Lesson

3-8 Solving Equations by Clearing Fractions

BIG IDEA Equations with fractions can be transformed into equivalent equations without fractions.

Choosing a Multiplier to Clear Fractions

With the techniques you have learned, you can solve any linear equation. However, when you want to solve an equation containing fractions, for example $\frac{t}{3} + \frac{t}{2} - 350 = 270$, you may want to *clear the fractions* before you do anything else. The Multiplication Property of Equality allows you to do this. If you make a wise choice of a number by which to multiply both sides, the result will be an equation with no fractions.

We will examine the results of different multipliers for the equation $\frac{t}{3} + \frac{t}{2} - 350 = 270$, as shown below. For example, to tell a CAS to multiply both sides of the equation by 2, type (t/3+t/2–350 = 270)∗2 ENTER.

Mental Math

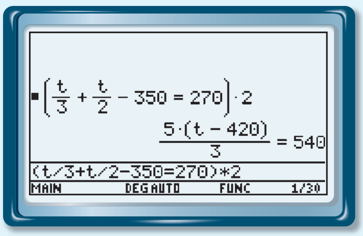
Let *n* be any real number. Determine if the statement is *always*, *sometimes but not always*, or *never true*.

a. $\frac{n}{3}$ is greater than n.

b. n is greater than $-n$.

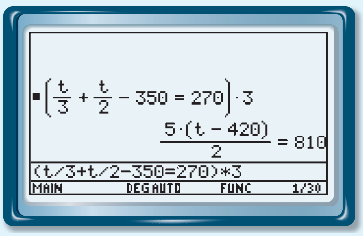
c. $5n$ is equal to $-5n$.

Multiply by 2.



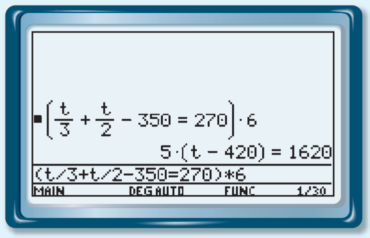
The CAS transformed the equation into $\frac{5(t-420)}{3} = 540$. But there is still a fraction in the equation. So 2 is not a useful multiplier.

Multiply by 3.



Again, the result is an equation that has fractions. So 3 is also not a good multiplier.

Multiply by 6.



Success! When 6 is a multiplier, the result is $5(t - 420) = 1{,}620$, an equation that has no fractions and is equivalent to $\frac{t}{3} + \frac{t}{2} - 350 = 270$.

On some CAS machines you must use the `expand` or `simplify` command to cause the multiplication to be carried out. You may need to type `expand((t/2 + t/3 – 350 = 270)*6)` or `simplify((t/2 + t/3 – 350 = 270)*6)` to multiply both sides by 6. So one multiplier that clears fractions in $\frac{t}{3} + \frac{t}{2} - 350 = 270$ is 6. But there are others, as you will see in the following activity.

Activity

Step 1 The table below shows the effect of three different multipliers on the equation $\frac{t}{3} + \frac{t}{2} - 350 = 270$. Experiment to find three more multipliers that clear the fractions. Record your results in the table.

Multiplier	Resulting Equation	Fractions Cleared?
2	$\frac{5(t-420)}{3} = 540$	No
3	$\frac{5(t-420)}{2} = 810$	No
6	$5(t-420) = 1{,}620$	Yes
?	?	?
?	?	?
?	?	?

Step 2 Consider the multipliers you tried in Step 1. Describe the relationship between the multipliers that eliminate fractions and the original equation $\frac{t}{3} + \frac{t}{2} - 350 = 270$.

Step 3 Use what you have learned about multipliers in Steps 1 and 2 to find an equation equivalent to $\frac{5n}{6} + \frac{n}{4} + \frac{2n}{3} = 42$ that contains no fractions.

a. Predict a value by which you could multiply each side of the equation to clear the fractions.

b. Test your prediction using a CAS. Multiply each side of the equation $\frac{5n}{6} + \frac{n}{4} + \frac{2n}{3} = 42$ by the value and write down the results.

Clearing Fractions in Equations

In the preceding activity you saw how to clear fractions in an equation. The idea is to multiply both sides of the equation by a common multiple of the denominators. The result is an equation in which all of the coefficients are integers.

Example 1

In 2004 the Washington Redskins and the Cleveland Browns had the highest earnings in the National Football League (NFL). The Redskins accounted for $\frac{1}{12}$ of the league's income and the Browns accounted for $\frac{1}{15}$ of the league's income. Their combined income was \$129 million. What was the total league income for 2004?

a. Write an equation to describe the situation.

b. Solve by clearing the fractions.

In 2005, the average NFL team was worth \$733 million.

