# Lesson 9-1

# The Function with Equation $y = ax^2$

▶ **BIG IDEA** The graph of any quadratic function with equation $y = ax^2$, with $a \neq 0$, is a parabola with vertex at the origin.
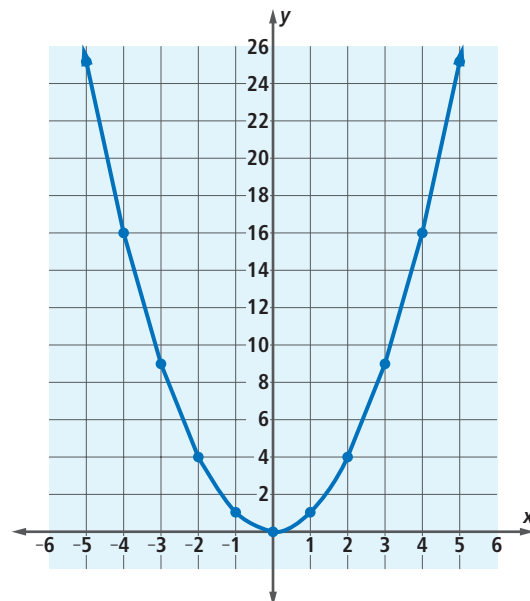
## Graphing $y = x^2$

The simplest quadratic function has equation $y = x^2$. A table of values for $y = x^2$ is given below. Notice the symmetry in the second row of the table. Each $x$ value and its opposite have the same square. For example, $3^2$ and $(-3)^2$ are both equal to 9. The bottom row of the table shows that the output of the function is positive for a pair of opposite positive and negative input values.

### Mental Math

**If ($a$, $b$) is in the 2nd quadrant, in which quadrant is:**

**a.** $(-a, -b)$?

**b.** $(-a, b)$?

**c.** $(a, -b)$?

opposite

| $x$ | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
|-----|----|----|----|----|---|---|---|---|---|
| $y$ | 16 | 9  | 4  | 1  | 0 | 1 | 4 | 9 | 16 |

equal

This symmetry can be seen in the graph of the equation $y = x^2$ at the right, which is a **parabola.** Every positive number is the $y$-coordinate of two points on the graph with opposite $x$-coordinates. For example, 25 is the $y$-coordinate of the points (5, 25) and (−5, 25). For this reason, the parabola is its own *reflection image* over the $y$-axis. For this reason we say the parabola is **reflection-symmetric** to the $y$-axis. The $y$-axis is called the **axis of symmetry** of the parabola.

The intersection point of a parabola with its axis of symmetry is called the **vertex** of the parabola. The vertex of the graph of $y = x^2$ is (0, 0).

The function $y = x^2$ is of the form $y = ax^2$, with $a = 1$. You should be able to sketch the graph of any equation of this form.

## Graphing $y = ax^2$

All equations of the form $y = ax^2$ have similar graphs.

### Activity

**Step 1** Use the window $-20 \le x \le 20$, and $-20 \le y \le 20$ to graph all three equations on your calculator. Sketch the graphs on a single grid on a separate sheet of paper.

    **a.** $f(x) = 3x^2$     **b.** $g(x) = -x^2$     **c.** $h(x) = -3x^2$

**Step 2** Evaluate $f(2)$, $g(2)$, and $h(2)$.

**Step 3** Is $(-2, -12)$ a point on the graph of $h(x)$? Explain how you know.

**Step 4** Use a graphing calculator to make a sketch of the following functions. Use the same window you used for Step 1.

    **a.** $j(x) = 0.2x^2$     **b.** $k(x) = -0.5x^2$

**Step 5** Evaluate $j(3)$ and $k(3)$.

**Step 6** Is $(-3, 2.9)$ a point on the graph of $j(x)$? Explain how you know.

**Step 7** If a parabola is opening up, what must be true about the value of $a$ in $y = ax^2$? If a parabola is opening down, what must be true about $a$ in $y = ax^2$?

## Properties of the Graph of $y = ax^2$

The graph of $y = ax^2$, where $a \ne 0$, has the following properties:

1.  It is a parabola symmetric to the $y$-axis.

2.  Its vertex is $(0, 0)$.

3.  If $a > 0$, the parabola opens up. If $a < 0$, the parabola opens down.

## Finding Points on the Graph of $y = ax^2$

If you know the $y$-coordinate of a point on the graph of a parabola and the equation, you can find the $x$-coordinate or coordinates. We illustrate this with a different parabola.

