

## Lesson

## 3-7

Solving  $ax + b < c$ 

► **BIG IDEA** Inequalities of the form  $ax + b < c$  can be solved in two steps, similar to those used in solving  $ax + b = c$ .

If you are  $x$  years old and an older friend's age is  $y$ , then you can write the inequality  $x < y$  to compare the ages.

Six years from now you will still be younger than your friend. The inequality that compares your ages then is  $x + 6 < y + 6$ .

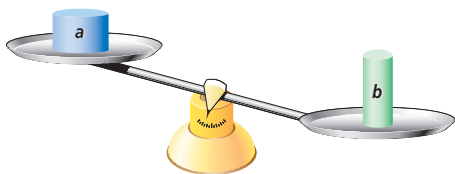
In general,  $H$  years from now you will still be younger than your friend. This is written as  $x + H < y + H$ .

These examples illustrate the *Addition Property of Inequality*.

### Addition Property of Inequality

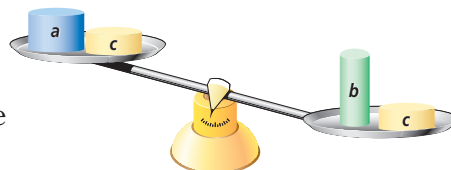
For all real numbers  $a$ ,  $b$ , and  $c$ , if  $a < b$ , then  $a + c < b + c$ .

The Addition Property of Inequality can be represented with a balance, as shown below. Suppose  $a$  and  $b$  represent the weights of two packages where  $a < b$ .



If the same weight  $c$  is added to each side of the balance at the right, then  $a + c < b + c$ .

Inequalities containing  $>$ ,  $\geq$ , or  $\leq$  signs work in the same way. Thus, sentences with  $=$ ,  $<$ ,  $>$ ,  $\leq$ , or  $\geq$  can all be solved in the same way.



### Mental Math

$n \parallel p$  and  $m\angle 1 = 140^\circ$ .

- What is  $m\angle 2$ ?
- What is  $m\angle 3$ ?

### Addition Property of Inequality (in words)

You may add the same number to both sides of an inequality or equation without affecting the set of solutions to the sentence.

Remember that  $b - a = b + -a$ , so you can convert any subtraction expression to an addition expression. This means you may subtract the same number from each side of an inequality or equation without affecting its solutions.

## Solving Inequalities with Positive Coefficients

The process of solving inequalities of the form  $ax + b < c$  is quite similar to that of solving equations.

### Example 1

A crate weighs 7 kilograms when empty. An orange weighs about 0.3 kilogram. For shipping, the crate and oranges must weigh at least 70 kilograms. How many oranges should be put in the crate?

**Solution** Let  $n$  be the number of oranges. Then the weight of  $n$  oranges is  $0.3n$ . The weight of the crate with  $n$  oranges is  $0.3n + 7$ , so the question can be answered by solving the inequality  $0.3n + 7 \geq 70$ .

This is of the form  $ax + b \geq c$  and is solved the same way as  $ax + b = c$ .

$$0.3n + 7 + -7 \geq 70 + -7 \quad \text{Add } -7 \text{ to both sides.}$$

$$0.3n \geq 63 \quad \text{Simplify.}$$

$$\frac{0.3n}{0.3} \geq \frac{63}{0.3} \quad \text{Divide each side by } 0.3.$$

$$n \geq 210 \quad \text{Simplify.}$$

At least 210 oranges should be put in the crate.

### Check

**Step 1** Check the boundary of  $n \geq 210$ .

$$\begin{aligned} \text{Is } 0.3(210) + 7 &= 70? \\ 70 &= 70 \quad \text{Yes} \end{aligned}$$

Therefore, 210 is the boundary point.

**Step 2** Pick a number that satisfies the inequality  $n \geq 210$ . We choose 250.

$$\begin{aligned} \text{Is } 0.3(250) + 7 &\geq 70? \\ 82 &\geq 70 \quad \text{Yes} \end{aligned}$$

Since both Steps 1 and 2 produce true statements,  $n \geq 210$  describes the solutions to the original inequality.



