

## Lesson

## 3-5

# Using the Distributive Property in Solving Equations

► **BIG IDEA** By collecting like terms or expanding expressions, you can transform many equations into the form  $ax + b = c$ .

## Solving Equations by Collecting Like Terms

We need to solve  $c + c + \frac{1}{2}c + \frac{1}{2}c + \frac{1}{2}c + 15,000 = 260,000$  in Example 1. It takes only one step more to solve this equation than to solve an equation in the form  $ax + b = c$ . The first step is to simplify the left side of the expression. Simplifying sides of equations is common in solving equations.

### Example 1

A \$260,000 estate is to be split among two children, three grandchildren, and a charity. Each child receives the same amount, while each grandchild receives half that amount. If the charity receives \$15,000, how much will each child receive?

**Solution** First identify the unknown. Let  $c =$  the portion a child receives. Then  $\frac{1}{2}c$  is a grandchild's portion of the estate.

Now translate the given information into an equation.

$$c + c + \frac{1}{2}c + \frac{1}{2}c + \frac{1}{2}c + 15,000 = 260,000$$

**Step 1** Use the Distributive Property to add the like terms.

$$3\frac{1}{2}c + 15,000 = 260,000$$

Now the equation is in the form  $ax + b = c$  and can be solved in two steps.

**Step 2** Add  $-15,000$  to each side. Rewrite  $3\frac{1}{2}$  as 3.5 to make the computation easier.

$$\begin{aligned} 3.5c + 15,000 + -15,000 &= 260,000 + -15,000 \\ 3.5c &= 245,000 \end{aligned}$$

**Step 3** Multiply each side by  $\frac{1}{3.5}$ .

$$c = 70,000$$

Each child will receive \$70,000.

(continued on next page)

### Mental Math

- How many cups is  $\frac{1}{3}$  of 2 cups?
- How many cups is  $\frac{1}{3}$  of 4 cups?
- How many cups is  $\frac{1}{3}$  of  $n$  cups?

**Check** Each grandchild receives half as much as a child. This amount is \$35,000. So the two children, three grandchildren, and the charity will receive  $2 \cdot 70,000 + 3 \cdot 35,000 + 15,000$  dollars.

Does  $2 \cdot 70,000 + 3 \cdot 35,000 + 15,000 = 260,000$ ? *Yes.*

Notice that in Example 1 we check the result not by substituting into the original equation, but by checking if the numbers work in the original statement of the problem.

Sometimes the use of the Distributive Property will result in an equation that requires only one step for its solution.

### Example 2

Solve  $9y - y = 40$ .

#### Solution

Write the equation.

$$9y - y = 40$$

Many students like to rewrite  $y$  as  $1 \cdot y$  to make the use of the Distributive Property more obvious.

$$\begin{array}{ll} 9y - 1y = 40 & y = 1 \cdot y \\ (9 - 1)y = 40 & \text{Distributive Property} \\ 8y = 40 & \text{Combine like terms.} \\ \frac{8y}{8} = \frac{40}{8} & \text{Divide each side by 8.} \\ y = 5 & \text{Simplify.} \end{array}$$

#### Check

Substitute 5 for  $y$  in the original equation.

$$\text{Does } 9(5) - 5 = 40?$$

$$45 - 5 = 40?$$

$$40 = 40? \text{ Yes.}$$

## Solving Equations by Removing Parentheses

You can use the Distributive Property to remove parentheses that appear in equations.

### Example 3

Solve  $2(x + 3) = 7$ .

**Solution**

$$\begin{array}{ll}
 2(x + 3) = 7 & \text{Write the equation.} \\
 2x + 6 = 7 & \text{Distributive Property} \\
 2x + 6 - 6 = 7 - 6 & \text{Subtraction Property of Equality} \\
 2x = 1 & \text{Simplify.} \\
 \frac{2x}{2} = \frac{1}{2} & \text{Division Property of Equality} \\
 x = \frac{1}{2} & \text{Simplify.}
 \end{array}$$

**Check**

$$\begin{array}{l}
 \text{Is } 2\left(\frac{1}{2} + 3\right) = 7? \\
 2\left(3\frac{1}{2}\right) = 7 \\
 7 = 7 \text{ Yes.}
 \end{array}$$

**Example 4**

A company charges \$29.95 each day to rent a truck, with the first 50 miles free. Subsequently, the cost is \$0.60 per mile. Therefore, if the truck is driven  $m$  miles (where  $m \geq 50$ ), the total rental cost is  $29.95 + 0.60(m - 50)$  dollars. Isabel rented a truck for one day and paid \$100.15. How many miles did Isabel drive?

