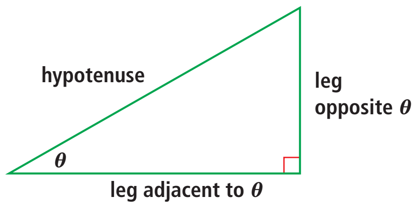
Chapter 10 Summary and Vocabulary

- Trigonometry is the study of relationships between sides and angles in triangles. In a right triangle, three important trigonometric ratios are the **sine**, **cosine**, and **tangent** of an acute angle θ, defined as follows for $0° < \theta < 90°$:

$$\sin \theta = \frac{\text{leg opposite } \theta}{\text{hypotenuse}};$$

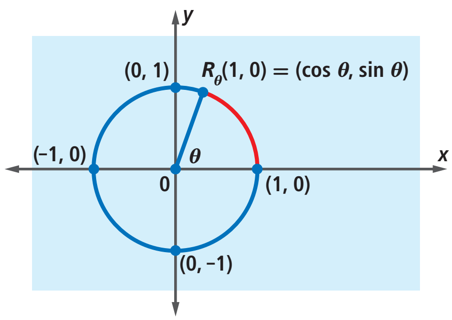
$$\cos \theta = \frac{\text{leg adjacent to } \theta}{\text{hypotenuse}};$$

$$\tan \theta = \frac{\text{leg opposite } \theta}{\text{leg adjacent to } \theta}.$$



- The sine, cosine, and tangent ratios are frequently used to find lengths in situations involving right triangles. Angle measures are found using inverses of the trigonometric functions: $\textbf{sin}^{-1}$, $\textbf{cos}^{-1}$, and $\textbf{tan}^{-1}$. Applications include finding **angles of elevation**, **depression**, and **parallax**.

- The trigonometric ratios can be generalized to find sines, cosines, and tangents for any real number θ. Every point on a **unit circle** is a rotation image of the point (1, 0) about the origin with magnitude θ. $\cos \theta$ is the x-coordinate of $R_\theta(1, 0)$, and $\sin \theta$ is the y-coordinate of $R_\theta(1, 0)$.



Vocabulary

10-1
right-triangle definitions of sine (sin), cosine (cos) and tangent (tan)
*sine function
*cosine function
*tangent function
*angle of elevation

10-2
*inverse sine function, $\sin^{-1}$
*inverse cosine function, $\cos^{-1}$
*inverse tangent function, $\tan^{-1}$
*angle of depression

10-3
*parallax angle

10-4
*unit circle
unit-circle definition of cosine and sine

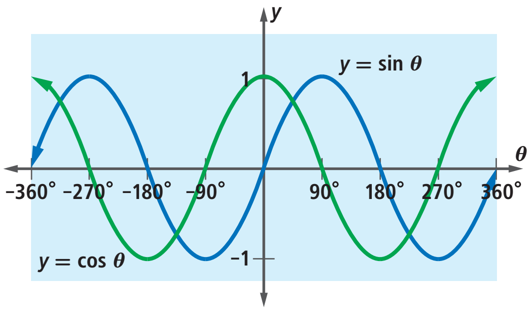
10-5
identity
tangent of θ (for all values of θ)

10-6
*periodic function, period
*sine wave
sinusoidal

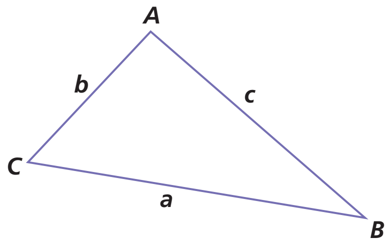
10-7
*solving a triangle

10-9
*radian

- The mappings $\theta \rightarrow \cos\theta$ and $\theta \rightarrow \sin\theta$ are functions whose domains are the set of real numbers and whose ranges are $\{y \mid -1 \le y \le 1\}$. When θ is in degrees, the graphs of these functions are **sine waves** with **period** 360°. When θ is in **radians**, the period is 2π, because radians are defined such that π radians = 180°.



- The Law of Cosines and the Law of Sines relate sides and measures of angles in triangles. In any triangle ABC,



$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (Law of Sines)

$c^2 = a^2 + b^2 - 2ab\cos C$ (Law of Cosines).

