Lesson 9-5

The Quadratic Formula

Vocabulary

quadratic equation

standard form of a quadratic equation

BIG IDEA If an equation can be put into the form $ax^2 + bx + c$, it can be solved using the Quadratic Formula.

One of the most exciting events in amateur sports is 10-meter platform diving. Once a diver leaves a platform, the diver becomes a projectile. Consequently, during a dive, a diver's height above the water at any given time can be determined using a quadratic equation. This is important because for a diver to practice spins and somersaults, he or she must know how much time will pass before entering the water.

Suppose a diver jumps upward at an initial velocity of 4.3 meters per second. Then the diver's height $h(t)$ in meters t seconds into the dive; can be estimated using the equation $h(t) = -4.9t^2 + 4.3t + 10$ and graphed below. When the diver hits the water, $h(t)$ is zero. So solving the equation $0 = -4.9t^2 + 4.3t + 10$ gives the number of seconds from departing the platform to entering the water.

The equation $0 = -4.9t^2 + 4.3t + 10$ is an example of a *quadratic equation*. A **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$ with $a \neq 0$. In this case, t is being used in place of x, $a = -4.9$, $b = 4.3$, and $c = 10$.

Mental Math

Write as a power of 10.

a. 65 ÷ 0.65

b. 483 ÷ 48.3

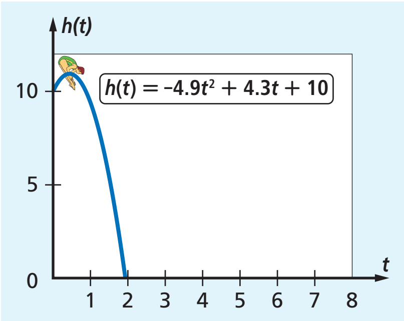
c. 7.2 ÷ 7,200

The Quadratic Formula

You can find the solutions to *any* quadratic equation by using the *Quadratic Formula*. This formula gives the value(s) of x in terms of a (the coefficient of x^2), b (the coefficient of x), and c (the constant term). The formula states that there are at most two solutions to a quadratic equation.

If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ or $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The calculations of the two solutions differ in only one way. $\sqrt{b^2 - 4ac}$ is added to $-b$ in the numerator of the first calculation, while $\sqrt{b^2 - 4ac}$ is subtracted from $-b$ in the second calculation.



The work in calculating the two solutions is almost the same. So, the two expressions can be written as one expression using the symbol $\pm$, which means "plus or minus." This symbol means you do two calculations: one using the + sign to add and one using the − sign to subtract.

The Quadratic Formula

If $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The quadratic equation $ax^2 + bx + c = 0$ and the Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are equivalent equations. The quadratic equation was solved for x to generate the quadratic formula. In Chapter 13, you can see how this formula was found.

The quadratic formula is one of the most often used and most famous formulas in all mathematics. *You should memorize it today!*

READING MATH

On some calculators there is a key that is labeled $\pm$ or $+/-$. That key takes the opposite of a number. It does not perform the two operations + and − required in the Quadratic Formula.

Applying the Quadratic Formula

Example 1

Solve $x^2 + 8x + 7 = 0$.

Solution Recall that $x^2 = 1x^2$. So think of the given equation as $1x^2 + 8x + 7 = 0$ and apply the Quadratic Formula with $a = 1$, $b = 8$, and $c = 7$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

Follow the order of operations. Work under the radical sign (with its unwritten parentheses) first.

$$= \frac{-8 \pm \sqrt{64 - 28}}{2}$$

$$= \frac{-8 \pm \sqrt{36}}{2}$$

So, $x = \frac{-8 + 6}{2}$ or $x = \frac{-8 - 6}{2}$

$x = \frac{-2}{2} = -1$ or $x = \frac{-14}{2} = -7$

Check Do −1 and −7 make the equation $x^2 + 8x + 7 = 0$ true? Substitute −1 for x.

Does $(-1)^2 + 8(-1) + 7 = 0$?

$1 + -8 + 7 = 0$ Yes, it checks.

(*continued on next page*)

Substitute −7 for x.

Does $(-7)^2 + 8(-7) + 7 = 0$?

$49 + -56 + 7 = 0$ Yes, it checks.

STOP QY

Sometimes you must decide whether or not a solution to a quadratic equation is reasonable, given the context of the problem. In Guided Example 2, we return to the diving situation described at the beginning of this lesson.

