Lesson

3-1 Graphing Linear Patterns

Vocabulary

constant-increase situation

collinear

constant-decrease situation

▶ **BIG IDEA** Constant-increase and constant-decrease situations lead to linear graphs and are represented by linear equations.

Constant-Increase Patterns

The size of Stephen's coin collection described on pages 128 and 129 increases by 10 coins each week. It provides an example of a **constant-increase situation** because his collection grows by the same amount each week. The graph of every constant-increase situation consists of points that lie on the same line. We call these points **collinear.**

But notice that the graph does not include every point on the line. Because Stephen receives coins at specific whole-number intervals, the domain of w is the set of whole numbers. It does not make sense to connect the points on the graph because numbers such as $2\frac{1}{2}$ are not in the domain of w.

Mental Math

Simplify.

a. $\frac{m}{x} + \frac{n}{x}$

b. $\frac{m}{x} - \frac{n}{kx}$

c. $\frac{5}{x} + \frac{3}{kx}$

Constant-Decrease Patterns

A **constant-decrease situation** involves a quantity that decreases at a constant rate. In Example 1 below, the variable can be any positive real number between two whole numbers. The graph is no longer a set of separate or *discrete* points; it is connected or *continuous.*

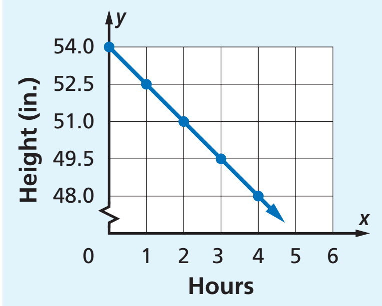
Example 1

After a flash flood, the level of water in a river was 54 inches above normal and dropping at a rate of 1.5 inches per hour. Let x equal the number of hours since the water started dropping and y equal the height, in feet, above normal of the lake. We can model this situation with the equation $y = 54 - 1.5x$. Graph this relationship.

Solution Find the lake level at various times and make a table. A table for 0, 1, 2, 3, and 4 hours is shown on the next page.

Flash floods result from a large amount of rain within a short amount of time and usually occur within 6 hours of a storm.

Time x (hr)	Height y (in.)	Ordered Pair (x, y)
0	$54 - 1.5 \cdot 0 = 54$	(0, 54)
1	$54 - 1.5 \cdot 1 = 52.5$	(1, 52.5)
2	$54 - 1.5 \cdot 2 = 51$	(2, 51)
3	$54 - 1.5 \cdot 3 = 49.5$	(3, 49.5)
4	$54 - 1.5 \cdot 4 = 48$	(4, 48)

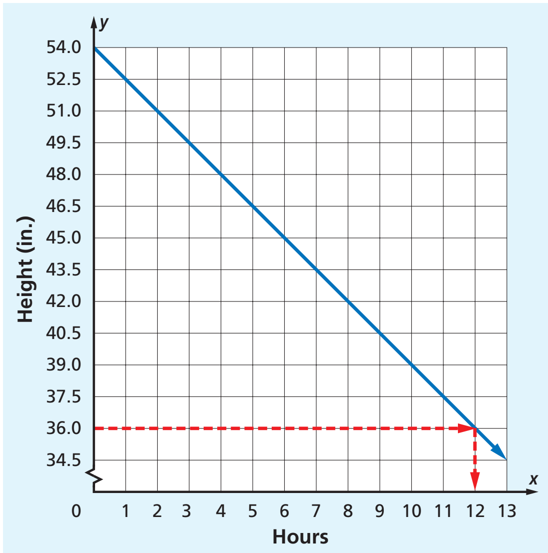


Plot the ordered pairs in the table and look for patterns. You should see that the five points lie on the same line. Time in hours can be any nonnegative real number, such as 1.75 or $3\frac{1}{2}$. This means that other points lie between the ones you have already plotted. So, draw the line through them for the domain $x \geq 0$.

Example 2

In Example 1, if the water level continues to drop at the same rate, how many hours will it take for the water level to fall to 3 feet above normal?