**Solution**

Let the total cost equal \$100.15 to determine how far the truck was driven.

$$\begin{array}{r}
 29.95 + 0.60(m - 50) = 100.15 \\
 29.95 + 0.60m - 0.60 \cdot 50 = 100.15 \\
 29.95 + 0.60m - 30 = 100.15 \\
 0.60m - 0.05 = 100.15 \\
 0.60m = 100.20 \\
 m = 167
 \end{array}$$

The truck was driven 167 miles.



In the U.S., one in five households moves every year, or 20% of the population.

Source: [www.ourtownamerica.com](http://www.ourtownamerica.com)

**GUIDED****Example 5**

$$\text{Solve } 2(3k + 4) - (9k - 7) = 6.$$

(continued on next page)

**QY**

In Example 4, how much does it cost to drive the truck 225 miles?

**Solution**

$$2(3k + 4) - (9k - 7) = 6$$

Write the equation.

$$\quad \quad \quad \underline{\quad ? \quad} = 6$$

Distributive Property

$$\quad \quad \quad \underline{\quad ? \quad}k + 15 = 6$$

Add like terms.

$$\quad \quad \quad \underline{\quad ? \quad}k + 15 - 15 = 6 - 15 \quad \underline{\quad ? \quad}$$

Add like terms.

$$\quad \quad \quad \underline{\quad ? \quad}k = \underline{\quad ? \quad}$$

Divide each side by  $-3$ .

$$\quad \quad \quad \underline{\quad ? \quad} = \underline{\quad ? \quad}$$

Multiplication Property of Equality

$$\quad \quad \quad k = \underline{\quad ? \quad}$$

**Check** Substitute 3 for  $k$  in the original equation.

Does  $2(3 \cdot \underline{\quad ? \quad} + 4) - (9 \cdot \underline{\quad ? \quad} - 7) = 6$ ?

**Questions****COVERING THE IDEAS**

In 1–10, solve the equation.

1.  $3r + r = 8$
  2.  $2x + 3x - 7 = 23$
  3.  $14 = 3(x + 2)$
  4.  $2(4x - 9) = -2$
  5.  $9 = 2(x + 2) + 2$
  6.  $5y - 3(5 - 2y) = -15$
  7.  $42 = t - 7t$
  8.  $3x - (x + 4) = 22$
  9.  $13(t + 1) - 3(2 + 4t) = 38$
  10.  $6(\frac{1}{2}b + 3) + 4(12 - b) = \frac{5}{6}$
11. The winner of a car race received a prize of \$150,000. Ten percent went to the driver and the rest was split among the 4 owners and the head mechanic, with the head mechanic getting half as much as the owners.
- a. Let  $E$  be the amount each owner received. Write an equation that can be solved to determine  $E$ .
  - b. Find  $E$  and the amount the head mechanic received.
12. In 2005, the federal income tax  $T$  for a single person whose taxable income  $I$  was between \$7,300 and \$29,700 was given by  $T = 730 + 0.15(I - 7,300)$ . This can be translated as \$730 plus 15% of the amount over \$7,300. If a single person paid \$2,150 in income tax, what was the person's taxable income to the nearest dollar?

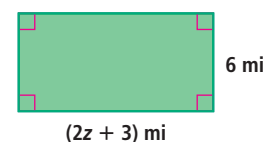
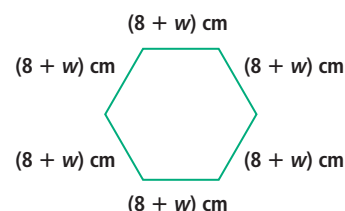
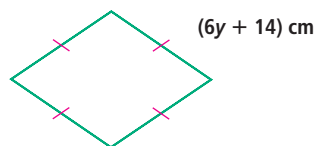
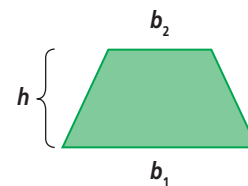