The x-axis represents a height of 0. The graph crosses the x-axis a little to the left of 2, which is close to 1.93. The INTERSECT feature on a graph is also helpful for finding a solution.

▶ QY

Solve $3y^2 - 10.5y + 9 = 0$.

GUIDED

Example 2

In 10-meter platform diving, the function $h(t) = -4.9t^2 + 4.3t + 10$ gives the approximate height $h(t)$ above the water in meters a diver is at t seconds after launching into the dive. How many seconds elapse from the time the diver leaves the 10-meter platform until the diver hits the water?

Solution The diver will hit the water when the diver's height above the water is zero, so solve the equation $0 = -4.9t^2 + 4.3t + 10$.

Apply the Quadratic Formula with $a = -4.9$, $b = 4.3$, and $c = 10$.

$$t = \frac{-4.3 \pm \sqrt{(\underline{\ ?\ })^2 \underline{\ ?\ } 4(\underline{\ ?\ })(\underline{\ ?\ })}}{2(\underline{\ ?\ })}$$

$$= \frac{-4.3 \pm \sqrt{\underline{\ ?\ }}}{-9.8}$$

So, $t = \frac{-4.3 + \sqrt{\underline{\ ?\ }}}{-9.8}$ or $t = \frac{-4.3 - \sqrt{\underline{\ ?\ }}}{-9.8}$.

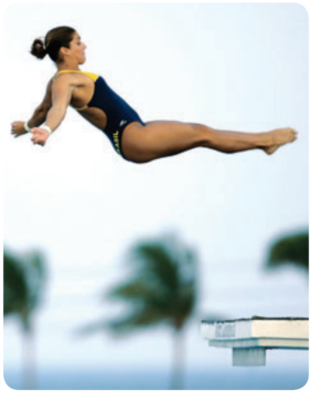
These are exact solutions to the quadratic equation. However, since the given information is not exact, it is more reasonable to want an approximation.

$t \approx \frac{-4.3 + \underline{\ ?\ }}{-9.8}$ or $t \approx \frac{-4.3 - \underline{\ ?\ }}{-9.8}$

$t \approx -1.1$ or $t \approx 1.9$

The diver cannot reach the water in negative time, so the solution −1.1 seconds does not make sense in this situation. We therefore eliminate this as an answer. The diver will hit the water about 1.9 seconds after leaving the diving platform.

Check Substituting 1.9 in for t in the equation $0 = -4.9t^2 + 4.3t + 10$ is one method you can use to check a solution. Other methods are looking at a table or a graph.



Juliana Veloso of Brazil competes in the women's 10-meter platform semifinals at the 2006 USA Grand Prix Diving Championships in Fort Lauderdale, Florida.

In Example 3, the equation has to be put in $ax^2 + bx + c = 0$ form before the Quadratic Formula can be applied. This form is called the **standard form of a quadratic equation.** When the number under the radical sign in the Quadratic Formula is not a perfect square, approximations are often used in the last step of the process.

Example 3

Solve $x^2 - 3x = 37$.

Solution Put $x^2 - 3x = 37$ into standard form.

$x^2 - 3x - 37 = 37 - 37$ Subtract 37 from both sides.

$x^2 - 3x - 37 = 0$

Apply the Quadratic Formula, with $a = 1$, $b = -3$, and $c = -37$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-37)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 + 148}}{2}$$

$$x = \frac{3 \pm \sqrt{157}}{2}$$

So $x = \frac{3 + \sqrt{157}}{2}$ or $x = \frac{3 - \sqrt{157}}{2}$.

These are exact solutions. You can approximate the solutions using a calculator.

Because $\sqrt{157} \approx 12.5$, $x \approx \frac{3 + 12.5}{2}$ or $x \approx \frac{3 - 12.5}{2}$. So $x \approx 7.75$ or $x \approx -4.75$.

Check Do the two values found work in the equation $x^2 - 3x = 37$?

Substitute 7.75 for x.

$(7.75)^2 - 3(7.75) = 36.8$. This is close to 37.

Substitute −4.75 for x.

$(-4.75)^2 - 3(-4.75) = 36.8$. This is close to 37.

In both cases the checks are not exact, but the solutions are approximations, so the check is close enough.

Questions

COVERING THE IDEAS

1. State the Quadratic Formula.
2. Is it true that the Quadratic Formula can be used to solve *any* quadratic equation?
3. Find the two values of $\frac{-3 \pm 9}{2}$.