Solution Look at the graph from Example 1. The level of the water above the normal level is given by y. Find the point for 3 feet, or 36 inches, on the y-axis. The x-coordinate of this point is 12. This is shown by the arrows on the graph. The water will be 3 feet above normal after 12 hours.



STOP QY

In Examples 1 and 2, notice that the ⌇ symbol appears on the graphs. This symbol indicates a break in the scale of the axis. It is often used so that patterns in graphs become more apparent.

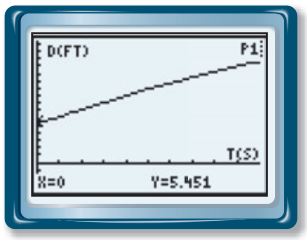
▶ QY

Based on the graph in Example 2, when is the water level 42 inches above normal?

Activity

You can create your own continuous graph using a motion detector that hooks up to a computer or your graphing calculator. These graphs are time-distance graphs. They plot the distance between the motion detector and a stationary solid object (like a wall) over time. If a person holds a motion detector and moves closer to or farther from a wall, those changes will be seen in the graph. An example of such a graph is shown at the right.

To create this graph, start about 5 feet away from the wall and move farther away from the wall at a constant speed.



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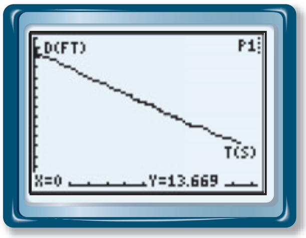
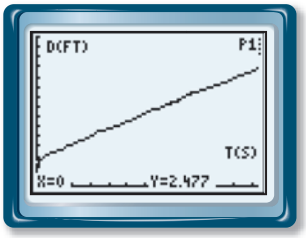
Step 1 Using a motion detector, create time-distance graphs as similar as possible to each graph below.

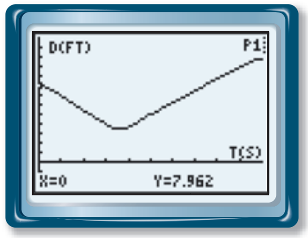
Graph A



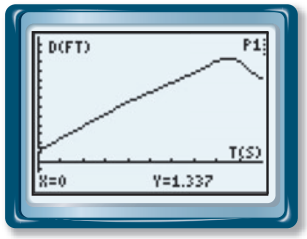
Graph B



Graph C



Graph D



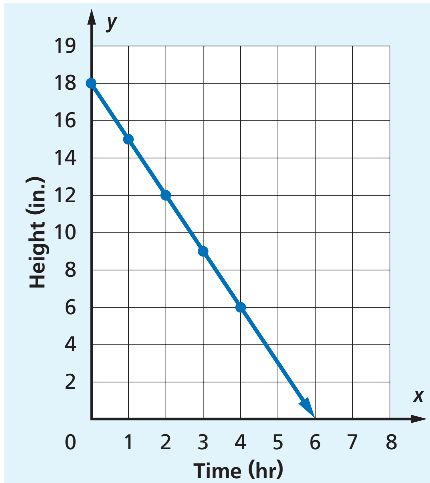
Step 2 Explain how you created each graph. Describe your starting point, direction, and speed while walking. Explain what quantity changes during the walk.

Questions

COVERING THE IDEAS

1. A flooded stream is 18 inches above its normal level. The water level is dropping 3 inches per hour. Its height y in inches above normal after x hours is given by the equation $y = 18 - 3x$. A table and a graph of the equation are shown at the right.

Time x (hr)	Height y (in.)
0	18
1	?
2	?
3	?
4	?



a. Complete the table at the right.

b. After how many hours will the stream be 12 inches above normal?

c. How high above normal will the stream be after 3 hours?

d. After how many hours will the stream be back to its normal level?

2. Suppose Miguel begins with \$500 in an account and adds \$20 per week.

a. Complete the table at the right, showing t, the total amount Miguel will have at the end of w weeks.

b. Graph the ordered pairs (w, t).

c. Write an equation that represents t in terms of w.

d. What is the domain of w?

