Lesson 9-3 Graphing $y = ax^2 + bx + c$

BIG IDEA The graphs of any quadratic function with equation $y = ax^2 + bx + c$, $a \neq 0$, is a parabola whose vertex can be found from the values of a, b, and c.

Mental Math

Simplify.

a. $\sqrt{18}$

b. $\sqrt{200}$

c. $\frac{\sqrt{18}}{\sqrt{200}}$

If $a \neq 0$, the graph of $y = ax^2$ is a parabola with vertex (0, 0). This lesson is about the graph of a more general function, $y = ax^2 + bx + c$. We begin with an important everyday use.

How Far Does a Car Travel after Brakes Are Applied?

When a driver decides to stop a car, it takes time to react and press the brake. Then it takes time for the car to slow down. The total distance traveled in this time is called the *stopping distance* of the car. The faster the car is traveling, the greater the distance it takes to stop the car. A formula that relates the speed x (in miles per hour) of a car and its stopping distance d (in feet) is $d = 0.05x^2 + x$.

This function is used by those who study automobile performance and safety. It is also important for determining the distance that should be maintained between a car and the car in front of it.

To find the distance needed to stop a car traveling 40 miles per hour, you can substitute 40 for x in the above equation.

$$\begin{aligned} d &= 0.05(40)^2 + 40 \\ &= 0.05(1{,}600) + 40 \\ &= 80 + 40 \\ &= 120 \end{aligned}$$

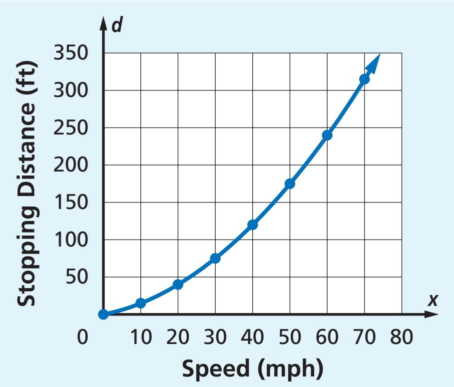
Thus, a car traveling 40 mph takes about 120 feet to come to a complete stop after the driver decides to apply the brakes.

A table of values and a graph for the stopping distance formula is shown on the next page. The situation makes no sense for negative values of x or d, so the graph has points in the first quadrant only. The graph is part of a parabola.

Traffic lights were used before the advent of the motorcar. In 1868, a lantern with red and green signals was used at a London intersection to control the flow of horse buggies and pedestrians.

Source: www.ideafinder.com

Speed x (mph)	Distance $d = 0.05x^2 + x$ (ft)
10	15
20	40
30	75
40	120
50	175
60	240
70	315



Properties of the Graph of $y = ax^2 + bx + c$

The equation $d = 0.05x^2 + x$ is of the form $y = ax^2 + bx + c$, with d taking the place of y, $a = 0.05$, $b = 1$, and $c = 0$. The graph of every equation of this form (provided $a \neq 0$) is a parabola. Moreover, every parabola with a vertical line of symmetry has an equation of this form. The values of a, b, and c determine where the parabola is positioned in the plane and whether it opens up or down. If $a > 0$ the parabola opens up, as in the situation above. If $a < 0$, the parabola opens down, as in Example 1.

Example 1

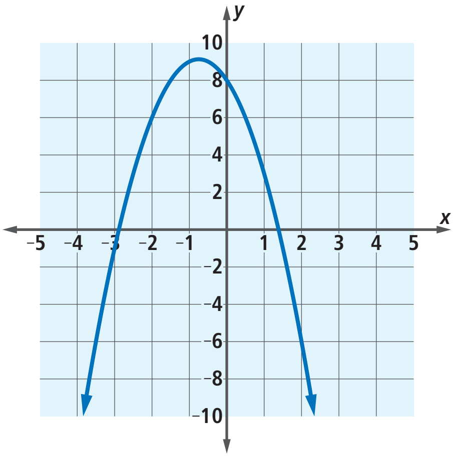
a. Graph $f(x) = -2x^2 - 3x + 8$. Use a window big enough to show the vertex of the parabola, the two x-intercepts, and the y-intercept.

b. Estimate its vertex, x-intercepts, and y-intercept.