The United States produced almost 295 million boxes of oranges in 2004.

Source: U.S. Department of Agriculture

## Solving Inequalities with Negative Coefficients

Remember the Multiplication Property of Inequality when solving inequalities with negative coefficients. Example 2 is about drought, a natural phenomenon that affects many communities.

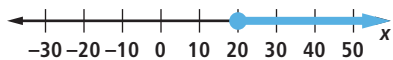
### Example 2

Suppose that during a particularly dry summer, the level of water in a local reservoir decreased 1.5 feet each week. At the start of summer, the water level was 60 feet. When the water level is 30 feet or less, a “water emergency” is put into effect, with various water conservation measures enacted. When will a water emergency be in effect?

**Solution** First, express the problem as an inequality. Let  $x$  represent the number of weeks from the start of summer. Since the water level is initially 60 feet and the water level decreases 1.5 feet each week, the water level in week  $x$  is  $60 - 1.5x$ . A water emergency is put into effect when the water level is 30 feet or less. The problem is solved with the inequality  $60 - 1.5x \leq 30$ . Solve this inequality.

$$\begin{aligned}
 60 - 1.5x + -60 &\leq 30 + -60 && \text{Add } -60 \text{ to each side.} \\
 -1.5x &\leq -30 && \text{Simplify.} \\
 -1.5x \cdot -\frac{1}{1.5} &\geq -30 \cdot -\frac{1}{1.5} && \text{Multiply each side by } -\frac{1}{1.5}. \\
 &&& \text{Reverse the inequality sign.} \\
 x &\geq 20 && \text{Simplify.}
 \end{aligned}$$

Here is a graph of the solutions.



**STOP** QY



The worst drought in 50 years affected at least 35 states during the long hot summer of 1988. Rainfall totals over the Midwest, Northern Plains, and the Rockies were 50–85% below normal.

Source: National Weather Service

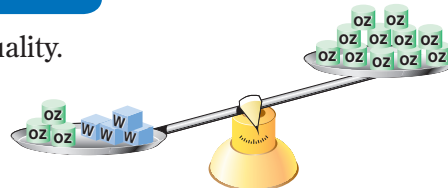
### ► QY

Check that  $\{x: x \geq 20\}$  is the set of solutions to Example 2 using the two-step check in Example 1.

## Questions

### COVERING THE IDEAS

- Use the symbol  $>$  to state the Addition Property of Inequality.
- What inequality is suggested at the right?
  - What is the solution to the inequality?



In 3–6, solve, graph, and check the inequality.

3.  $3x + 4 < 19$

4.  $6 \leq 4b + 10$

5.  $5 \leq -3n + 2 - 7n + 203$

6.  $-101 - 102x < 103$

7. T-shirts can be ordered from the Sports Central catalog for \$7.50 each, with a shipping fee of \$4 for the order. A team wants to spend \$300 on shirts to give away to fans. How many shirts can they buy?

### APPLYING THE MATHEMATICS

In 8–13, solve the inequality.

8.  $6(-4x - 23) - 73 < 77$

9.  $-0.2y + \frac{1}{2} \geq 0.48$

10.  $15 \geq 12 + \frac{1}{3}a$

11.  $\frac{-5d}{6} + 30 < 120$

12.  $11 - 5(16 - 7p) \geq 49$

13.  $594 > -2(q - 9) + 3(7 - 8q) - 17$

14. Using the graph of  $y = 13x - 20$  at the right, find each of the following for  $13x - 20 \leq 45$ .
- the boundary point
  - a value of  $x$  in the solution set
  - a value of  $x$  *not* in the solution set
  - an inequality using  $x$  that describes all solutions