Source: *Forbes*

Solution 1

a. Let T be the NFL's total earnings, in millions of dollars.

Redskins' earnings + Browns' earnings = 129

$$\frac{1}{12}T + \frac{1}{15}T = 129$$

b. Multiply each side by a common multiple of 12 and 15. We use 60.

$60\left(\frac{1}{12}T + \frac{1}{15}T\right) = 60 \cdot 129$	Multiply each side by 60.
$5T + 4T = 7{,}740$	Distributive Property
$9T = 7{,}740$	Combine like terms.
$T = 860$	Divide each side by 9.

The total income for the NFL in 2004 was \$860 million.

Solution 2

b. Add the fraction coefficients.

$\frac{1}{12}T + \frac{1}{15}T = 129$	Write the equation.
$\frac{5}{60}T + \frac{4}{60}T = 129$	Find equivalent fractions with the same denominator.
$\frac{9}{60}T = 129$	Add like terms.
$60 \cdot \frac{9}{60}T = 60 \cdot 129$	Multiply each side by 60 to clear the fractions.
$9T = 7{,}740$	Simplify.
$T = 860$	Divide each side by 9.

STOP QY1

QY1

Use a CAS to solve $\frac{1}{12}T + \frac{1}{15}T = 129$.

Clearing Fractions in Inequalities

When solving an inequality, you can multiply to clear fractions just like you do when solving an equation.

GUIDED

Example 2

Solve $\frac{x}{4} - 8 > \frac{1}{6}$.

Solution The two denominators in the sentence are 4 and 6. The least common denominator is _?_. So multiply each side of the inequality by _?_.

$$\underline{\ ?\ }\left(\frac{x}{4} - 8\right) > \underline{\ ?\ } \cdot \frac{1}{6}$$

$$\underline{\ ?\ } \cdot \frac{x}{4} + \underline{\ ?\ } \cdot 8 > 2$$

$$\underline{\ ?\ }x - \underline{\ ?\ } > 2$$

$$\underline{\ ?\ }x > \underline{\ ?\ }$$

$$x > \underline{\ ?\ }$$

To Clear Fractions in an Equation or Inequality

1. Choose a common multiple of all of the denominators in the sentence.
2. Multiply each side of the sentence by that number.

STOP QY2

QY2

To clear the fractions, what number could you use to multiply each side of $\frac{m}{6} - \frac{3}{8}m \leq 5$?

Clearing Decimals

Like fractions, decimals can be cleared from an equation to give a simpler equation with integer coefficients. A decimal can be thought of as a fraction whose denominator is a power of 10. For example, 0.4 can be written as $\frac{4}{10}$, so the "denominator" of 0.4 is 10. Similarly, the "hidden denominators" of 9.38 $\left(\text{or } 9\frac{38}{100}\right)$ and 6.022 $\left(\text{or } 6\frac{22}{1{,}000}\right)$ are 100 and 1,000, respectively.

Example 3

Solve $5.85n - 9 = 2.7$.

Solution The equation involves two decimals: 5.85 and 2.7. Their "hidden denominators" are 100 and 10. Since 100 is divisible by both 100 and 10, multiply each side of the equation by 100.

$5.85n - 9 = 2.7$	Write the equation.
$100(5.85n - 9) = 100 \cdot 2.7$	Multiply each side by 100.
$585n - 900 = 270$	Simplify.
$585n - 900 + 900 = 270 + 900$	Add 900 to each side.
$585n = 1{,}170$	Simplify.
$\frac{585n}{585} = \frac{1{,}170}{585}$	Divide each side by 585.
$n = 2$	Simplify.

Questions

COVERING THE IDEAS

1. Suppose $\frac{3}{5}w + 2 = 26$.
   a. Multiply each side of the equation by 5.
   b. Solve the resulting equation.
   c. Check your answer.
2. Consider the equation $\frac{m}{9} + \frac{m}{3} = 16$.
   a. Multiply each side by 9 and solve the resulting equation.
   b. Multiply each side by 27 and solve the resulting equation.
   c. What conclusions can you make from your work in Parts a and b?
3. Consider the equation $0.152 = 0.3m - 0.43$.
   a. What are the "hidden denominators" in the equation?
   b. Multiply each side by 100 and solve the resulting equation.
   c. Convert the decimals in the equation to fractions and solve the resulting equation.
   d. What conclusions can you make from your work in Parts b and c?

In 4 and 5, an inequality and a number are given.
a. Write the inequality that results if both sides of the inequality are multiplied by the given number.
b. Solve the inequality.
c. Graph the solution set on a number line.
d. Check your work.

4. $\frac{3n}{2} - \frac{n}{4} < 6$; 4
5. $\frac{2}{3}a + \frac{a}{5} \geq 21$; 15

In 6–13, solve and check the sentence.