Solutions

a. Here is a graph of $y = -2x^2 - 3x + 8$ on the window $-5 \leq x \leq 5$; $-10 \leq y \leq 10$.

b. From the window we have shown, we can only estimate the location of the vertex. The vertex is near (-0.8, 9.1). You do not have to estimate the y-intercept. It is $f(0)$, the value of y when $x = 0$, and is easily calculated. The graph has y-intercept 8. On the other hand, you can only estimate its x-intercepts. They are the values of x when the graph intersects the x-axis. The x-intercepts are near -2.9 and 1.4.



Using Tables to Determine the Vertex of a Parabola

We could only estimate the vertex in Example 1 from the graph. Even if you use the trace function on a calculator or computer, you might not happen to find the vertex exactly. But in some cases, the symmetry of a parabola can be used to determine its vertex.

Example 2

Find the exact location of the vertex of the parabola that is the graph of $f(x) = -2x^2 - 3x + 8$ from Example 1.

Solution Because we know that the vertex is near the point (−0.8, 9.1), we find y for values of x between −1 and 0. Look in the table at the right for a pair of points which have the same y-coordinates. Notice that there are three pairs of points whose y-coordinates are the same. These pairs occur on either side of −0.75. This indicates that the vertex is the point for which $x = -0.75$. Because $f(-0.75) = 9.125$, the vertex is (−0.75, 9.125).

x	y
−1.0	9
−0.9	9.08
−0.8	9.12
−0.7	9.12
−0.6	9.08
−0.5	9
−0.4	8.88
−0.3	8.72
−0.2	8.52
−0.1	8.28
0	8

From a graph, it is often not as easy to locate the x-intercepts of a parabola as it is the vertex. The next two activities explore how the intercepts of the parabolas change as the values of b and c change in the equation $y = ax^2 + bx + c$.

Activity 1

Step 1 Graph $y = 2x^2$ and $y = 2x^2 - 10$ on the same axes.

a. What is the y-intercept of $y = 2x^2$?

b. What is the y-intercept of $y = 2x^2 - 10$?

c. Describe how the two graphs are related to each other.

Step 2 Graph $y = -0.75x^2$ and $y = -0.75x^2 + 1$ on the same axes.

a. What is the y-intercept of $y = -0.75x^2$?

b. What is the y-intercept of $y = -0.75x^2 + 1$?

c. Describe how the two graphs are related to each other.

Step 3 Graph $y = 2x^2 + c$ for three different values of c, with at least one of these values negative.

a. Give the equations that you graphed.

b. What is the effect of c on the graphs?

Activity 2

Step 1 To determine the effect of b on the graph of a quadratic function, consider the function $y = x^2 + bx - 4$.

a. Graph this function for the three different values of b given in the table below. Fill in the table.

Equation $y = ax^2 + bx + c$	b	Vertex of Parabola	y-intercept of Parabola	x-intercepts (if any)
$y = x^2 + 2x - 4$	?	?	?	?
$y = x^2 + 4x - 4$	?	?	?	?
$y = x^2 - 3x - 4$	?	?	?	?

b. What features of the graph of $y = x^2 + bx - 4$ does the value of b affect?

c. What feature of the graph of $y = x^2 + bx - 4$ is not affected by the value of b?

Step 2 Find a window that shows a graph of $y = \frac{1}{2}x^2 - 6x + 4$, including its vertex, its y-intercept and its x-intercept(s).

a. Describe your window.

b. Use the trace feature of the calculator to estimate or determine the vertex, the y-intercept, and the x-intercepts.

c. Tell how the x-coordinate of the vertex is related to the x-intercepts.

Questions

COVERING THE IDEAS

In 1–4, use the formula for automobile stopping distances given in this lesson, $d = 0.05x^2 + x$.

1. Define stopping distance.
2. Find the stopping distance for a car traveling 45 miles per hour.
3. Find the stopping distance for a car traveling 55 miles per hour.
4. **True or False** The stopping distance for a car traveling 50 mph is exactly double the stopping distance of a car traveling 25 mph.
5. The equation $d = 0.05x^2 + x$ is of the form $f(x) = ax^2 + bx + c$. What are the values of a, b, and c?
6. Explain how you can tell by looking at an equation of the form $y = ax^2 + bx + c$ whether its graph will open up or down.