In 4–7, use the Quadratic Formula to solve the equation. Give the exact solutions and check both solutions.

4. $x^2 + 15x + 54 = 0$
5. $t^2 + 4t + 4 = 0$
6. $3m^2 + 2m = 4$
7. $3y^2 = 13y + 100$

In 8 and 9, use the Quadratic Formula to solve the equations. Round the solutions to the nearest hundredth and check both solutions.

8. $20n^2 - 6n - 2 = 0$
9. $3p^2 + 14 = -19p$

10. If a diver dives from a 20-foot platform with an initial upward velocity of 14 feet per second, then the diver's approximate height can be represented by the function $h(t) = -16t^2 + 14t + 20$, where $h(t)$ is the height and t is the time in seconds. (This formula is different from the one in this lesson because meters have been converted to feet.)
    a. Find $h(1)$. Write a sentence explaining what it means.
    b. Estimate to the nearest tenth of a second how much time the diver will be in the air before hitting the water.

APPLYING THE MATHEMATICS

11. The solutions to $ax^2 + bx + c = 0$ are the x-intercepts of the graph of $y = ax^2 + bx + c$.
    a. Use the Quadratic Formula to find the solutions to $3x^2 - 6x - 45 = 0$.
    b. Check your answers to Part a by graphing an appropriate function.

12. The graphs of $y = -0.5x^2 + 6$ and $y = 4$ intersect at two points.
    a. Find the x-coordinate of each of the intersecting points.
    b. Find both coordinates of the two points of intersection.
    c. Check your answers to Part b by graphing these equations.

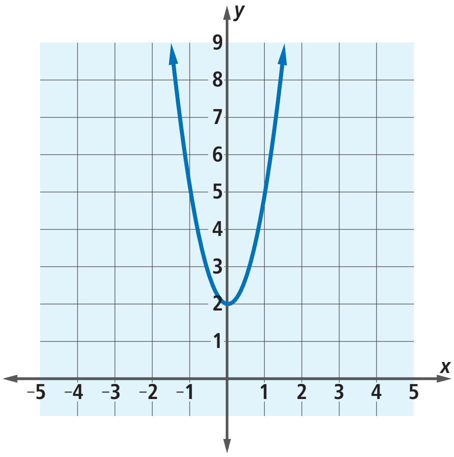
13. In 1971, the astronaut Alan Shepard (who had been the first U.S. man in space 9 years earlier) snuck a collapsible golf club and a golf ball onto *Apollo 14*. Just before taking off from the moon to return to Earth, he hit two golf balls. In doing so, he vividly showed the difference between gravity on the moon and on Earth. On the moon the approximate height $h(t)$ of the ball (in feet) after t seconds is given by the function $h(t) = -0.8t^2 + 12t$.
    a. At what two times would the golf ball reach a height of 20 feet? (Round your answer to the nearest hundredth.)
    b. How long would it take for the ball to come back to the surface of the moon?

14. The area of a rectangle is 240 cm^2. The length is 14 cm more than the width. What are the length and width of the rectangle?

REVIEW

15. If the x-intercepts of a parabola are 8 and –4, what is the x-coordinate of its vertex? **(Lesson 9-3)**

16. **Multiple Choice** Which equation is graphed at the right? **(Lessons 9-3, 9-2)**



A $y = 2x^2$ **B** $y = -2x^2$

C $y = 2x^2 + 2$ **D** $y = -2x^2 - 2$

17. Find the radius of a circle if the center of the circle is at (–2, 6) and the point (1, 2) is on the circle. **(Lesson 8-8)**

18. Kristen is buying a new car. She can choose from 4 models, 2 transmission types, 9 exterior colors, and 6 interior colors. How many combinations of models, transmissions, exteriors, and interiors are possible? **(Lesson 8-1)**

19. ***Skill Sequence*** If $f(x) = \frac{5x - 6}{x}$, find each value. **(Lesson 7-6)**

a. $f(2)$ **b.** $f(3)$ **c.** $f(x + 1)$

EXPLORATION

20. **a.** Solve the equation $ax^2 + bx + c = 0$ for x using a CAS. In what form does the CAS put the two solutions?

b. Add the two solutions using the CAS. What is the sum?

c. Multiply the two solutions using the CAS. What is the product?

d. Use the results from Parts b and c to check the *exact* answers to Example 3 of this lesson.

QY ANSWER

$y = 1.5$ or $y = 2$