

## Lesson

## 3-4

Solving  $ax + b = c$ 

► **BIG IDEA** An equation of the form  $ax + b = c$  can be solved in two major steps.

In 1637, the French philosopher and mathematician René Descartes started the practice of identifying known quantities by the letters  $a$ ,  $b$ , and  $c$  from the beginning of the alphabet and unknown quantities by the letters  $x$ ,  $y$ , and  $z$  from the end of the alphabet. Following the practice of Descartes, when we write “solving  $ax + b = c$ ” we mean that  $a$ ,  $b$ , and  $c$  are known numbers and  $x$  is unknown. For example, when  $a = -\frac{3}{2}$ ,  $b = -53$ , and  $c = 7$ , we obtain the equation  $-\frac{3}{2}x - 53 = 7$ .

In general, any equation of the form  $ax + b = c$ , with  $a$  not equal to zero, can be solved in two steps. First add the opposite of  $b$  to both sides. Then multiply both sides by the reciprocal of  $a$ .

## GUIDED

## Example 1

Solve  $-\frac{3}{2}x - 53 = 7$ .

## Solution

$$-\frac{3}{2}x - 53 = 7$$

Write the equation.

$$-\frac{3}{2}x - 53 + \underline{\quad?} = 7 + \underline{\quad?}$$

Add  $\underline{\quad?}$  to each side.

$$-\frac{3}{2}x = \underline{\quad?}$$

Simplify.

$$\underline{\quad?} \left(-\frac{2}{3}\right) = \underline{\quad?} \quad (60)$$

Multiply each side by the reciprocal of  $-\frac{3}{2}$ .

$$x = \underline{\quad?}$$

Simplify.

Be sure to check your solution by substituting it back into the original equation.

## Equations That Require Simplifying First

Equations are often complicated, but they can be simplified into ones that you can solve.

## Mental Math

- Which is greater,  $\frac{1}{3}$  or 0.33?
- Which is greater, 1.4 or  $\frac{33}{22}$ ?
- Which is greater,  $-\frac{5}{4}$  or  $-\frac{4}{5}$ ?



Descartes was a French scientist, mathematician, and philosopher. His statement, “I think; therefore I am,” is very famous.

Source: *Discourse on Method*

**Example 2**

When Val works at the zoo on Saturday, she earns \$10.80 per hour. She is also paid \$8 for meals and \$3 for transportation. Last Saturday she received \$83.90. How many hours did she work?

**Solution** Let  $h$  = the number of hours Val worked.  
In  $h$  hours she earned  $10.80h$  dollars. So,  
 $10.80h + 8 + 3 = 83.90$ .

Next, 8 and 3 are added. The resulting equation has the form  $ax + b = c$ . Solve for  $h$ .

$$10.80h + 11 = 83.90 \quad \text{Write the equation.}$$

$$10.80h + 11 + -11 = 83.90 + -11 \quad \text{Addition Property of Equality}$$

$$10.80h = 72.90 \quad \text{Simplify.}$$

$$\frac{1}{10.80} \cdot 10.80h = \frac{1}{10.80} \cdot 72.90 \quad \text{Multiplication Property of Equality}$$

$$h = 6.75 \quad \text{Simplify.}$$

Val worked 6.75 hours.

**Check** If Val worked 6.75 hours at \$10.80 per hour, she earned  $6.75 \cdot \$10.80$ , or \$72.90. Now add \$8 for meals and \$3 for transportation. The total comes to \$83.90.

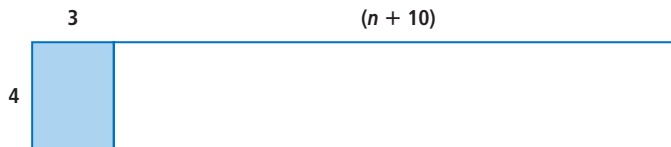


Zookeepers take care of wild animals in zoos and animal parks. They feed the animals, clean their living spaces, work to keep them healthy, and keep them cool in the summer months.

Source: Bureau of Labor Statistics

**GUIDED****Example 3**

The area of the largest rectangle is 94 square centimeters. What is the value of  $n$ ?



**Solution** Write an equation to represent the area. (*Hint*: You can use the sum of areas or the length and width of the big rectangle.) Then solve.

Area of left rectangle + Area of right rectangle = 94

$$\underline{\quad} \cdot \underline{\quad} + \underline{\quad} \cdot \underline{\quad} = 94 \quad \text{Write the equation.}$$

$$12 + \underline{\quad} = 94 \quad \text{Distributive Property}$$

$$\underline{\quad}n + \underline{\quad} = 94 \quad \text{Simplify.}$$

$$\underline{\quad}n + \underline{\quad} + \underline{\quad} = 94 + \underline{\quad} \quad \text{Addition Property of Equality}$$

$$\underline{\quad}(\underline{\quad}) = \underline{\quad}(\underline{\quad}) \quad \text{Multiplication Property of Equality}$$

$$n = \underline{\quad} \quad \text{Simplify.}$$

Be sure to check your solution.

