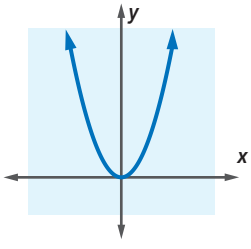
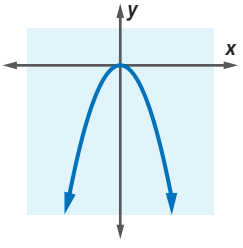
Chapter 9

Summary and Vocabulary

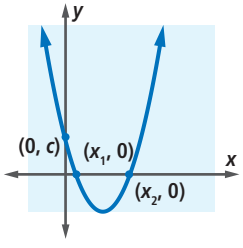
- A **quadratic function** is a function f whose equation can be written in the form $f(x) = ax^2 + bx + c$ with $a \neq 0$. The simplest quadratic function is $f(x) = ax^2$. The **vertex** of the graph of $y = ax^2$ is at $(0, 0)$. The graph of $y = ax^2 + bx + c$ is a parabola symmetric to the vertical line through its vertex. If $a > 0$, the parabola opens up. If $a < 0$, the parabola opens down. To determine where this parabola crosses the horizontal line $y = k$, you can solve $ax^2 + bx + c = k$.



$y = ax^2$, if $a > 0$



$y = ax^2$, if $a < 0$



$y = ax^2 + bx + c$, if $a > 0$

- Quadratic expressions, equations, and functions appear in a variety of situations. The word "squaring" comes from applications of quadratic expressions in such formulas as $A = s^2$ and $A = \pi r^2$. The **path of a projectile** can be described by a quadratic equation. The function whose input is the time t (in seconds) since launch and whose output is the height h (in feet) of the projectile has the formula $h = -16t^2 + vt + s$, where s is the height at launch and v is the upward launch velocity (in feet per second).
- These situations, certain counting problems, and many geometric situations give rise to problems that can be solved by quadratic equations. The values of x that satisfy the equation $ax^2 + bx + c = 0$, where $a \neq 0$, can be found using the **Quadratic Formula,** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- The **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$. If the discriminant is positive, there are two real solutions to the equation; if it is zero, there is one solution; and if it is negative, there are no real solutions.

Theorems and Properties

General Formula for the Height of a Projectile over Time (p. 546)
The Quadratic Formula (p. 553)
Discriminant Property (p. 561)

Vocabulary

9-1
parabola
reflection-symmetric
axis of symmetry
vertex

9-4
force of gravity
initial upward velocity
initial height

9-5
quadratic equation
standard form of a quadratic equation

9-6
discriminant

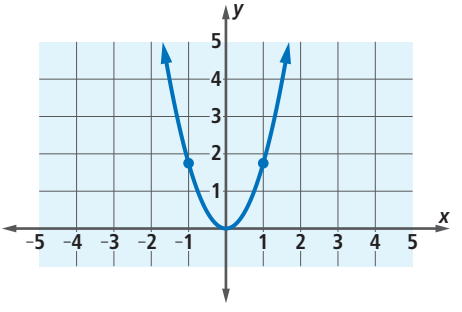
Chapter 9 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

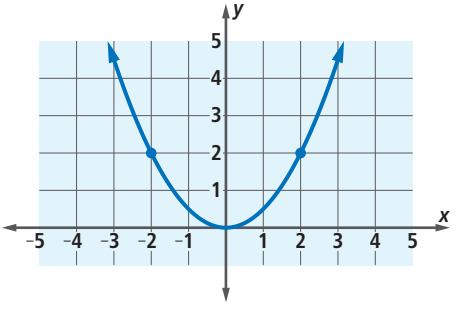
In 1–6, find all real solutions. Round your answers to the nearest hundredth. If there are no real solutions, write that.

1. $2x^2 = 162$
2. $n^2 - 8n - 10 = 0$
3. $5y^2 - 1 = 11y$
4. $24 = \frac{1}{6}z^2$
5. $v^2 = 16v - 64$
6. $3p^2 - 9p + 7 = 0$
7. If the discriminant of a quadratic equation is 6, how many solutions does the equation have?
8. **Multiple Choice** Which of these graphs is of the equation $y = 1.75x^2$?