In 7–9, an equation for a function is given.

a. Make a table of *x* and *y* values when *x* equals –3, –2, –1, 0, 1, 2, and 3.

b. Graph the equation.

c. Identify the *y*-intercept, *x*-intercept(s), and vertex.

d. Describe the range of the function.

7. $y = x^2 - 6$

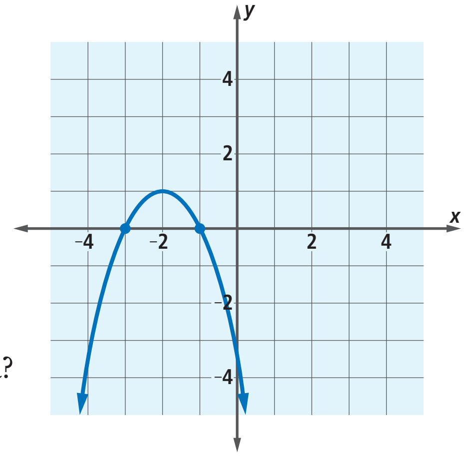
8. $y = x^2 - 2x + 5$

9. $y = 3x^2 + 4$

10. The parabola at the right contains (–3, 0) and (–1, 0) and its vertex has integer coordinates.

a. Find the coordinates of its vertex.

b. Write an equation of its axis of symmetry.



11. Consider this table of values for a parabola.

x	–8	–7	–6	–5	–4	–3	–2	–1	0
y	–28	0	20	32	36	32	?	?	?

a. What are the coordinates of the vertex of the parabola?

b. Use symmetry to find the coordinates of the points whose *y* values are missing in the table.

APPLYING THE MATHEMATICS

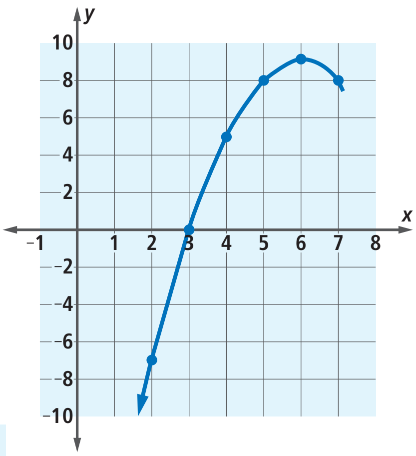
12. The parabola at the right contains points with integer coordinates as shown by the dots.

a. Copy this graph on graph paper. Then use symmetry to graph more of the parabola.

b. Give the coordinates of the vertex.

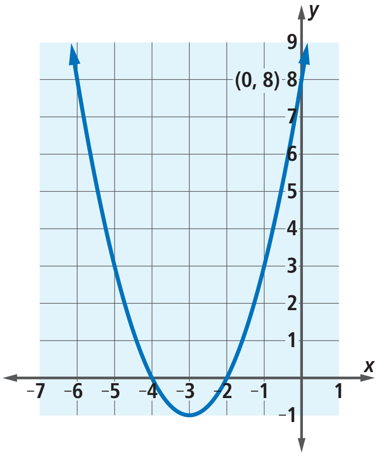
c. Give an equation for the axis of symmetry.

d. At what points does the parabola intersect the *x*-axis?

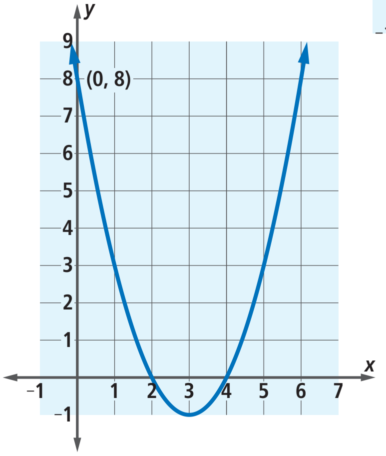


13. ***Multiple Choice*** Which of the two graphs is the graph of $y = x^2 - 6x + 8$? Justify your answer.

A



B



Matching In 14–17, match the graph with its equation.

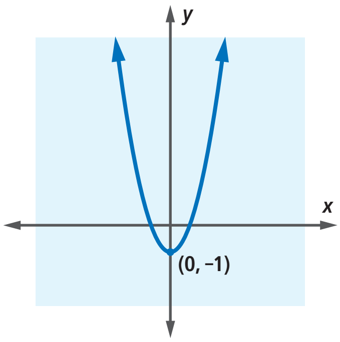
a. $y = 2x^2$

b. $y = 2x^2 + 1$

c. $y = 2x^2 - 1$

d. $y = -2x^2 - 1$

14.



