative frequency of a circle landing completely in a target. For example, if *M* has 16 points, then its relative frequency of points per drop is $\frac{16}{25}$. Record the frequencies in a table like the one at the right. The radii of circles on a standard CD have been filled in for you, but you should measure your CD to check.

Circle	Radius (in.)	Relative Frequency
I	$\frac{5}{16}$	?
M	$\frac{7}{8}$	?
O	2	?

Step 4 Create a scatterplot with three data points (radius, frequency) for *I*, *M*, and *O* on your calculator.

Choose the quadratic regression option from the appropriate menu. A sample is shown at the right. On this calculator, the graph of a quadratic model for the data is added to the scatterplot.

The calculator displays the equation for the quadratic model.



Step 5 Examine the scatterplot with the graph of the regression equation on it. How well does your model fit your data?

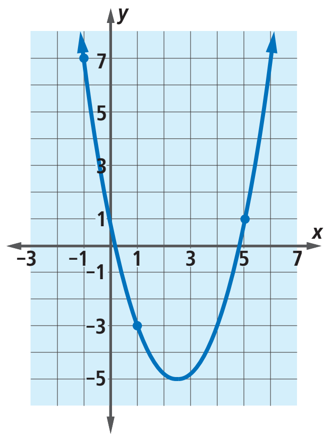
Step 6 Measure the diameter of a quarter and use your regression equation to predict the relative frequency of the quarter landing inside a target rectangle. Drop the quarter 25 times and see if the relative frequency is close to your prediction. Combine your results with other classmates. Compare the combined data with the prediction. Which value is closer to the predicted value—your own data or the combined data?

Finding the Equation of a Given Parabola

You can apply the techniques of solving systems of equations you learned in Chapter 5 and the regression technique in the Activity to find an equation for any parabola on which you know three points.

Example

The parabola at the right contains the points (–1, 7), (1, –3) and (5, 1). Find its equation.



Solution 1 Use a system of equations.

Because the ordered pairs (x, y) are solutions of the equation $y = ax^2 + bx + c$, substitute to get 3 linear equations in a, b, and c.

When $x = -1$, $y = 7$: $\quad 7 = a(-1)^2 + b(-1) + c$

When $x = 1$, $y = -3$: $\quad -3 = a(1)^2 + b(1) + c$

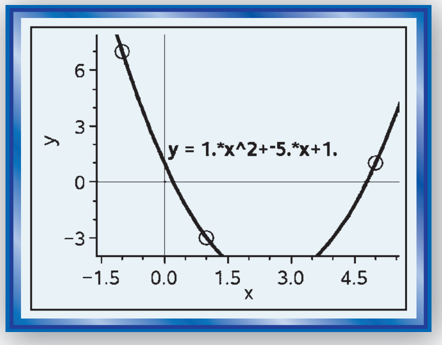
When $x = 5$, $y = 1$: $\quad 1 = a(5)^2 + b(5) + c$

So a, b, and c are solutions to the system $\begin{cases} 7 = a - b + c \\ -3 = a + b + c \\ 1 = 25a + 5b + c \end{cases}$.

You are asked to solve this system in Question 2.

Solution 2 Use quadratic regression.

Enter the x–coordinates and y–coordinates into lists in your calculator and apply quadratic regression. One calculator gives the solution at the right, so the parabola has equation $y = x^2 - 5x + 1$.



Check Substitute the points into the equation.

Does $(-1)^2 - 5(-1) + 1 = 7$? Yes.

Does $(1)^2 - 5(1) + 1 = -3$? Yes.

Does $(5)^2 - 5(5) + 1 = 1$? Yes; it checks.

Questions

COVERING THE IDEAS

1. Refer to the Activity.
   a. Write an equation for the function your quadratic regression describes.
   b. Use the model from Part a to predict the probability of a mini CD with a radius of 4 centimeters landing completely inside one of the two rectangular targets.

2. Solve the system of the Example to find an equation for the parabola that contains the given points.

In 3 and 4, solve a system of equations to write an equation for the parabola that contains the given points.

3. (4, –6), (–2, 30), (0, 10)
4. (–1, –3), (5, 81), (2, 12)

In 5 and 6, use quadratic regression to find an equation for the parabola that contains the given points.