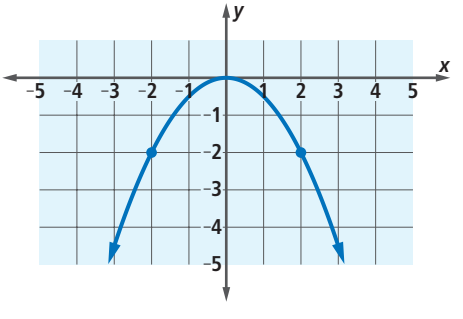
A



B



C



In 9 and 10, an equation is given.

a. Make a table of values of x and y for integer values of x from $x = -3$ to $x = 3$.

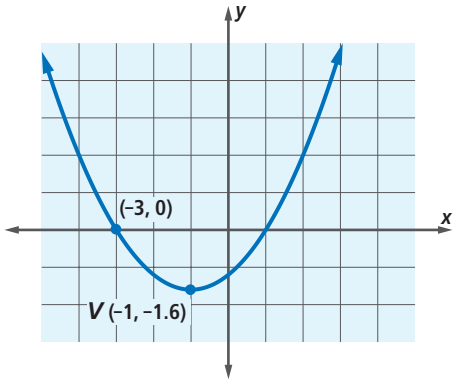
b. Graph the equation.

9. $y = 2x^2$
10. $y = -x^2 + 4x - 3$

In 11–13, consider the following. When a roller coaster goes down a hill, then $h = 0.049v^2$, where v is the velocity of the coaster (in meters per second) when it is h meters below the top of the hill.

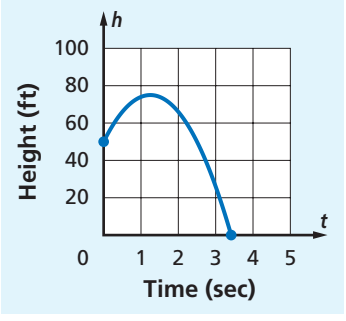
11. Use the equation to determine at what distance below the top of the hill the roller coaster will reach a velocity of 20 meters per second.
12. Suppose the designer of the roller coaster builds the hill to be 44 meters high. At what velocity will the roller coaster be traveling when it reaches the bottom of the hill?
13. Currently the fastest wooden roller coaster is Son of Beast located at Kings Island in Cincinnati, Ohio. The maximum speed of the roller coaster is 35 meters per second. What is the height of the top of the hill?
14. The product of two consecutive integers, n and $n + 1$, is 1,722. If the integers are both negative, what are the numbers?
15. A circle and a rectangle have equal areas. One side of the rectangle is 6 inches longer than the other side, and the perimeter of the rectangle is 24. Calculate the radius of the circle, to the nearest hundredth.

In 16–18, use the parabola with vertex *V* below.



16. Find the minimum y value of the parabola.
17. Find the x-intercepts.
18. Find an equation for the axis of symmetry of the parabola.

In 19 and 20, a tennis ball is thrown from the top of a building. The graph below shows $h = -16t^2 + 40t + 50$, giving the height h of the ball in feet after t seconds.



19. To the nearest hundredth of a second, how long does it take the ball to reach the ground?
20. At what times is the ball 70 feet above the ground? Give your answer to the nearest hundredth of a second.

21. **True or False** The parabola $y = \frac{1}{2}x^2 - 7x - 35$ opens down.
22. Suppose that the quadratic equation $ax^2 - 5x + 3 = 0$ has a discriminant of 1. Find the value of a.

In 23 and 24, use the discriminant to give the number of real solutions to the equation.

23. $-3x^2 + 12x - 7 = 0$
24. $x^2 - 4x = -4$

Chapter 9 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A **Solve quadratic equations of the form $ax^2 = b$.** **(Lesson 9-2)**

In 1–8, solve without using the Quadratic Formula.

1. $4x^2 = 676$
2. $9 = \frac{1}{4}h^2$
3. $k^2 + 15 = 100$
4. $t^2 - 11 = 11$
5. $2(m + 3)^2 = 72$
6. $69 = 5 + 2y^2$
7. $\frac{63}{16} = 7(4 - v)^2$
8. $(6w - 1)^2 = \frac{25}{4}$

OBJECTIVE B **Solve quadratic equations using the Quadratic Formula.** **(Lessons 9-5, 9-6)**

In 9–18, solve the equation using the Quadratic Formula. Round your answers to the nearest hundredth.