### GUIDED

#### Example 1

Consider the following situation: You know the area of a circle and want to find its radius.

*(continued on next page)*

**Solution** Sketch a graph of the familiar formula $A = \pi r^2$, where $A$ is the area of a circle with radius $r$.

**Step 1** Make a scale on each axis. In doing this, ask yourself: What are the possible values of $r$? What are the possible values of $A$?

**Step 2** Make a table of values for $A$ when $r = 1, 2, 3$, and 4. Estimate each to the nearest hundredth. The first value has been done for you.

**Step 3** Graph the points $(r, A)$ from the table.

**Step 4** Put an open circle at $(0, 0)$ because 0 is not in the domain of $r$. Connect $(0, 0)$ and the other points with a curve like a parabola.
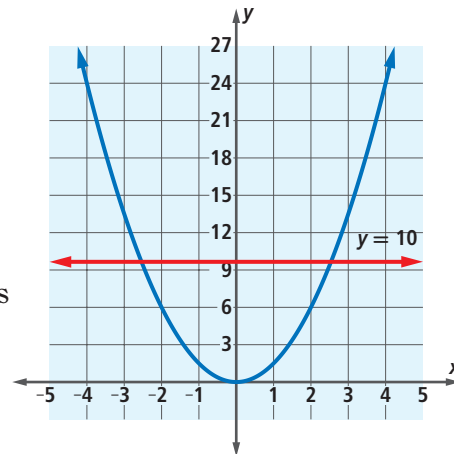
**Step 5** Use your graph to estimate the radius of a circle whose area is 12 square units.

| r | A |
|---|---|
| 1 | 3.14 |
| 2 | ? |
| 3 | ? |
| 4 | ? |

## Example 2

The graph of $f(x) = 1.5x^2$ is shown at the right. Estimate $x$ if $f(x) = 10$.

**Solution** Draw the horizontal line $y = 10$. The graph intersects this line at two points. The $x$-coordinates of these points are approximately **2.5** and **−2.5**.



In the next lesson, you will see how to obtain the exact values of $x$ with $f(x) = 10$.

## Questions

### COVERING THE IDEAS

In 1 and 2, an equation of a function is given.

    a. Make a table of $x$ and $y$ for integer values of $x$ from −4 to 4.

    b. Graph the equation.

    c. Tell whether the graph opens up or down.

1. $g(x) = \frac{1}{2}x^2$            2. $f(x) = -\frac{1}{2}x^2$

3. Refer to the parabola at the right.

    a. Does the parabola open up or down?

    b. The parabola is the graph of a function. Which does the function have, a maximum value or a minimum value?

    c. Give the coordinates of the vertex.

    d. Give an equation of the axis of symmetry of the parabola.

4. How are the graphs of $y = 7x^2$ and $y = -7x^2$ related to each other?
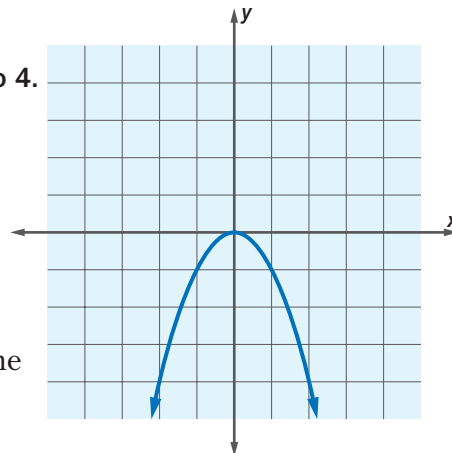
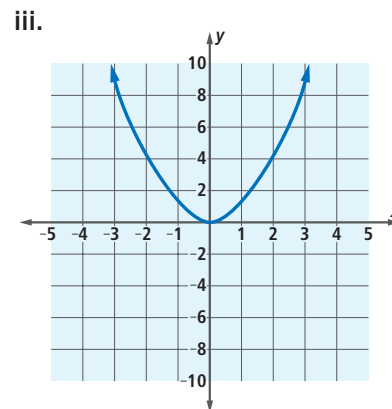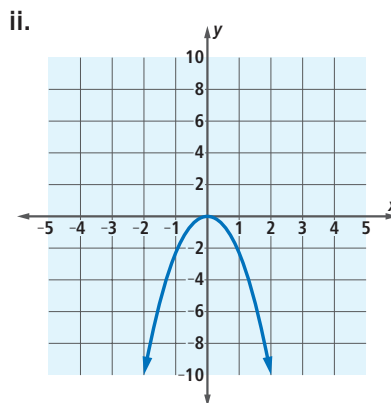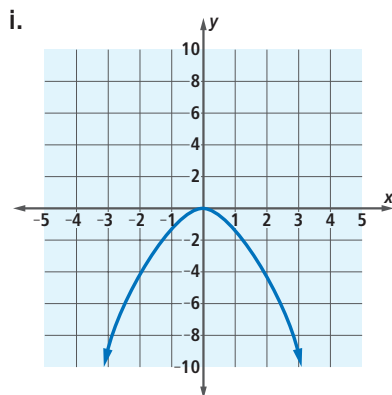5. Match each table with the graph it most accurately represents.