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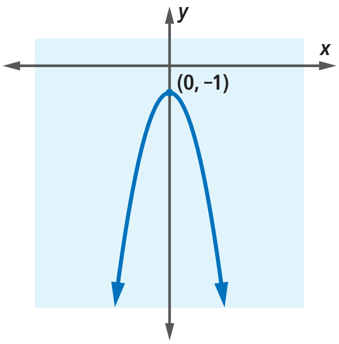

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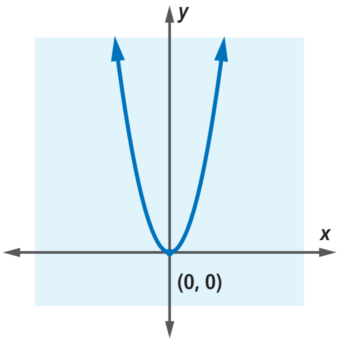
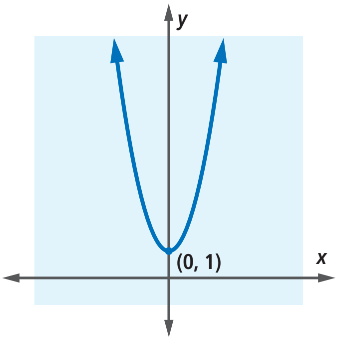


17.



18. An insurance company reports that the equation $y = 0.4x^2 - 36x + 1{,}000$ relates the age of a driver x (in years) to the accident rate y (number of accidents per 50 million miles driven) when $16 \le x \le 74$.

a. Graph this equation on your calculator. Give the window you used, the vertex, and the intercepts of the graph.

b. Use the trace function of the graph to determine the age in which drivers have the fewest accidents per mile driven. About how many accidents do drivers of this age have per 50 million miles driven?

c. According to this model, an 18-year-old driver is how many times as likely to have an accident as a 45-year-old driver?

REVIEW

19. Consider Galileo's equation $d = 16t^2$. (**Lesson 9-2**)

a. Find t when $d = 350$.

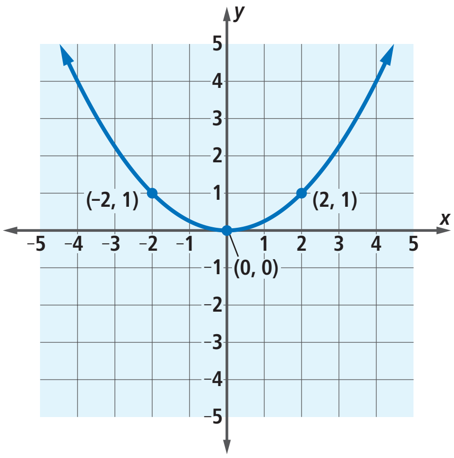
b. Write a question involving distance and time that can be answered using Part a.

20. Below are Charlotta's answers to questions about $y = \frac{x^2}{4}$. After she wrote this, she realized that she copied the equation incorrectly. It should be $y = -\frac{x^2}{4}$. What does Charlotta need to change to correct her work? **(Lesson 9-1)**

x	−4	−2	0	2	4
y	4	1	0	1	4

vertex = (0, 0)

axis of symmetry: $x = 0$



21. ***Skill Sequence*** If $a = -6$, $b = 8$, and $c = 12$, find the value of each expression. **(Lessons 8-6, 1-1)**

a. $-4ac$ **b.** $b^2 - 4ac$ **c.** $\sqrt{b^2 - 4ac}$

22. A giant tortoise is walking at an average rate of 0.17 $\frac{\text{mile}}{\text{hour}}$. Assume the tortoise continues walking at this rate.

a. How long will it take the tortoise to travel 50 feet?

b. How long will it take the tortoise to travel f feet? **(Lessons 5-4, 5-3)**

EXPLORATION

23. Begin with $y = -x^2$. With your calculator, experiment to find an equation for the parabola whose graph is used in Question 12.

In the past, giant species of *Geochelone* (tortoise) were once found on all continents except Australasia, but today the giant forms are restricted to *G. elephantopus* in the Galapagos and *G. gigantea* on the island of Aldabara.

Source: Rochester Institute of Technology