9. $m^2 + 7m + 12 = 0$
10. $14x = x^2 + 49$
11. $y^2 - 6y = 3$
12. $r^2 - \frac{11}{7} = \frac{4}{5}r$
13. $0 = p^2 + 10(p + 2.5)$
14. $\frac{3}{4}x^2 - \frac{2}{3}x = 2$
15. $5n^2 + 9n = 2$
16. $2a^2 - 8a = -8$
17. $b^2 + 5.4b - 19.75 = 0$
18. $30 + 5(2z^2 - 10z) = 0$

PROPERTIES The principles behind the mathematics

OBJECTIVE C **Identify and use the properties of solutions to quadratic equations.** **(Lesson 9-6)**

19. Give the values of x that satisfy the equation $ax^2 + bx + c = 0$.
20. **True or False** If a quadratic equation has two solutions, then it has two x-intercepts.

In 21 and 22, calculate the discriminant.

21. $x^2 + 4x - 8 = 0$
22. $7y^2 - y = 1$
23. If the discriminant of the quadratic equation $x^2 + bx + 2 = 0$ is 8, find the possible value(s) of b.

In 24–27, find the number of real solutions to the equation by using the discriminant.

24. $g^2 - 3g - 8 = 0$
25. $3v = 2v^2 + 4$
26. $m^2 = 6m - 9$
27. $w(w - 2) = -8$

USES Applications of mathematics in real-world situations

OBJECTIVE D **Use quadratic equations to solve problems about paths of projectiles.** **(Lessons 9-2, 9-4)**

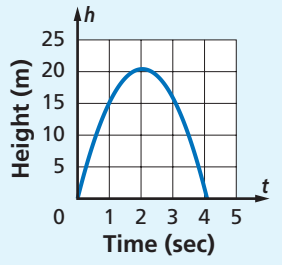
28. Regina is a track and field athlete competing in the shot put, an event that requires "putting" (throwing in a pushing motion) a heavy metal ball (the "shot") as far as possible. The height of the ball h when it is x feet from Regina can be described by the quadratic equation $h = -0.021x^2 + 0.6x + 6$.
    a. At what distances from Regina will the shot put be at a height of 8 feet? Round your answers to the nearest hundredth.
    b. Will Regina's shot put travel 38 feet, the distance needed to win the event? Justify your answer.

In 29 and 30, when an object is dropped near the surface of a planet or moon, the distance d (in feet) it falls in t seconds is given by the formula $d = \frac{1}{2}gt^2$, where g is the acceleration due to gravity. Near Earth $g \approx 32$ ft/sec^2, and near Earth's moon $g \approx 5.3$ ft/sec^2.

29. A skydiver jumps from a plane at an altitude of 10,000 feet. She begins her descent in "free fall," that is, without opening the parachute.
    a. How far will she fall in 15 seconds?
    b. The diver plans to open the parachute after she has fallen 6,000 feet. How many seconds after jumping will this take place?

30. An astronaut on the moon drops a hammer from a height of 6 feet.
    a. How long will it take the hammer to hit the ground?
    b. Suppose the astronaut is back on Earth and drops a hammer from a height of 6 feet. How long will it take the hammer to hit the ground?

31. Refer to the graph below of $h = -4.9t^2 + 20t$, which shows the height (in meters) of a ball t seconds after it is thrown from ground level at an initial upward velocity of 20 meters per second.

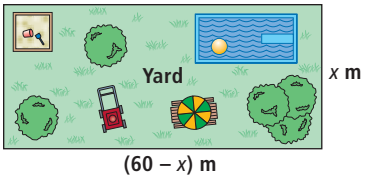


    a. Give the height of the ball after 1 second.
    b. Find when the ball will reach a height of 15 meters.
    c. Use the Quadratic Formula to calculate how long the ball will be in the air.

OBJECTIVE E Solve geometric problems involving quadratic equations. (Lessons 9-2, 9-7)

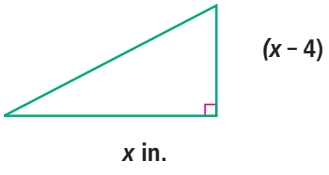
32. Suppose a rectangle has length $2x$ inches and width $x + 3$ inches. Find the length and width given that the area of the rectangle is 5.625 square inches.

33. Consider the rectangular region below. Luisa has 120 meters of fencing to build a fence around her yard.



    a. Use the diagram to write an equation for the area A enclosed by the fencing.
    b. What value of x will result in the greatest possible area enclosed by the fencing?

34. Refer to the triangle below. If the area of the triangle is 18 square inches and the base of the triangle is 4 inches shorter than its height, find the length of the base of the triangle. Round your answer to the nearest hundredth of an inch.



