Lesson 9-2

Solving $ax^2 = b$

BIG IDEA When $\frac{b}{a}$ is not zero, the equation $ax^2 = b$ has two solutions, $x = \sqrt{\frac{b}{a}}$ and $x = -\sqrt{\frac{b}{a}}$.

Graphs and tables can be very helpful in seeing the behavior of equations of the form $ax^2 = b$.

Mental Math

Evaluate.

a. 5^0

b. $4^3 - 3^3$

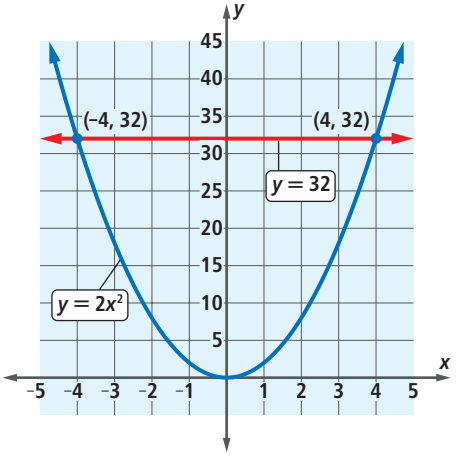
c. $10^2 - 5^3 + 2^4$

Example 1

Solve $2x^2 = 32$.

Solution 1 Create and graph two functions from the equation. One is the parabola $y = 2x^2$. The other is the horizontal line $y = 32$. Notice that the graphs intersect at two points. This is because two different values of x make the equation true. Trace on the graph of the parabola to find the intersection points, $(-4, 32)$ and $(4, 32)$. Use the x-coordinates.

The solutions are $x = -4$ and $x = 4$.



Solution 2 Create a table of values for the equation. Notice that there are two places where the expression $2x^2$ is equal to 32, when $x = -4$ and when $x = 4$. The solutions are $x = -4$ and $x = 4$.

x	$2x^2$
−5	50
−4	32
−3	18
−2	8
−1	2
0	0
1	2
2	8
3	18
4	32
5	50

Check

Does $2(4)^2 = 32$?
$2(16) = 32$
$32 = 32$

Does $2(-4)^2 = 32$?
$2(16) = 32$
$32 = 32$

Both −4 and 4 check.

In Example 1, the graph and table helped to show that an equation like $2x^2 = 32$ has two solutions.

You can solve equations of the form $x^2 = b$ symbolically by recalling the meaning of *square root*. If $x^2 = b$, then $x = \sqrt{b}$ or $x = -\sqrt{b}$. Notice that it takes only one step to solve an equation of the form $x^2 = b$. With just one additional step, you can solve an equation of the form $ax^2 = b$.

Example 2

A quarter is dropped 60 feet from the roof of a school building. To determine how long the quarter will be in the air, use Galileo's equation $d = 16t^2$. In the equation, t is the time, in seconds, that it takes a heavier-than-air object to fall d feet.

Solution Here $d = 60$, so we need to solve $60 = 16t^2$.

$\frac{60}{16} = t^2$ Divide both sides by 16.

$t = \pm\sqrt{\frac{60}{16}}$ Take the square roots of both sides.

$t \approx \pm 1.936$ Approximate the square root.

$t \approx 1.936$ Only the positive solution makes sense in this situation.

The quarter will be in the air for approximately 1.9 seconds.

STOP **QY**

▶ **QY**

Find how long it will take an object to hit the ground if it falls from the top of Chicago's John Hancock Center, which is 1,127 feet tall.

You can combine your knowledge of solving linear equations with what was done in Example 2 to solve some equations that look quite complicated. In the next example, you should think of $2n + 11$ as a single number. Psychologists call this idea *chunking*. Chunking is what you do when you read an entire word without thinking of the individual letters.

Example 3

Solve $3(2n - 11)^2 = 75$.

Solution Think of $2n - 11$ as a single number, say x. Then this equation is $3x^2 = 75$. Some people like to write the x in place of $2n - 11$, but you do not have to do that.

$3(2n - 11)^2 = 75$

Divide both sides by 3.

$(2n - 11)^2 = 25$

(continued on next page)

Take the square roots of both sides of the equation.

$(2n - 11) = \pm\sqrt{25}$

$2n - 11 = \pm 5$

Thus either $2n - 11 = 5$ or $2n - 11 = -5$.

Now there are two linear equations to be solved. It is good to separate the two processes.

$2n - 11 = 5$	or	$2n - 11 = -5$
$2n = 16$		$2n = 6$
$n = 8$		$n = 3$

So there are two solutions, $n = 8$ or $n = 3$.

Check Substitute 8 for *n* in the original equation.

Does $3(2 \cdot 8 - 11)^2 = 75$? Yes, because $3(5)^2 = 75$.

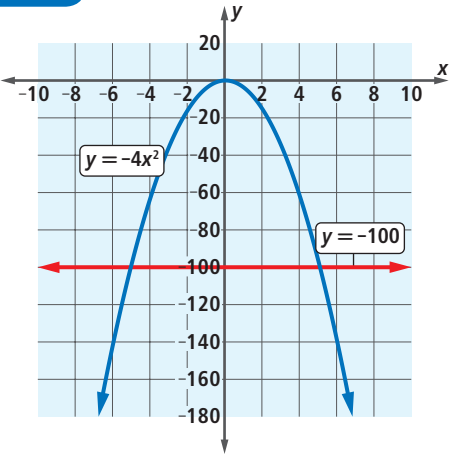
Substitute 3 for *n* in the original equation.

Does $3(2 \cdot 3 - 11)^2 = 75$? Yes, because $3 \cdot (-5)^2 = 75$.