These theorems can be used to **solve a triangle**, that is, to find unknown sides and angle measures in triangles. The Law of Cosines is most useful when an SAS, SSS, or SsA condition is given; the Law of Sines can be used in all other situations that determine triangles.

Theorems and Properties

Chapter 10 Self-Test

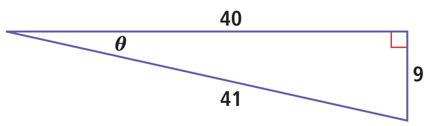
Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

1. Let $\theta = 17.4°$. Approximate to the nearest thousandth.

a. $\cos\theta$ **b.** $\sin\theta$

2. Suppose $0 < \theta < 90°$ and $\tan\theta = 0.64$. Approximate θ to the nearest thousandth of a degree.

3. Use the triangle below. Evaluate.



a. $\cos\theta$ **b.** $\tan\theta$

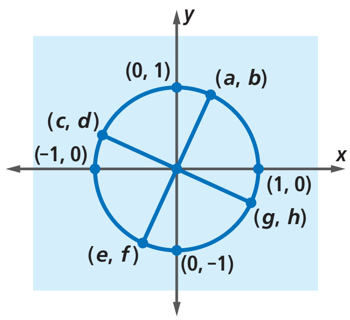
4. If $\sin\theta = 0.280$ and $90° < \theta < 180°$, what is $\cos\theta$?

5. a. What are the exact coordinates of $R_{-423}(1, 0)$?

b. Justify your answer to Part a.

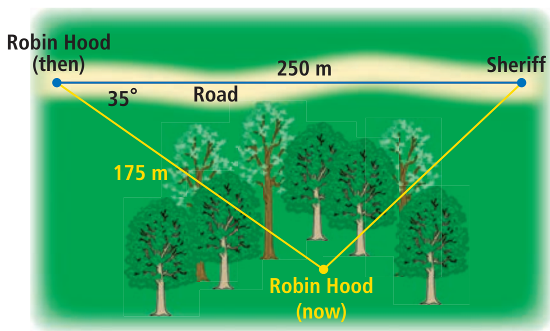
6. For what value of x such that $0° < x \leq 180°$ and $x \neq 17°$ does $\sin 17° = \sin x$?

In 7 and 8, refer to the unit circle below. Name the letter that could be equal to the value of the trigonometric function.



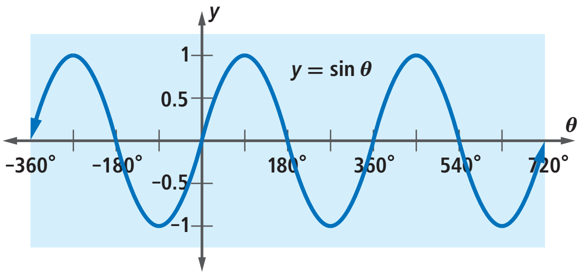
7. $\cos 70°$ **8.** $\sin 250°$

9. Ten minutes ago, Robin Hood spied the Sheriff of Nottingham 250 meters down the road. In order to sneak around, Robin left the road at a 35° angle and traveled 175 meters into the forest as shown below. If the Sheriff has not moved, how far is Robin Hood from the Sheriff now, to the nearest meter?



10. In $\triangle SLR$, $m\angle L = 12°$, $s = 425$, and $\ell = 321$. Approximate $m\angle S$ to the nearest 0.1 degree.

In 11 and 12, consider the function graphed below.

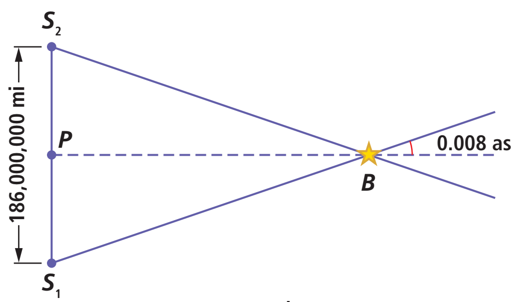


11. What is the period of this function?

12. Name 2 intervals on which the function increases as θ increases.

13. A parallelogram has sides of length 20 and 30. If the shorter diagonal has length 15, find the measures of the angles of the parallelogram.

