Solving Equations by Creating Equivalent Equations

Vocabulary

equivalent equations

> ▶ **BIG IDEA** Performing the same arithmetic operations on both sides of an equation can create an equation that is easier to solve.

In the previous lesson, you used a table and a graph to find the solution to an equation of the form $ax + b = c$. Those methods allow you to visualize what it means to solve an equation. But in practice, they are sometimes awkward to use. Other times they yield solutions that are not exact. In this lesson, we discuss a method that enables you to find exact solutions to many types of equations.

Mental Math

a. If $x = 4z - 9y$, what is $3x$?

b. If $x = 4z - 9y$, what is $-x$?

c. If $x = 4z - 9y$, what is $x + 9y$?

Solving with a Balance

A balance illustrates the meaning of the "=" sign in an equation by placing equal weights on both sides. The scale will still balance as long as changes made to one side are also made to the other side. This is the idea behind solving an equation algebraically.
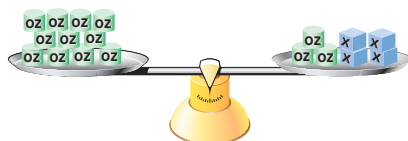
The equation $11 = 4x + 3$ is shown above with 11 identical 1-ounce weights on the left side and 3 one-ounce weights with 4 unknown weights on the right side. You can find the weight x of one box in two steps. Each step keeps the scale balanced.
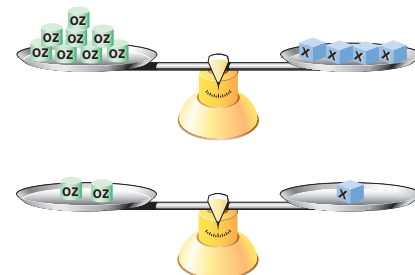

Step 1 Remove 3 one-ounce weights from each side of the scale.

Step 2 Leave $\frac{1}{4}$ of the contents on each side.

From the original equation two more equations were formed.

$$11 = 4x + 3$$
$$8 = 4x$$
$$2 = x$$

These three equations are called *equivalent equations* because 2 is the solution to each of them. **Equivalent equations** are equations with exactly the same solutions.

Exploring $ax + b = c$ Equations with a Computer Algebra System (CAS)

The goal when solving equations of the form $ax + b = c$ is to add, subtract, multiply, or divide both sides of the equation to eventually get an equation of the form $x =$ a number. In the activity below, you will use a CAS to analyze the effect of performing various operations to the two sides of an equation.
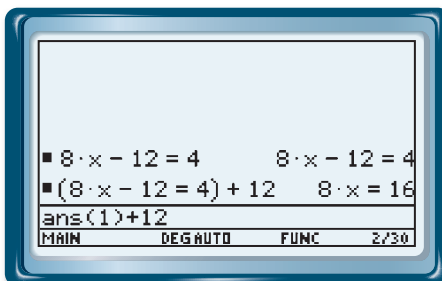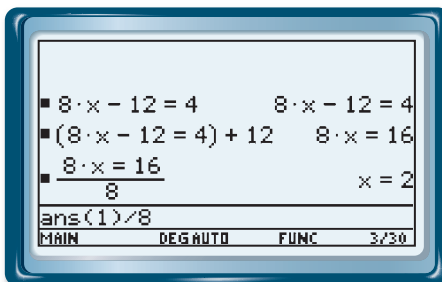
Activity

Solve $8x - 12 = 4$.

Step 1 Enter $8x - 12 = 4$ into the calculator.
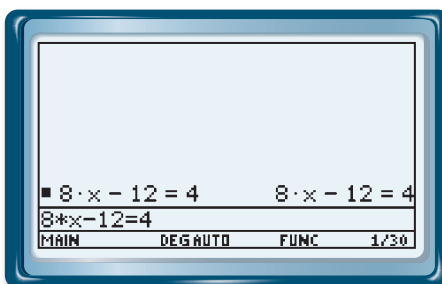
Step 2 Next enter $\boxed{+}$12 to instruct your CAS to add 12 to both sides of the equation. The result is $8x = 16$, which is simpler than $8x - 12 = 4$. So adding 12 was a good choice.

Step 3 Enter $\boxed{\div}$8 to divide each side by 8. The result is an equation that is as simple as possible. So $x = 2$.

STOP QY

Solving a Linear Equation Using a CAS

Step 1 Enter your equation into your CAS.

Step 2 Decide which operation to perform on each side. For example, to add 9 to both sides enter $\boxed{+}$9.

Step 3 Do you get an equation that is simpler than the previous one? If the answer is "yes," continue until the equation is solved. If the answer is "no," re-enter the equation and try a different operation.

Step 4 You are finished solving when the equation has the form *variable = number.*

▶ **QY**

Solve each question using a CAS.

 a. $\frac{4}{7}z + \frac{2}{3} = \frac{1}{7}$

 b. $-3y + 14 = 98$
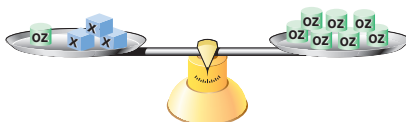
Questions

COVERING THE IDEAS

1. Why are the equations below called equivalent equations?

$$2x + 5 = 11$$
$$2x = 6$$
$$x = 3$$

2. **a.** What equation is represented by the diagram below?

   b. What two steps can be taken with the weights on each side of the scale to find the weight of a single box?

   c. How much does a single box weigh?