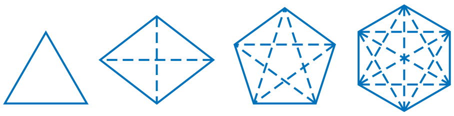
5. (3, 2), (–1, 5), (8, 7)
6. (–6, 5), (–1, 10), (3, 4)

APPLYING THE MATHEMATICS

7. A quarterback threw a ball from 5 yards behind the line of scrimmage and a height of 6 feet. The ball was 10 feet high as it crossed the line of scrimmage. It was caught 20 yards past the line of scrimmage at a height of 5 feet off the ground.
   a. Find an equation that gives the height y of the ball (in feet) when it was x yards beyond the line of scrimmage.
   b. Graph the equation.

Peyton Manning throwing for the Indianapolis Colts

8. In *The Greedy Triangle* by Marilyn Burns, an equilateral triangle keeps changing shape into a regular polygon with more and more sides. As the number of sides of a polygon increase, so do the number of diagonals of the polygon. Recall the formula for the number of diagonals in a polygon. If you have forgotten, you can derive it using quadratic regression.
   a. Count the number of diagonals in each polygon below and complete the table to the right.



Shape	Number of Sides	Number of Diagonals
Triangle	?	?
Quadrilateral	?	?
Pentagon	?	?
Hexagon	?	?

   b. Use quadratic regression to find a model for the number of diagonals in a polygon.
   c. Is this model exact or approximate?
   d. Use the model to determine the number of diagonals in a 50-sided polygon.

9. Shown below is a table of data on the eight major planets in our solar system. For each planet, the period of the orbit is given in Earth years and the average distance from the Sun is in astronomical units (AU). One AU is approximately the mean distance between Earth and the Sun. Each planet's distance from the Sun is not constant due to its elliptical orbit, so the average distance is given.

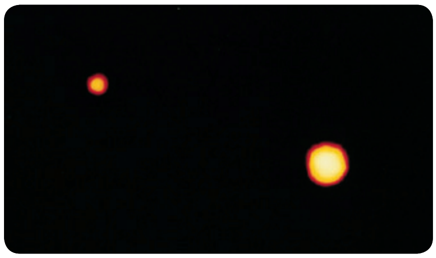
Planet	d = Avg. Distance From the Sun (AU)	P = Period of Orbit (in Earth years)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1.00	1.00
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.58	29.46
Uranus	19.20	84.01
Neptune	30.05	164.79

a. Create a scatterplot of the data. Use average distance d as the independent variable and period P as the dependent variable.

b. Fit a quadratic model to these data. Graph the model on the same axes as your scatterplot. Does the model seem like it fits the data? Why or why not?

c. Pluto, a dwarf planet, is an average 39.48 AU from the Sun. It takes Pluto 248.54 Earth years to orbit the Sun. Does Pluto fit your model?

d. Another dwarf planet, Sedna, or 2003 VB12, as it was originally designated, orbits the sun at an average distance of 90 AU. Use your model from Part b to predict how long it takes Sedna to orbit the Sun.



Pluto and one of its moons, Charon, are shown above. Sunlight takes about $5\frac{1}{2}$ hours to reach Pluto. Sunlight reaches Earth in about 8 minutes.

REVIEW

In 10–12, rewrite the equation in vertex form. (Lesson 6-5)

10. $y = x^2 - 14x + 53$

11. $y = 3x^2 - 9x + 9$

12. $y = x^2 - 4gx + 3g^2$

13. Find an equation in standard form for the image of the parabola with equation $y = -2x^2$ under the translation $T_{-4, -0.5}$. (Lessons 6-3, 6-1)

14. a. Find an equation for the line through points (a, b) and (b, a).

b. Use a rotation matrix to find an equation for the image of the line from Part a under a rotation of 90° about the origin. **(Lessons 4-9, 3-4)**

15. The Moon is much less massive than Earth, and exerts less gravity. Near the surface of the Moon, the acceleration due to gravity is about $5.31 \frac{\text{ft}}{\text{sec}^2}$. Suppose an astronaut drops an object on the Moon from a height of 5 feet.

a. How long will it take the object to fall?

b. How long would it have taken the object to fall if it was dropped from the same height on Earth? **(Lesson 6-4)**

EXPLORATION

16. Follow these steps to determine the theoretical probabilities that the circles of the CD in the Activity will be in one of the rectangular target areas.

a. Find the area of the region in which the center point of the CD can land so that the CD is touching the paper.

b. Find the area of the region in which the center point of the CD can land so that circle I lands completely in one of the target areas.

c. Let a be your answer to Part a, and b your answer to Part b. Calculate $\frac{b}{a}$. This number is the probability that the smallest circle is in the target area.

d. Repeat Parts b and c for the other two circles.

e. Do the probabilities seem to agree with the relative frequencies your class found?