## Variations of $ax + b = c$

If an equation has the variable on the right side, as in  $c = ax + b$ , the solution can still be obtained by adding the opposite of  $b$ , and multiplying by the reciprocal of  $a$ . The Commutative Property of Addition also implies that  $ax + b = c$  is equivalent to  $b + ax = c$ . For example, the following equations can be solved with the same major steps.

$$7 = \frac{3}{2}x - 53 \qquad -53 + \frac{3}{2}x = 7$$

$$\frac{3}{2}x - 53 = 7 \qquad 7 = -53 + \frac{3}{2}x$$

## Questions

### COVERING THE IDEAS

- Fill in the Blanks** When solving  $7t - 57 = 97$ , first add   ? to both sides. Then   ? each side by   ?.
  - Solve and check  $7t - 57 = 97$ .
- Steps in solving  $73y - 432 = 1,101$  are shown here.  
 Given:  $73y - 432 = 1,101$   
 Step 1  $73y = 1,533$   
 Step 2  $y = 21$ 
  - What was done to arrive at Step 1?
  - What was done to arrive at Step 2?

In 3 and 4, the equation is in the form  $ax + b = c$ . Find the values of  $a$ ,  $b$ , and  $c$ .

- $73y - 432 = 1,101$
- $17 - 4y = 88$
- Multiple Choice** How do the solutions to  $50x - 222 = 60$  and  $60 = 50x - 222$  compare?
  - They are equal.
  - They are opposites.
  - They are reciprocals.
  - None of these are true.

In 6–13, solve and check the equation.

- $6x + 42 = 126$
- $31 = 11A - 24$
- $-20y - 2 = 8$
- $18 = 16 + 5B$
- $7 + \frac{3}{5}d = -5$
- $2.4n - 2.4 = 2.4$
- $1.06P + 3.25 = 22.86$
- $200 = 4 - \frac{7}{2}m$

14. Write directions to teach a friend how to solve the equation given in Question 6 in a step-by-step process, but do not tell your friend the exact equations.

### APPLYING THE MATHEMATICS

In 15–17, a situation is given.

- a. Write an equation of the form  $ax + b = c$  to describe the situation.

Be sure to identify what the unknown variable represents.

- b. Solve the equation and answer the question.

15. Ms. Toshio bought gas for \$2.39 per gallon and a drink for \$1.50. The total bill was \$31.15 before sales tax. How many gallons of gas did she buy?
16. Bena lives in Delaware, where there is no sales or meals tax. She bought three chicken sandwiches with a \$10 bill and received \$1.45 change. What was the price of one sandwich?
17. Eighty students from a school went to the taping of a television program. They filled 7 rows of seats and there were 3 students in the eighth row. How many seats were in each row?
18. **Skill Sequence** Solve for  $n$  in each equation.

a.  $17n + 38 = 4$       b.  $-2n + 15 = 7$       c.  $3n + a = 9$

In 19–22, solve and check the equation.

19.  $\frac{11}{17}x + \frac{17}{11} = 11\frac{11}{17}$       20.  $0.003 = 0.02y - 0.1$
21.  $\frac{4}{3}\left(a - \frac{1}{2}\right) = 3\frac{1}{3}$       22.  $2.08 + 4.2n = 41.56$

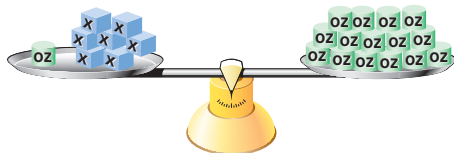


The average price per gallon of unleaded gasoline in 2004 in the United States was \$1.92.

Source: U.S. Department of Energy

### REVIEW

23. Consider the balance below. (Lesson 3-3)



- a. What equation is shown?
- b. What two steps can be done with the weights on the balance to find the weight of a single box?
- c. How much does a single box weigh?

24. **True or False** Determine whether the given equation is equivalent to  $\frac{1}{2}x + 1 = 21$ . (**Lesson 3-3**)
- $x + 2 = 42$
  - $-x + 2 = -38$
  - $x = 40$
  - $\frac{1}{4}x + \frac{1}{4} = \frac{21}{4}$
25. a. Graph the following set of points:  $\{(1, 2), (3, 4), (5, 6), (7, 8), (9, 10)\}$ . (**Lessons 3-1, 1-4**)
- b. These points are collinear. The point  $(99, 63a + 10)$  is on that line. What is  $a$ ?
- c. **Fill in the Blank** The point  $(m, \underline{\quad ? \quad})$  is also on this line.

In 26–28, apply the Distributive Property to rewrite the expression with fewer terms. (**Lessons 2-4, 2-2, 2-1**)

26.  $7p + q - 7(q + 6p)$
27.  $-(a - b) + 2(a + b)$
28.  $3(-2n - 5) - 2(-3n + 5)$
29. Suppose  $y = 11 - (9 - 14x)$ . If  $x$  is 9, what is  $y$ ? (**Lesson 1-1**)

### EXPLORATION

30. Solve the general equation  $ax + b = c$  for  $x$ .
31. Consider equations of the form  $ax + b = c$ , where  $a \neq 0$ .
- Write a program for a calculator or a computer that accepts values of  $a$ ,  $b$ , and  $c$  as input and gives the value of  $x$  as output.
  - Run your program with different values of  $a$ ,  $b$ , and  $c$  that lead to both positive and negative solutions.