6. $\frac{3}{7}x + 2 = \frac{2}{5}$

7. $\frac{3}{4}y - \frac{1}{3} = 5$

8. $0.05n + 3.75 = 22.50$

9. $40{,}000 = 138{,}000 - 2{,}000c$

10. $\frac{d}{3} + \frac{3d}{5} < \frac{3}{4}$

11. $1 - \frac{n}{10} \geq -\frac{4}{5}$

12. $5 - \frac{t}{3} = -7$

13. $\frac{m}{5} - \frac{1}{13} \geq \frac{3}{22}$

14. Philo Dendrum owns $\frac{3}{8}$ of the stock in Blossom Industries and his wife Rhoda Dendrum owns $\frac{1}{4}$ of it. This means that they receive $\frac{3}{8}$ and $\frac{1}{4}$, respectively, of the dividends paid to the stockholders.

   a. Last year the Dendrums together earned \$25,400 from the stock. What was the total amount of dividends paid to the stockholders?

   b. How much did stockholders other than Philo and Rhoda receive in dividends?

Nine of the top ten greatest one-day point gains on the Dow Jones Industrial Average occurred in 2000 or later.

Source: Dow Jones & Company, Inc.

APPLYING THE MATHEMATICS

15. When solving $4{,}000 = 8{,}000 - 2{,}000x$, a student first multiplies both sides by $\frac{1}{1{,}000}$. Is this a good idea? Why or why not?

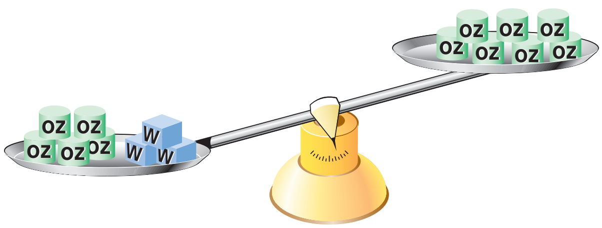
In 16 and 17, solve the sentence.

16. $\frac{1}{6}\left(\frac{17}{3} - \frac{y}{4}\right) < -7$

17. $\frac{x}{6} + \frac{17}{36} - \frac{x}{4} = -7$

REVIEW

18. What inequality is suggested by the balance below? What is the solution to the inequality? **(Lesson 3-7)**



In 19–21, solve and check the inequality. (Lesson 3-7)

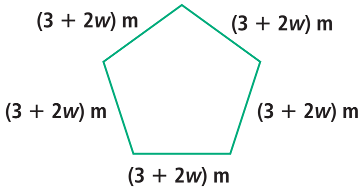
19. $5t - 3(7t + 1) < 93$

20. $77 \leq -3n + 29$

21. $5y - 16 > 49$

22. Use the formula $C = \frac{5}{9}(F - 32)$ to determine which Fahrenheit temperatures are between 50°C and 70°C. **(Lessons 3-7, 1-1)**

23. Find the value of w in the pentagon below if the perimeter is 105 meters. **(Lessons 3-5, 1-1)**



24. Felipe has been trying to lower his cell phone bill by limiting the length of his calls to an average of 2.5 minutes. His calls on November 30th were 2, 3, 6, 1, 1, 2, 1, 3, 4, 1, and 7 minutes long. **(Lessons 3-4, 1-7)**
   a. Was his average call less than 2.5 minutes long?
   b. What is the mean absolute deviation for the calls?

In 25–27, combine like terms. (Lesson 2-2)

25. $\frac{5}{t} + \frac{-4}{7t}$

26. $\frac{3x + y}{3} - \frac{2z + 8y}{5}$

27. $\frac{4x^2}{9} - \frac{7x^2}{18}$

In 2006, about 22% of people in the world owned a cellular phone.

Source: International Telecommunication Union

EXPLORATION

28. Diophantus, a Greek mathematician who lived in the third century, was the first known person to use variables to stand for unknown numbers. About 200 years after his death, an algebraic riddle was written to honor him. Here is one version of that riddle, written as a rhyming poem. Decipher the riddle to find an equation and solve the equation to determine how long Diophantus lived.

> "Here lies Diophantus." The wonder behold
> Through art algebraic, the stone tells how old.
> "God gave him his boyhood one-sixth of his life,
> One-twelfth more as youth while whiskers grew rife;
> And then yet one-seventh ere marriage begun;
> In five years there came a bounding new son.
> Alas, the dear child of master and sage
> Met fate at just half his dad's final age.
> Four years yet his studies gave solace from grief,
> Then leaving scenes earthly he, too, found relief."

QY ANSWERS

1. $T = 860$

2. Answers vary. Sample answer: 24