OBJECTIVE F Solve other real-world problems involving quadratic functions. (Lesson 9-7)

35. The relationship between elevation above sea level in kilometers, e, and the boiling point of water in degrees Celsius, t, can be approximated by the equation $e = t^2 - 200.58t + 10{,}058$. Water boils at lower temperatures at higher elevations. Find the boiling point of water at the top of Mt. Ararat in Turkey, which is 5,166 meters high.

36. Consider the formula $d = \frac{n(n-3)}{2}$, where d is the number of diagonals of an n-sided convex polygon.
    a. How many diagonals does a dodecagon (12-sided polygon) have?
    b. Is it possible for a polygon to have 27 diagonals? If so, how many sides does that polygon have?

37. A financial analyst working for an investment company projects the net profit P (in millions of dollars) of the company to be modeled by the equation $P = 4.23t^2 - 5.32t + 3.86$, where t is the number of years since 2005.
    a. Use the analyst's model to predict whether the net profit will reach 100 million dollars in 2011.
    b. Calculate, to the nearest tenth, when the company's net profit is projected to reach 75 million dollars.

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE G **Graph equations of the form $y = ax^2$ and interpret these graphs.** (Lesson 9-1)

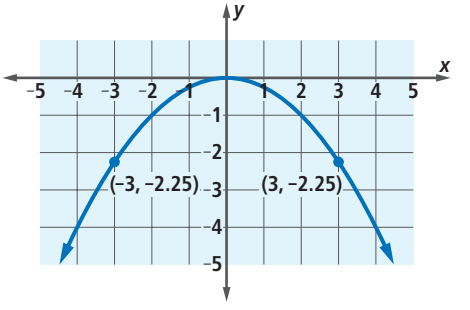
In 38 and 39, an equation is given.
a. **Make a table of values.**
b. **Graph the equation.**

38. $y = 7x^2$

39. $y = \frac{3}{5}x^2$

40. Consider the quadratic equation $y = -1.5x^2$.
    a. Graph the equation.
    b. Determine the coordinates of the vertex.
    c. Tell whether the vertex is a maximum or a minimum.

41. **Multiple Choice** Which equation is graphed below?

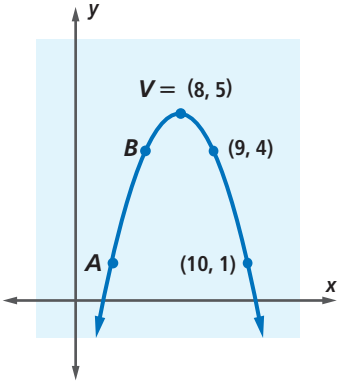


A $y = -4x^2$
B $y = -\frac{1}{4}x^2$
C $y = \frac{1}{4}x^2$
D $y = 4x^2$

42. **True or False** The axis of symmetry for the parabola with equation $y = 2x^2$ is the line $x = 0$.

OBJECTIVE H **Graph equations of the form $y = ax^2 + bx + c$ and interpret these graphs.** (Lesson 9-3)

43. Use the parabola with vertex V below.



    a. What is the maximum value of the function?
    b. What is an equation for its axis of symmetry?
    c. Find the coordinates of points A and B, the reflection images of the named points over the parabola's axis of symmetry.

In 44 and 45, answer *true* or *false*.

44. Every parabola has a minimum value.

45. The parabola $y = -4x^2 + 2x - 13$ opens down.

46. What equation must you solve to find the x-intercepts of the parabola $y = ax^2 + bx + c$?

47. The parabola $y = \frac{1}{8}x^2 - 6x + 22$ has x-intercepts 4 and 44. Find the coordinates of its vertex without graphing.

48. A table of values for a parabola is given below.

x	0	2	4	6	8	10	12
y	?	?	−6	−8	−6	0	10

a. Complete the table.

b. Write an equation for the parabola's axis of symmetry.

c. What are the coordinates of its vertex?

49. Consider the quadratic equation $y = -x^2 - 4x + 3$.

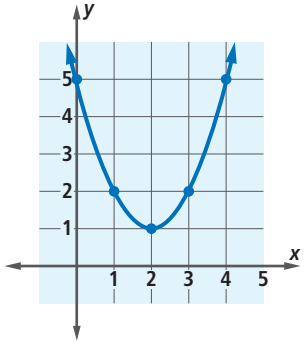
a. Make a table of x and y values for integer values of $-5 \le x \le 1$.

b. Graph the equation.

c. Determine whether the vertex is a minimum or a maximum.

50. Which of these is the graph of $y = x^2 - 4x + 5$?

A



B