a.

| x | y |
|----|----|
| −2 | 4 |
| 0 | 0 |
| 2 | 4 |

b.

| x | y |
|----|----|
| −2 | −4 |
| 0 | 0 |
| 2 | −4 |

c.

| x | y |
|----|-----|
| −2 | −10 |
| 0 | 0 |
| 2 | −10 |

i.

ii.

iii.

6. Match each graph at the right with one of the equations below.

a. $y = x^2$

b. $y = -0.25x^2$

c. $y = -3x^2$

7. **Fill in the Blanks** Consider the graph of the function $f(x) = ax^2$.

a. If $a$ is positive, the graph is a parabola that opens _____?_____.

b. If $a$ is negative, the graph is a parabola that opens _____?_____.

8. Use the graph of $A = \pi r^2$ to estimate the radius of a circle whose area is 20 square units.

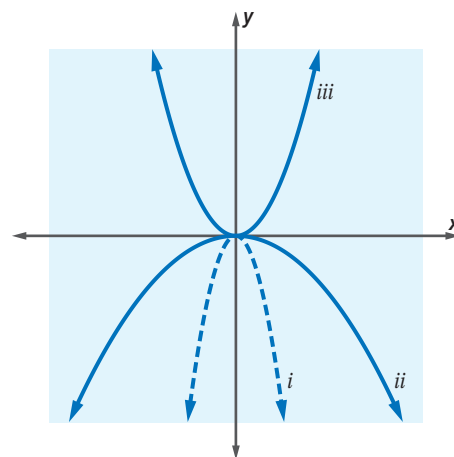9. Consider the graph of the function defined by $y = 5x^2$.

a. Without plotting any points, sketch what you think the graph of this function looks like.

b. Make a table of values satisfying this function. Use $x = -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5,$ and $2$.

c. Draw a graph of this function from your table.

d. From the graph, estimate the values of $x$ for which $y = 14$.

The Function with Equation $y = ax^2$    **529**

10. Consider the formula $A = s^2$ for the area $A$ of a square with a side of length $s$.

    a. Graph all possible values of $s$ and $A$ on a coordinate plane.

    b. Explain how the graph in Part a is like and unlike the graph of $y = x^2$ at the start of this lesson.

11. The parabola at the right has equation $y = -5x^2$.

    a. Find $y$ if $x = 0$.

    b. Find $x$ if $y = -5$.

    c. Find $x$ if $y = -20$.
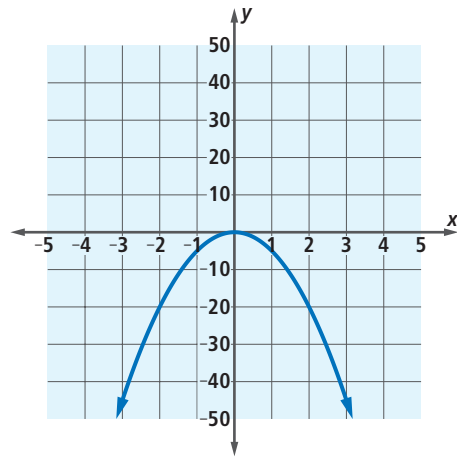
### APPLYING THE MATHEMATICS

12. Refer to the parabola at the right. Points $P$ and $Q$ are reflection images of each other over the $y$-axis. What are the coordinates of $Q$?
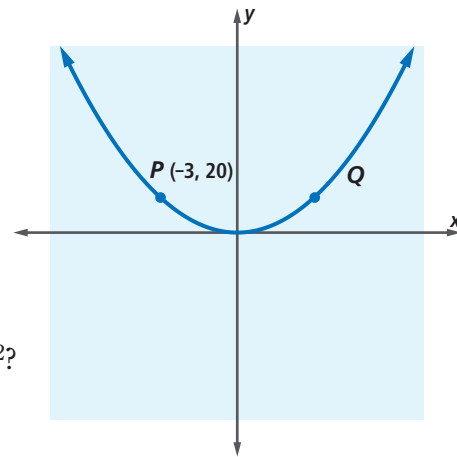
In 13 and 14, fill in the blanks with *negative, zero,* or *positive.*