14. The star Betelgeuse has a parallax angle of about 0.008 arc-second when viewed from the endpoints of Earth's orbit. How many light-years away is it? Use the facts that 1 arc-second is $\frac{1}{3600}$ degree, 1 light-year is $5.88 \cdot 10^{12}$ miles, and Earth's orbit has a radius of about 93,000,000 miles.



15. Convert to radians. Give your answer as a rational number times π.

a. 60° **b.** 25°

16. Convert to degrees. Round your answer to the nearest tenth.

a. 4π radians **b.** $\frac{\pi}{7}$ radian

17. A photographer is in a hot air balloon 1500 feet above the ground. She tilts her camera down 36° from horizontal to aim it at an African elephant.

a. How far away from the elephant is the photographer?

b. How far away from the elephant is her assistant, who is standing directly beneath the balloon?

Chapter 10 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Approximate values of trigonometric functions using a calculator. (Lessons 10-1, 10-9)

In 1–6, evaluate to the nearest thousandth.

1. $\sin 34°$
2. $\cos^2 125°$
3. $\sin\left(-\frac{\pi}{8}\right)$
4. $\cos 3$
5. $\sin\left(\frac{8\pi}{5}\right)$
6. $\tan 167°$

OBJECTIVE B Determine the measure of an angle given its sine, cosine, or tangent. (Lessons 10-2, 10-7)

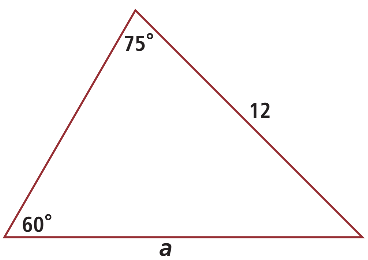
In 7–10, find all θ between 0° and 180° satisfying the given equation.

7. $\sin \theta = 0.5$
8. $\cos \theta = \frac{\sqrt{2}}{2}$
9. $\cos \theta = 0$
10. $\tan \theta = 1$

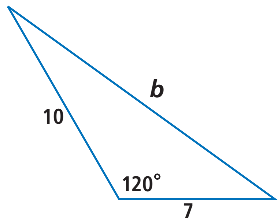
OBJECTIVE C Find missing side lengths and angle measures of a triangle using the Law of Sines or the Law of Cosines. (Lessons 10-7, 10-8)

In 11–15, use the Law of Sines or the Law of Cosines to solve for the variable. Round your answers to the nearest tenth.

11.



12.



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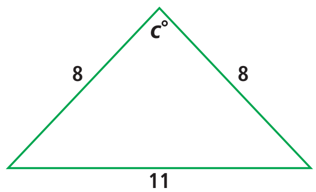

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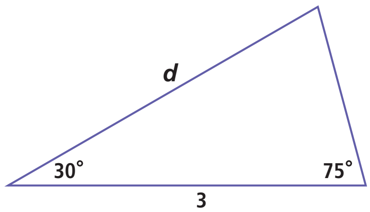
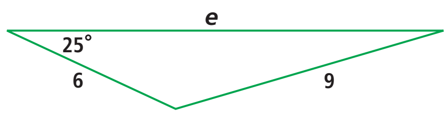


15.



OBJECTIVE D Convert angle measures from radians to degrees or from degrees to radians. (Lesson 10-9)

In 16–19, convert to radians. Express your answers in terms of π.

16. 30°
17. –105°
18. 360°
19. 405°

In 20–23, convert the radian measure to degrees. Round your answers to the nearest tenth.

20. π
21. $\frac{3\pi}{2}$
22. $-\frac{\pi}{8}$
23. $\frac{7\pi}{6}$

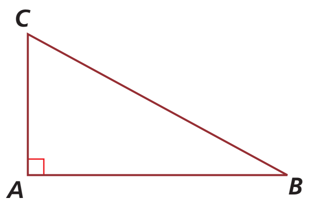
PROPERTIES Principles behind the mathematics

OBJECTIVE E **Identify and use the definitions of sine, cosine, and tangent.** **(Lessons 10-1, 10-4, 10-5)**

Multiple Choice In 24–27, use the diagram below. Identify the given trigonometric function as one of the following ratios.