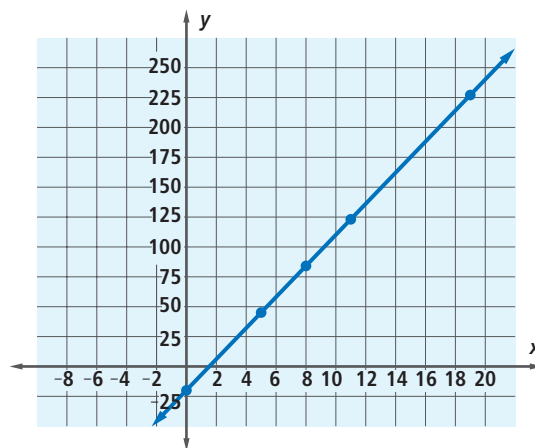
15. Using the table of values for  $y = -9.6x - 41.4$  below, find each of the following for  $-9.6x - 41.4 > -108.6$ .

$x$	-3	-1	1	3	5	7	9	11	13
$y$	-12.6	-31.8	-51	-70.2	-89.4	-108.6	-127.8	-147	-166.2

- the boundary point
  - a value of  $x$  in the solution set
  - a value of  $x$  *not* in the solution set
  - an inequality using  $x$  that describes all solutions
16. Use the formula  $C = \frac{5}{9}(F - 32)$  to determine which Celsius temperatures are below  $68^\circ\text{F}$ .



During sporting events, T-shirts are often launched into crowds with slingshots.



17. A membership to Jenny's Gym costs \$22 per month plus an initial \$49.95 fee to join. If Arturo has \$300 budgeted to join and pay monthly fees, for how many months can he be a member?
18. You are allowed to fly with a suitcase that weighs no more than 50 pounds. Suppose your suitcase and clothes total 35.2 pounds, and your shoes weigh 3.7 pounds per pair. How many pairs of shoes can you bring on the flight?
19. Compare and contrast the solutions to  $-7w - 1 \leq -15$  and  $7w + 1 \leq 15$ .
20. a. Solve the sentence  $ax + b < c$  for  $x$  when  $a > 0$ .  
b. How does the result from Part a change if  $a < 0$ ?

## REVIEW

21. Consider the inequality  $8 > -2$ . What inequality (if any) results if you multiply both sides of the inequality by each number? (Lesson 3-6)
- a. 100      b. -5      c.  $\frac{1}{2}$       d. 0
22. The area of the foundation of a rectangular building is not to exceed 15,000 square feet. The width of the foundation is to be 150 feet. Write an inequality that should be solved to find how long the foundation can be. (Lesson 3-6)

In 23–25, solve the inequality. (Lesson 3-6)

23.  $5j > 17$       24.  $4.2 < 8.4k$       25.  $\frac{5}{6}x \leq 12$
26. Solve and check  $3x + (-5x) + 12(x - 15) = -4$ . (Lesson 3-5)
27. Nomar gets paid \$2 per pizza that he delivers in his car. He also gets paid \$0.31 for every mile he drives while making deliveries. Last month he delivered 512 pizzas. If his monthly paycheck was \$1,482.80, how many miles did he drive last month delivering pizzas? (Lesson 3-4)
28. **Skill Sequence** Simplify the expression. (Previous Course)
- a.  $\frac{5}{6} + \frac{1}{9}$       b.  $\frac{5}{6}c + \frac{1}{9}c$       c.  $\frac{5c}{6} + \frac{c}{9}$

## EXPLORATION

29. Create five inequalities of the form  $ax + b < c$  equivalent to  $x < 24$ .
30. Create five inequalities of the form  $ax + b < c$  equivalent to  $x > 4$ .



Some people recommend that a person participate in an aerobic activity at least 3–5 times per week for 20–30 minutes per session as part of a healthy lifestyle.

Source: Aerobics and Fitness Association of America

## QY ANSWER

$$x = 20: 30 = 30;$$

$$x > 20 (x = 30): 15 < 30$$