13. **Fill in the Blanks** Consider the expression $-1x^2$.

    a. If $x$ is negative, $-1x^2$ is ___?___.

    b. If $x$ is zero, $-1x^2$ is ___?___.

    c. If $x$ is positive, $-1x^2$ is ___?___.

    d. What do Parts a–c tell you about the graph of $y = -1x^2$?

14. **Fill in the Blanks** Consider the expression $4x^2$.

    a. If $x$ is negative, $4x^2$ is ___?___.

    b. If $x$ is zero, $4x^2$ is ___?___.

    c. If $x$ is positive, $4x^2$ is ___?___.

    d. What do Parts a–c tell you about the graph of $y = 4x^2$?

15. What is the only real number whose square is not a positive number?

In 16 and 17, a graph of a function $f$ with $f(x) = ax^2$ is shown. Find the value of $a$.

16.

(3, 10)

17.

(−2, −6)
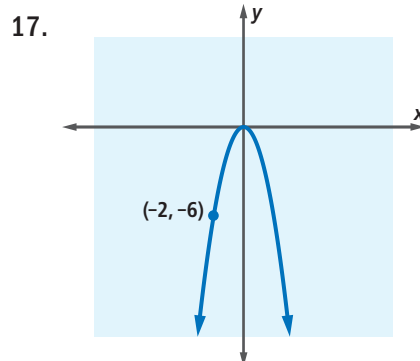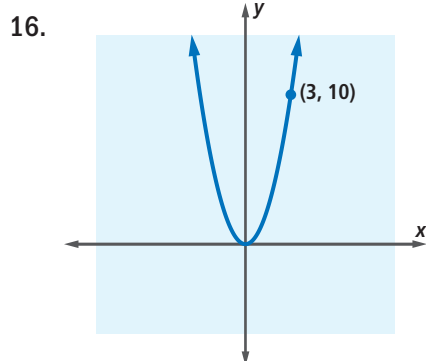
18. **Fill in the Blank** If $a = 0$, the graph of the function $y = ax^2$ is ___?___.

19. Consider the equation $d = 16t^2$, which gives the distance $d$ in feet that an object dropped at time $t = 0$ will have fallen after $t$ seconds.

    a. If an object falls 400 feet in $t$ seconds, find $t$.

    b. Estimate how long it will take an object to fall 200 feet.
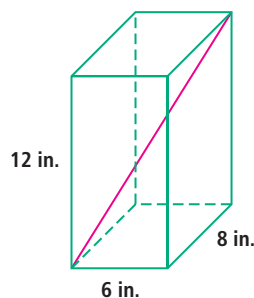
## REVIEW

20. **Skill Sequence** Simplify each expression. (**Lesson 8-7**)

    a. $\sqrt{3} \cdot \sqrt{27}$      b. $\sqrt{3x} \cdot \sqrt{27x}$      c. $\sqrt{3x} + \sqrt{27x}$

21. Suppose a box has sides of length 6 inches, 8 inches, and 12 inches. Find the length of the longest thin pole, like the one shown at the right, which can fit inside the box. (*Hint:* First find the diagonal of the base.) (**Lesson 8-6**)

    12 in.

    8 in.

    6 in.

22. Suppose that $t$ years ago, Kendra deposited $P$ dollars into a savings account with an annual yield of 3%. If she has not deposited or withdrawn any additional money and the account now contains $500, write an equation involving $t$ and $P$. (**Lesson 7-1**)

23. Derek is a tennis instructor at his local gym. He gives lessons to 3 people twice a week. If he charges $25 per person for a lesson, and he works for 15 straight weeks, how much money will Derek earn? (**Lesson 5-4**)

The United States Tennis Association is the largest tennis organization in the world with more than 665,000 individual members and 7,000 organizational members.

Source: USTA

## EXPLORATION

24. Draw a set of axes on graph paper. Aim a lit flashlight at the origin up the *y*-axis. What is the shape of the lit region? Keep the lit end of the flashlight over the origin but tilt the flashlight to raise its bottom. How does the shape of the lit region change?