Weeks (w)	Total (t)
0	?
1	?
2	?
3	?
4	?

3. A train consists of an engine that is 60 feet long and cars that are each 40 feet long. There is a distance of 2.5 feet between two cars and between the first car and the engine. Let T be the total length, in feet, of a train with c cars.
   a. What is the total length of a train with 1 car?
   b. What is the total length of a train with 2 cars?
   c. **Multiple Choice** How are T and c related?
      A $T = 60 + 40c$ B $T = 62.5 + 40c$
      C $T = 60 + 42.5c$ D $T = 62.5 + 42.5c$
   d. Graph the equation you found in Part c for values of c from 1 to 5.
   e. Find the length of the train if it has 12 cars.

John Stevens built and operated the first steam locomotive in the United States in 1825.

Source: *The World Almanac*

APPLYING THE MATHEMATICS

4. A tree has a trunk with a 12-centimeter radius. The radius increases by 0.5 centimeter per year. Its radius y, in centimeters, after x years is described by $y = 12 + 0.5x$.
   a. Make a table of values for this relationship.
   b. Draw a graph of this situation.
   c. After how many years will the radius equal 20 centimeters?

5. a. Draw the graph of $y = 4x$. Choose your own values for x.
   b. On the same grid from Part a, draw the graph of $y = -4x$.
   c. At which points do the graphs of $y = 4x$ and $y = -4x$ intersect?
   d. Describe any patterns you observe in these graphs.

6. A flooded stream is now 30 centimeters above its normal level. The water level is dropping at a rate of 3 centimeters per hour. Let x equal the number of hours from now and y equal the water level above normal after x hours.
   a. Suppose the water level continues to drop at the same rate. Write an equation for y in terms of x.
   b. Graph your equation from Part a.
   c. Use your graph to estimate when the stream is expected to drop to a level of 6 centimeters above normal.

In 7–10, graph the equation using a graphing calculator, and determine whether the graph is linear. You may have to adjust the window to view the graph.

7. $y = -0.02(x - 3)$
8. $y = \frac{1}{100}x^2$
9. $y = \frac{x - 3}{5} - x$
10. $y = |x|$

REVIEW

In 11–14, compute in your head. (Lesson 2-8)

11. $\frac{9}{11}h \cdot \left(\frac{9}{11}h \cdot \frac{11}{9h}\right)$

12. $95{,}620 \cdot 657 - 95{,}620 \cdot 656$

13. $85.69 \cdot 6.514 \cdot 0 \cdot 12$

14. $\frac{4}{13} \cdot 9.85 \cdot \frac{13}{40}$

In 15–17, use the table at the right and the formula $d = rt$ to calculate the following. (Lesson 2-8)

15. At top speed, how far can an ostrich run in 15 seconds?

16. a. At top speed, how long will it take a cheetah to run 0.78 mile?

b. How long will it take a cheetah to run 100 yards at top speed?

17. If a greyhound runs at top speed for 8 seconds and a giraffe runs at top speed for 17 seconds, which animal has run farther?

18. Consider the equation $-0.6(5x) = 90$. (Lesson 2-8)

a. Simplify the left side of the equation.

b. Solve for x.

c. Check your solution.

19. ***Skill Sequence*** Evaluate each expression. (Lesson 2-4)

a. 6^2 **b.** -6^2

c. $(-6)^2$ **d.** -6^3

e. $(-6)^3$ **f.** $-(-6)^4$

20. Is $x = -9$ a solution to $28 = -3x - 1$? Explain your reasoning. (Lesson 1-1)

Fast Land Animals

Animal	Top Speed (mi/hr)
Cheetah	70
Ostrich	40
Greyhound	39
Giraffe	32

Source: American Museum of Natural History

EXPLORATION

21. In Question 3, approximate lengths are given for an engine, train cars, and the distance between them.

a. Find the lengths of an engine and of a car for an actual train. Identify the type of train and where you found the information.

b. Give an equation relating the total length T of the train and the number of cars c.

QY ANSWER

after 8 hr