**Race cars can reach speeds in excess of 230 mph.**

Source: National Aeronautics and Space Administration

### APPLYING THE MATHEMATICS

13. The area  $A$  of a trapezoid with parallel bases  $b_1$  and  $b_2$  and height  $h$  is given by the formula  $A = 0.5h(b_1 + b_2)$ , as shown at the right.
- If  $h = 6$  in.,  $b_2 = 4$  in., and  $b_1 = 8$  in., calculate  $A$ .
  - If  $A = 26$  in<sup>2</sup>,  $h = 4$  in., and  $b_2 = 5$  in., calculate  $b_1$ .
  - If  $A = 48$  in<sup>2</sup>,  $h = 4$  in., and  $b_1 = 8$  in., calculate  $b_2$ .
14. If the perimeter of the rhombus at the right is 940 centimeters, find  $y$ .
15. Find the value of  $w$  in the regular hexagon at the right if each side has length  $(8 + w)$  cm and the perimeter is 186 centimeters.
16. Find the value of  $z$  if the area of the rectangle at the right is 96 square miles.
17. Suppose you have \$100,000 to invest. You decide to put part of the money in a certificate of deposit (CD) that pays 6% annual interest and the rest in a savings account that pays 4% per year. If  $d$  dollars are invested in the CD, then  $E = 0.06d + 0.04(100,000 - d)$  gives the interest earned  $E$  in one year. How much should you put in each place to earn \$4,800 in the first year?



### REVIEW

In 18–20, solve and check the equation. (Lesson 3-4)

18.  $\frac{4}{7}d + 9 = -11$       19.  $6.21 = 3.4 + -c$       20.  $\frac{1}{2}t + 4 = -2$

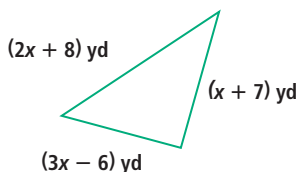
21. There had already been 5 inches of snow on the ground when it started snowing at midnight. Snow continued to fall throughout the night, accumulating at three-quarters of an inch per hour. (Lesson 3-4)
- How much snow was on the ground at 4 A.M.?
  - Winterville Junior High cancels school when there are more than 10 inches of snow on the ground. At what time was school canceled?



In the U.S., snow depth is usually reported to the nearest 1 inch. 24-hour snowfall is reported to the nearest 0.1 inch.

Source: National Oceanic & Atmospheric Administration

22. Sareeta is planning a party and she wants to serve sandwiches to her guests. Each sandwich costs \$4.50. (Lessons 3-4, 3-1)
- Labeling the  $x$ -axis *Number of Sandwiches* and the  $y$ -axis *Total Cost*, draw a graph to represent possible costs for the sandwiches. Use  $0 \leq x \leq 20$ .
  - Suppose Sareeta paid \$50 for the sandwiches and received \$0.50 in change. How many sandwiches did she buy? (Assume there is no sales tax.)
23. Sherman was twice as late arriving home today as he was yesterday. If he is supposed to be home at 5:00 P.M. each day and didn't arrive until 7:30 P.M. today, at what time did he get home yesterday? (Lesson 3-3)
24. The perimeter of the triangle below is 54 yards. Find  $x$  and the length of the shortest side. (Lesson 3-2)



25. The equation  $3\frac{1}{2}c = 245,000$  in Example 1 was solved by changing  $3\frac{1}{2}$  to 3.5 then multiplying both sides by  $\frac{1}{3.5}$ . The equation can also be solved by multiplying by the reciprocal of  $3\frac{1}{2}$ . What is the reciprocal of  $3\frac{1}{2}$ ? (Lesson 2-8)

### EXPLORATION

26. a. Solve the equation  $mx + (m + 1)x = 8mx + 4$  for  $x$ .  
 b. Check your solution by substituting a number for  $m$  and solving the resulting equation.
27. You have \$64 and you want to buy a pair of jeans and a \$20 T-shirt. There is a 7% sales tax. If  $x$  represents the cost of the jeans, then the equation  $x + 20 + 0.07(x + 20) = 64$  shows how much you can spend on jeans. What is the price of the most expensive jeans you can afford?



Consumers in the United States spent about \$329 billion on clothing and shoes in 2004.

Source: *The World Almanac and Book of Facts*

### QY ANSWER

\$134.95