

## Lesson

## 9-7

# More Applications of Quadratics: Why Quadratics Are Important

► **BIG IDEA** Quadratics have applications in engineering, geometry, and counting problems.

You may wonder why you are asked to memorize the Quadratic Formula or even why you need to know how to solve quadratic equations. There is a simple reason. Quadratic expressions, equations, and functions appear in a wide variety of situations. Furthermore, these situations are unlike those that lead to linear expressions. Many problems involving linear expressions can be solved by some people without algebra, using just intuition from arithmetic. Few people can solve problems that lead to quadratic expressions without using algebra.

You have seen applications of quadratic expressions in areas of squares and of circles, and to describe paths of projectiles. There are too many other applications to describe them all here. In this lesson, we give just a few.

## Parabolas as Important Curves

In your future mathematics courses, you will study perhaps the most important property of a parabola: its reflective property. Because of its reflective property, a parabola is the shape of a cross-section of automobile headlights, satellite dishes, and radio telescopes.

Parabolas also appear on suspension bridges. When a chain is suspended between two fixed points, the curve it describes is a *catenary*. A catenary looks much like a parabola but is slightly deeper. But in a suspension bridge where the roadway is hung by support cables from the main cables, the shape of the main cable is a parabola.

### Example 1

Suppose a team of engineers and construction workers are repairing a suspension bridge to strengthen it for use with increased traffic flow. The engineer uses scale models, such as the graph on the next page, to make decisions about repairs.

(continued on next page)

### Mental Math

Solve.

a.  $E = mc^2$  for  $m$ .

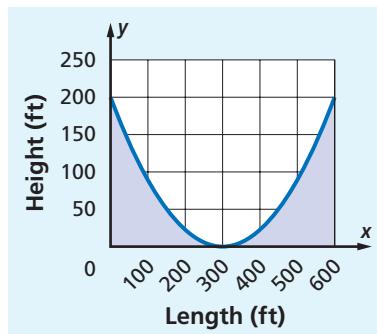
b.  $A = \pi r^2$  for  $\pi$ .



The total length of wires in the cables of the Brooklyn Bridge is approximately 3,600 miles.

Source: Endex Engineering, Inc.

Let the roadway be along the  $x$ -axis. Place the  $y$ -axis at one end of the roadway. Then the ends of the bridge are at  $(0, 0)$  and  $(600, 0)$ . The graph at the right shows that the parabola passes through points  $(0, 200)$ ,  $(300, 0)$ , and  $(600, 200)$ . By using quadratic regression, an equation for the parabola through these points can be found. In standard form, the equation is given by  $y = \frac{1}{450}x^2 - \frac{4}{3}x + 200$ , where  $y$  is the length of each vertical cable at the distance  $x$  from the end of the bridge. Suppose a support cable 82.5 feet long is delivered to the construction site. How far from the left end of the bridge should the cable be placed?



**Solution 1** Substitute 82.5 for  $y$  in the equation of the parabola. Then solve the equation for  $x$ .

$$82.5 = \frac{1}{450}x^2 - \frac{4}{3}x + 200$$

$$37,125 = x^2 - 600x + 90,000 \quad \text{Multiply both sides by 450.}$$

$$0 = x^2 - 600x + 52,875 \quad \text{Subtract 37,125 from both sides.}$$

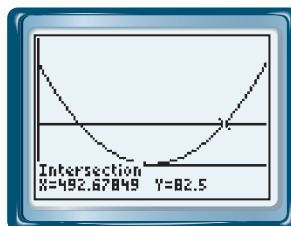
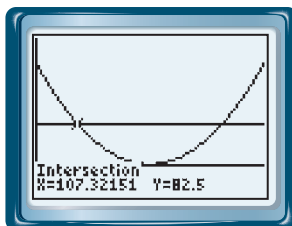
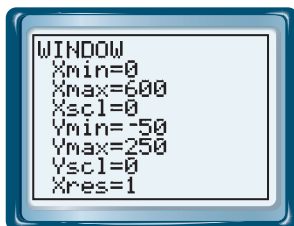
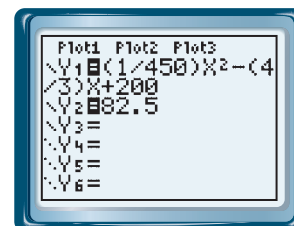
Now the equation  $x^2 - 600x + 52,875 = 0$  can be solved using the Quadratic Formula, with  $a = 1$ ,  $b = -600$ , and  $c = 52,875$ .

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-600) \pm \sqrt{(-600)^2 - 4(1)(52,875)}}{2(1)} \\ &= \frac{600 \pm \sqrt{360,000 - 211,500}}{2} \\ &= \frac{600 \pm \sqrt{148,500}}{2} \\ &\approx \frac{600 \pm 385.4}{2} \\ x &\approx 107.3 \text{ ft or } x \approx 492.7 \text{ ft} \end{aligned}$$

The cable can be placed either 107.3 ft or 492.7 ft from the left end of the bridge.