3. **a.** **Fill in the Blanks** When solving $5x - 27 = 13$, first ___?___ to each side, then ___?___ on both sides.

   b. Solve $5x - 27 = 13$ and check your result.

4. Consider the steps used in the solution of $82n - 51 = 441$ below.

   Given: $82n - 51 = 441$
   Step 1 $\qquad 82n = 492$
   Step 2 $\qquad\quad n = 6$

   a. What was done to go from the given equation to Step 1?

   b. What was done to go from Step 1 to Step 2?

In 5–8, a pair of equations is given. Determine what was done to each side of the first equation to arrive at the second equation.

5. $4x - 11 = 12$
   $\quad\; 4x = 23$

6. $72 - 18t = 864$
   $\qquad -18t = 792$

7. $\frac{3}{5}n = 30$
   $\quad n = 50$

8. $0.004v = 1.2$
   $\qquad v = 300$

9. What might be done first to each side of $75x - 100 = 800$ to begin the process of solving the equation?

APPLYING THE MATHEMATICS

10. In solving $5x + 430 = 315$, Paula instructed her CAS to divide both sides of the equation by 5.

    a. What result did she get?

    b. Is this a reasonable first step to solve the equation? If so, use it to solve the equation. If not, explain why.

11. In solving $-3y + 14 = 98$, Paula instructed her CAS to divide both sides by -3.

    a. What result did she get?

    b. Is this a reasonable first step to solve the equation? If so, use it to solve the equation. If not, explain why.

12. A student showed the following work to solve $4x - 32 = 20$.

$$4x - 32 = 20$$
$$4x - 32 - 32 = 20 - 32$$
$$4x = -12$$
$$x = -3$$

    But substituting -3 in the original equation results in an equation that is not true. What did the student do wrong?

13. a. Solve $3x + 5y = c$ for x by hand or with a CAS.

    b. Check your answer to Part a by substituting numbers for y and c and solving that equation.

REVIEW

14. a. Using the equation $y = 7x - 2$, complete the table at the right. (Lesson 3-2)

    b. Use the table to find a solution to the equation $7x - 2 = -16$

x	y
−2	?
−1	?
0	?
1	?
2	?

15. Jordan collects basketball cards. Last Sunday he had 200 cards. He then bought one pack of cards on each weekday, and two packs on Saturday. Jordan now has 284 cards. (Lesson 3-2)

    a. Define a variable and represent the relationship with an equation.

    b. Use a table or graph to find the number of cards in one pack.

16. Consider the following situation. A bathtub has 11 gallons of water in it. Rachana adjusts the faucet so it is now filling at approximately 4.5 gallons per minute. (Lesson 3-2)

    a. Create a table and a graph showing the amount of water in the tub in terms of the amount of time that has passed from when Rachana adjusted the faucet.

    b. How much water will be in the tub after 7 minutes?

    c. If the bathtub holds 56 gallons of water, how long does it take until the bathtub is full?

17. Death Valley, California once reached a record high of 134 degrees Fahrenheit. Suppose the temperature was 85 degrees at 9 A.M. and increased by 7 degrees every hour until 4 P.M., when it reached the record high. (**Lesson 3-1**)

    a. Use this information to complete the table at the right.
    b. Graph the data using the table.

18. Consider $\frac{3}{4}x = 16$. (**Lesson 2-8**)

    a. Solve this equation.
    b. Think of a real-world problem that can be solved with this equation.

19. Below is a dot plot of the number of mosquitoes caught in a trap over 15 days in Buckinghamshire, England. (**Lesson 1-7**)
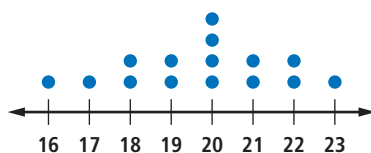
    a. Is the shape symmetric, skewed right, skewed left, or uniform?
    b. Calculate the mean and mean absolute deviation of the data.

Time of Day	Temperature (°F)
9:00 A.M.	?
10:00 A.M.	?
11:00 A.M.	?
12:00 P.M.	?
1:00 P.M.	?
2:00 P.M.	?
3:00 P.M.	?
4:00 P.M.	?

The average rainfall in Death Valley is about 1.65 inches per year.

Source: National Park Service

EXPLORATION

20. Enter $4(3x - 17) + 12 = 6(x + 2) + 6x - 68$ into your CAS as if you were going to solve the equation as in this section. Then press ENTER.

    a. Describe what the screen shows.
    b. Explain why. (*Hint:* Use algebra to simplify each side of the equation by hand.)
    c. Write down the other two equations that produce the same result when entered into a CAS.

QY ANSWERS

a. $z = \frac{-11}{12}$

b. $y = -28$