A $\frac{AB}{BC}$ B $\frac{AC}{BC}$

C $\frac{AB}{AC}$ D $\frac{AC}{AB}$



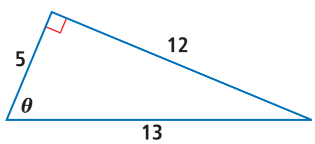
24. $\sin B$

25. $\tan B$

26. $\cos C$

27. $\tan C$

In 28–30, use the triangle below. Evaluate each trigonometric function.



28. $\sin \theta$

29. $\cos \theta$

30. $\tan \theta$

31. **True or False** $\sin \frac{4\pi}{7} = \tan \frac{4\pi}{7} \cdot \cos \frac{4\pi}{7}$. Justify your answer without a calculator.

32. **Multiple Choice** What is the image of (1, 0) under R_{-600}?
   A $(-\sin 600°, -\cos 600°)$
   B $(-\cos 600°, -\sin 600°)$
   C $(\sin -600°, \cos -600°)$
   D $(\cos -600°, \sin -600°)$

33. **Fill in the Blanks** Because $R_{253}(1, 0) \approx (-0.292, -0.956)$, $\cos 253° \approx$ __?__ and $\sin 253° \approx$ __?__.

OBJECTIVE F **Identify and use theorems relating sines and cosines.** **(Lessons 10-5, 10-7).**

34. **True or False** For all real numbers θ, $\sin \theta - \sin(180° - \theta) = 0$. Justify your answer.

35. **True or False** For all real numbers θ, $\cos \theta - \cos(180° - \theta) = 0$. Justify your answer.

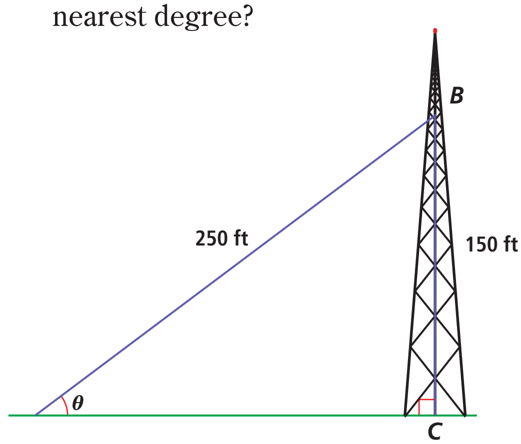
36. Suppose $0° \le \theta < 90°$. If $\sin \theta = \frac{3}{5}$, use the Pythagorean Identity to determine $\cos \theta$.

37. Suppose $\sin x = 0.25$. What are all possible values of $\cos x$?

USES Applications of mathematics in real-world situations

OBJECTIVE G **Solve real-world problems using the trigonometry of right triangles.** **(Lessons 10-1, 10-2, 10-3)**

38. A tower is anchored by a 250-foot guy wire attached to the tower at a point 150 feet above the ground. What is the angle of elevation of the wire, to the nearest degree?



39. A wheelchair ramp is built with slope $\frac{1}{12}$. To the nearest tenth of a degree, what angle does the ramp make with the horizontal?

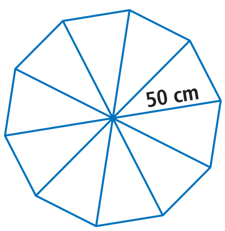
40. A ship sails 560 kilometers at an angle 35° clockwise from north. How far east of its original position is the ship?

41. Neil and Buzz are exploring the Moon. At one moment, the Sun is shining directly over Neil. Meanwhile, Buzz has just set up a 10-meter tall flagpole 150 kilometers away from Neil. The flagpole casts a shadow that is 86 centimeters long. Using this information, compute the circumference of the Moon to the nearest 100 kilometers.

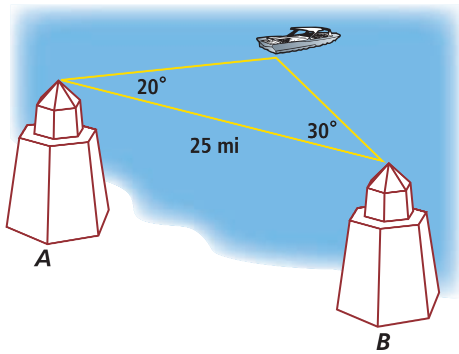
OBJECTIVE H **Solve real-world problems using the Law of Sines or Law of Cosines.** (Lessons 10-7, 10-8)

42. Observers in two ranger stations 8 miles apart spot a fire. The observer in station A spots the fire at an angle of 44° with the line between the two stations, while the observer in station B spots the fire at a 105° angle with the same line. Which station is closer to the fire, and how far away is the fire from this station?