**Solution 2** Enter the equation of the parabola and the desired  $y$  value into a calculator, as shown at the right.

Graph each equation over the domain  $0 \leq x \leq 600$ . The solutions to the problem are intersection points of the two functions.



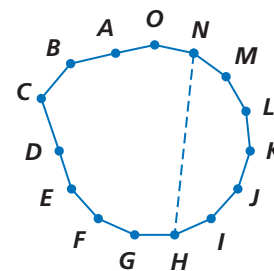
The x-coordinates of the intersections indicate that the cable is 82.5 feet long when  $x \approx 107.3$  ft and  $x \approx 492.7$  ft.

This makes sense because these distances are equally far from the center of the bridge at 300 ft.

## A Geometry Problem Involving Counting

Many counting problems lead to quadratic equations. For example, the number  $d$  of diagonals of an  $n$ -sided convex polygon is given by

the formula  $d = \frac{n(n-3)}{2}$ .



$\overline{NH}$  is one diagonal of this convex 15-sided polygon.

### Example 2

- How many diagonals does a convex polygon of 15 sides have?
- Can a convex polygon have exactly 300 diagonals? If so, how many sides must that polygon have?

#### Solutions

- When  $n = 15$ ,  $d = \frac{n(n-3)}{2} = \frac{15(15-3)}{2} = \frac{15 \cdot 12}{2} = 90$ .  
A 15-sided polygon has 90 diagonals.

- Substitute 300 for  $d$  in the formula.

$$300 = \frac{n(n-3)}{2}$$

$$600 = n(n-3) \quad \text{Multiply both sides by 2.}$$

$$600 = n^2 - 3n \quad \text{Distributive Property}$$

To solve, write the equation in standard form.

$$0 = n^2 - 3n - 600 \quad \text{Subtract 600 from both sides.}$$

This equation is in standard form with  $a = 1$ ,  $b = -3$ , and  $c = -600$ .

Use the Quadratic Formula.

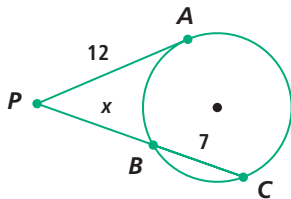
$$\begin{aligned} n &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-600)}}{2 \cdot 1} \\ &= \frac{3 \pm \sqrt{9 + 2,400}}{2} \\ &= \frac{3 \pm \sqrt{2,409}}{2} \end{aligned}$$

Because the discriminant 2,409 is not a perfect square, the values of  $n$  that we get are not integers. But  $n$  has to be an integer because it is the number of sides of a polygon. So the discriminant signals that there is no polygon with exactly 300 diagonals.

## Questions

### COVERING THE IDEAS

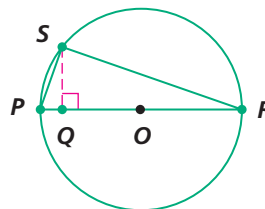
1. A cable of length 100 feet is brought to the construction site in the situation of Example 1. How far from the left end of the bridge can that cable be placed?
2. Draw a convex decagon (10-sided polygon) and one of its diagonals. How many other diagonals does this polygon have?
3. Can a convex polygon have exactly 21 diagonals? If so, how many sides does that polygon have?
4. Can a convex polygon have exactly 2,015 diagonals? If so, how many sides does that polygon have?
5. The sum of the integers from 1 to  $n$  is  $\frac{n(n+1)}{2}$ . If this sum is 7,260, what is  $n$ ?
6. The sum  $5 + 6 + 7 + 8 + 9 + 10 = 45$  is an instance of the more general pattern that the sum of the integers from  $n$  to  $2n$  is  $1.5(n^2 + n)$ .
  - a. What is the sum of the integers from 100 to 200?
  - b. If the sum of the integers from  $n$  to  $2n$  is 759, what is  $n$ ?
7. In the figure below,  $\overline{PA}$  is *tangent* to the circle at point  $A$ . (It intersects the circle only at that point.) Another segment from  $P$  intersects the circle at points  $B$  and  $C$ . When you study geometry, you will learn that  $PA^2 = PB \cdot PC$ . Suppose  $PA = 12$ ,  $BC = 7$ , and  $PB = x$ .



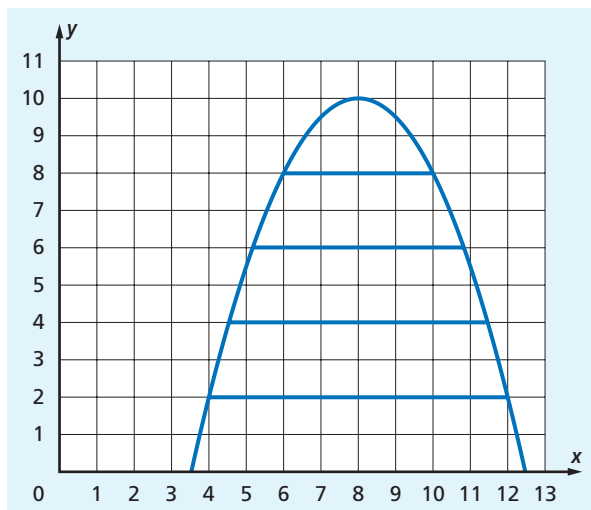
- a. Write an algebraic expression for  $PC$ .
- b. Substitute into  $PA^2 = PB \cdot PC$  and solve the resulting quadratic equation to find  $PB$ .