Questions

COVERING THE IDEAS

1. Solve $-4x^2 = -100$ using the graph at the right.



2. Solve $5x^2 + 7 = 7.8$ using the table below.

x	−0.6	−0.4	−0.2	0	0.2	0.4	0.6
$5x^2 + 7$	8.8	7.8	7.2	7	7.2	7.8	8.8

In 3–8, solve the equation.

3. $x^2 = 40$
4. $3{,}705 = y^2 + 436$
5. $12v^2 - 24 = 36$
6. $5w^2 = 400$
7. $3(a + 5)^2 = 12$
8. $(7v - 2)^2 = 81$
9. **a.** You drop a stone into a deep well and carefully time how long it takes until you hear the stone plop into the water. If it takes 2.4 seconds, about how far from the top is the water in the well?

   b. If the water was 64 feet from the top of the well, how long would it take until the stone hit the water?
10. A 30-foot ladder is placed against a wall so that its bottom is 9 feet away from the wall. How high up the wall is the top of the ladder?

APPLYING THE MATHEMATICS

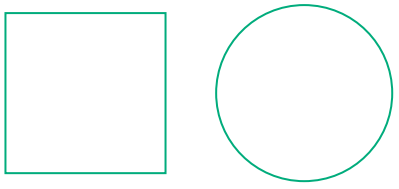
11. If the area of a circle is 100 square units, then what is the radius of the circle, to the nearest hundredth of a unit?

In 12 and 13, solve the quadratic equation.

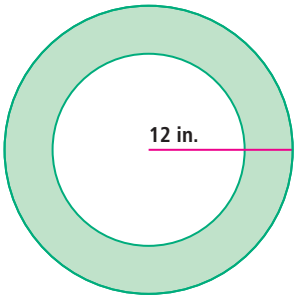
12. $\frac{1}{2}(2z + 3)^2 = 18$

13. $3v^2 + 10 = 7v^2 - 15$

14. Consider the figures drawn at the right. Suppose that the circle has the same area as the square. If the diameter of the circle is 4 feet, what is the length of the side of the square, to the nearest inch? (*Hint:* You will need to use the formulas for the area of a square and the area of a circle.)



15. You wish to make a bull's-eye target so that the area of the inner circle equals the area of the outer ring between the two circles. If the radius of the outer circle is 12 inches, what should the radius of the inner circle be?



REVIEW

16. **True or False** 0 is in the range of $y = ax^2$ for all values of a. **(Lesson 9-1)**

In 17–19, simplify the expression. (Lessons 8-5, 8-3, 8-2)

17. $n \cdot m^3 \cdot n^5 \cdot m^2$

18. $\frac{12x^8}{8x^2}$

19. $\left(\frac{3}{5a}\right)^2$

20. A company that manufactures combination locks wants each lock to have a unique 3-number combination. There are 36 numbers on each lock. **(Lesson 8-1)**

    a. How many locks can the company produce without having to use the same combination twice?

    b. Suppose a worker at the company forgets the combination of one of the locks, but he knows that it does not begin with 1. How many different combinations might he have to try to open the lock?

21. Let $f(x) = 2x^2$. **(Lessons 7-6, 7-5)**

    a. What is the domain of f? b. What is the range of f?

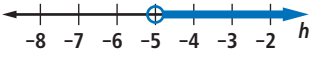
22. a. Determine a real situation that can be answered by solving $\frac{18}{24} = \frac{x}{32}$.

    b. Answer your question from Part a. **(Lesson 5-9)**

23. Tickets for a school play cost \$6 for adults and \$4 for children. The organizers of the play have determined that they must sell at least \$975 worth of tickets in order to cover their expenses. **(Lesson 6-9)**
    a. Write an inequality that represents this situation.
    b. Graph the inequality.
    c. Suppose 86 adult tickets are sold. If exactly \$1,000 were raised from ticket sales, how many children tickets must have been sold?

24. Jessica is driving between Cleveland and Chicago, a distance of about 350 miles. From Cleveland to Chicago, she averages 65 miles per hour. On the return trip, she averages x miles per hour. If the round trip takes Jessica 12 hours, find her average speed x on the way back. **(Lessons 5-3, 3-4)**

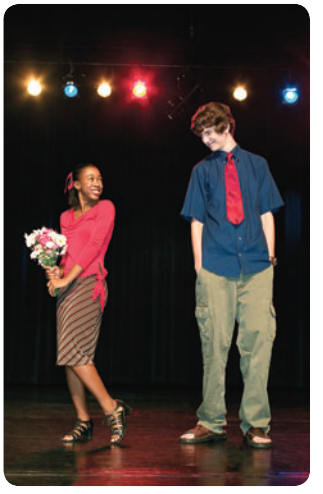
25. **Multiple Choice** The graph below pictures solutions to which inequality? **(Lessons 3-7, 3-6)**



A $h - 3 \leq -8$ **B** $h + 7 > 2$ **C** $h - 5 \geq -10$ **D** $h + 5 < 0$

EXPLORATION

26. On a piece of graph paper, carefully draw a circle with center at (0, 0) and radius 10 units. This circle will contain the point (10, 0) and three other points on the axes.
    a. What are the coordinates of those other three points?
    b. The circle will contain 8 other points whose coordinates are both integers. Identify those 8 points.
    c. Find the coordinates of three other points in the 1st quadrant that are on the circle. (*Hint:* You may need to describe the coordinates with square roots.)



Since 1929, more than two million students have been honored for excellence in theater arts with invitations to join the International Thespian Society.

Source: Educational Theatre Association

QY ANSWER

about 8.39 sec