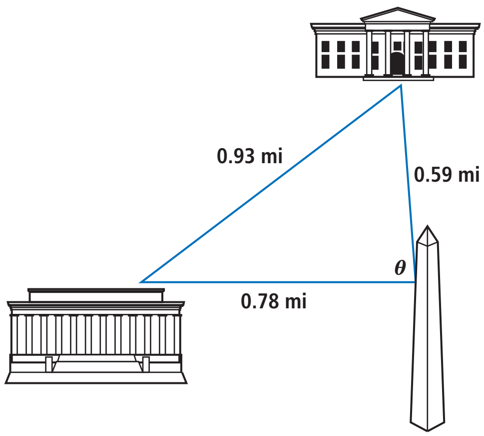
43. Pictured below is the top view of a chandelier with 10 spokes equally spaced around a central point in the same plane. If each spoke is 50 cm long, what is the perimeter of the chandelier?



44. Two observers are in lighthouses 25 miles apart, as shown below. The observer in lighthouse A spots a ship in distress at an angle of 20° with the line between the lighthouses. The observer in lighthouse B spots the ship at an angle of 30° with that line. How far is the ship from each lighthouse?



45. The White House, the Washington Monument, and the Lincoln Memorial form the triangle shown below. To the nearest degree, find the angle θ between the Lincoln Memorial and the White House at the Washington Monument.



REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE I **Use the properties of a unit circle to find values of trigonometric functions.** (Lessons 10-4, 10-5)

In 46–51, use the fact that

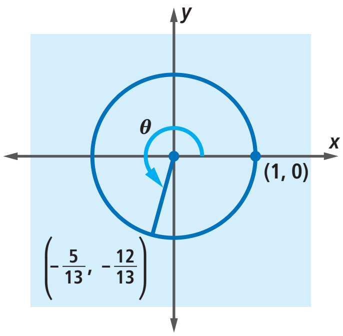
$\sin 15° = \cos 75° = \frac{\sqrt{6} - \sqrt{2}}{4}$,

and $\sin 75° = \cos 15° = \frac{\sqrt{6} + \sqrt{2}}{4}$.

Evaluate without a calculator. Draw a picture to explain your result.

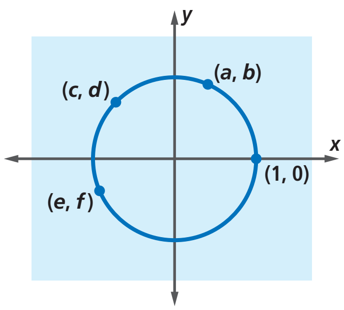
46. $\sin 345°$
47. $\cos 105°$
48. $\sin 285°$
49. $\cos 165°$
50. $\sin(-15°)$
51. $\cos(-435°)$

In 52 and 53, use the diagram below.



52. What is the value of $\sin \theta$?
53. Find θ to the nearest radian.

In 54–56, use the unit circle below. Which letter could stand for the given number?



54. $\sin 67°$
55. $\cos 560°$
56. $\sin(-222°)$

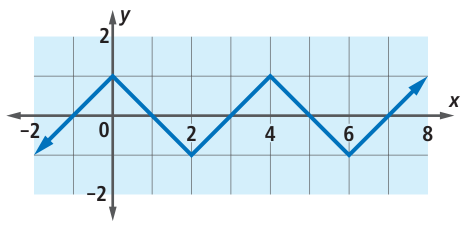
OBJECTIVE J **Identify properties of the sine and cosine functions using their graphs.** (Lesson 10-6)

57. a. Graph the sine function on the interval $0° \le \theta \le 360°$.
    b. State the domain and range of the sine function.

58. a. Graph the cosine function on the interval $-2\pi \le \theta \le 2\pi$.
    b. At what points does the graph of the cosine function intersect the x-axis?
    c. What is the period of the cosine function?

59. The graph of $y = \cos x$ is the image of the graph of $y = \sin x$ under what translation?

In 60 and 61, use the graph below.



60. What is the period of this graph?
61. Is this graph a sine wave? Why or why not?