### APPLYING THE MATHEMATICS

8. In any circle  $O$  with diameter  $\overline{PR}$ ,  $SQ^2 = PQ \cdot RQ$ , as shown at the right.
  - a. If  $PR = 10$  and  $PQ = x$ , write an algebraic expression for  $QR$ .
  - b. Substitute the values from Part a into the formula  $SQ^2 = PQ \cdot RQ$  to find  $SQ$  when  $PR = 10$  and  $PQ = 3$ .



9. Suppose an architect is designing a building with arched windows in the shape of a parabola. The area under the arch will be divided into windowpanes as shown in the diagram below. The architect needs to know the lengths of the four horizontal bars at heights 2, 4, 6, and 8 units. If the parabola has equation  $y = -0.5x^2 + 8x - 22$ , find the length, to the nearest tenth of a unit, of the bar at the 6-unit height.



10. Secure telephone networks (ones in which each person is connected to every other person by a direct line) require  $\frac{n(n-3)}{2} = \frac{1}{2}n^2 - \frac{1}{2}n$  cable lines for  $n$  employees. A company is interested in setting up a secure phone networking group. Each employee in the group is provided with a secure connection to all other employees in the group. Suppose enough money is allotted to provide cable for 500 such connections. How many employees can be enrolled in the group?



Radio and telecommunications equipment installers and repairers held about 222,000 jobs in 2004.

Source: U.S. Department of Labor

## REVIEW

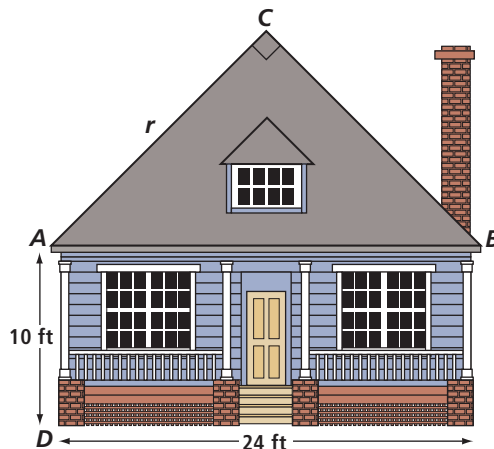
11. Suppose a quadratic equation has only one real solution. What can you conclude about its discriminant? (Lesson 9-6)

In 12–15, determine whether the equation has 0, 1, or 2 real number solutions. (You do not need to find the solutions.) (Lesson 9-6)

12.  $-a^2 + 3a - 5 = 0$                       13.  $b^2 + 4b + 4 = 0$   
 14.  $2c(c - 5) = 11$                       15.  $3(d - 6) = 5(d^2 + 11)$   
 16. Find an equation for a parabola with vertex  $(0, 0)$  that opens down. (Lesson 9-1)  
 17. **Skill Sequence** Solve the equation. (Lessons 9-1, 8-6)  
 a.  $\sqrt{a} = 6$                       b.  $\sqrt{b + 8} = 6$                       c.  $\sqrt{4c - 5} = 6$

In 18 and 19, use the figure at the right, which represents the front view of a building plan for a cottage. The cottage is to be 24 feet wide. The edges  $\overline{AC}$  and  $\overline{BC}$  of the roof are to be equal in length and to meet at a right angle. (Lessons 8-8, 8-7)

18. a. Find the length  $r$  of each edge as a simplified radical.  
b. Round the length of an edge to the nearest tenth of a foot.
19. Find  $BD$ .
20. **True or False** If the growth factor of an exponential growth situation is 2, then an equation that represents this situation is  $y = 2 \cdot b^x$ . (Lesson 7-2)
21. Consider the three points  $(2, 1)$ ,  $(-4, 31)$ , and  $(7, -24)$ . (Lessons 6-8, 6-3)
  - a. Show that these points lie on the same line.
  - b. Write an equation for the line in standard form.
22. A school begins the year with 250 reams of paper. (A ream contains 500 sheets.) The teachers are using an average of 18 reams per week, and the school receives a shipment of 10 additional reams each week. (Lesson 2-2)
  - a. How many reams will the school have after  $w$  weeks?
  - b. Suppose the school year lasts 36 weeks. Assuming these rates continue, will the school run out of paper before the year ends?



### EXPLORATION

23. Some telescopes use parabolic mirrors. Look on the Internet or in reference books to find out why it is useful to have mirrors shaped like parabolas and summarize your findings.