

Lesson

4-1

Storing Data in Matrices

► **BIG IDEA** A variety of types of data, from numerical information to coordinates of points, can be stored in *matrices*.

A rectangular arrangement of objects or numbers is called a **matrix**. The plural of *matrix* is *matrices*. Each object in a matrix is called an **element** of the matrix. Matrices are useful for storing data of all kinds.

For example, the median salaries of collegiate head coaches for three different sports, based on the highest degree an institution grants, are shown in the matrix below. Entries are in dollars.

	column 1	column 2	column 3	column 4
	↓	↓	↓	↓
	Doctoral	Master's	Bachelor's	Associate's
row 1 → Football	185,000	72,070	62,499	31,314
row 2 → Baseball	72,975	46,654	41,105	44,799
row 3 → Basketball	157,500	63,347	51,092	45,982

Source: Chronicle of Higher Education, March 2006

Dimensions of a Matrix

The elements of the above matrix are enclosed by large square brackets. (Sometimes large parentheses are used in place of brackets.) This matrix has 3 *rows* and 4 *columns*. Because of this, it is said to have the **dimensions** 3 by 4, written 3×4 . In general, a matrix with m rows and n columns has dimensions $m \times n$. Each element of a matrix is identified first by its row location, then by its column location. For example, the element in the 3rd row and 2nd column of this matrix is 63,347. Headings are placed outside the matrix, like the sports and degrees above.

A rectangular block of cells in a spreadsheet also constitutes a matrix. Spreadsheets use the reverse order for identifying an element—column first (a letter) and row second (a number). Like matrices, spreadsheets can have headings to identify their row(s) and column(s).

Vocabulary

matrix
 element
 dimensions
 equal matrices
 point matrix

Mental Math

Find an equation for a line satisfying the conditions.

- slope 4 and y-intercept 2.5
- undefined slope and passing through $(-7, 2)$
- slope $\frac{1}{3}$ and passing through $(0, \frac{9}{10})$
- passing through $(17, 12)$ and $(0.4, 12)$

	A doct...	B mas...	C bac...	D asso...
1	185000	72070	62499	31314
2	72975	46654	41105	44799
3	157500	63347	51092	45982
4				
5				
6				

Many calculators let you enter and manipulate matrices. Use the Guided Example to see how to enter a matrix into a CAS and to store a matrix as a variable.

GUIDED

Example

According to the *Statistical Abstract of the United States*, in 1980, approximately 3.5 million males and 1.9 million females participated in high school athletic programs. Ten years later, 3.4 million males and 1.9 million females participated. In 2000, 3.9 million males and 2.8 million females participated.

- Store the high school athletic participation information in a matrix M .
- What are the dimensions of the matrix?
- Enter the matrix from Part a into a CAS and store it as a variable.

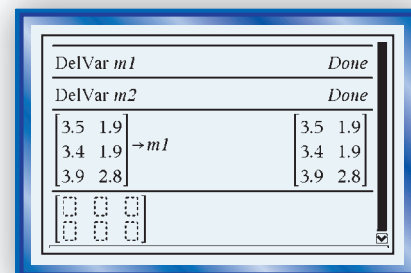
Solution

- You can write either of the two matrices below. Matrix $M1$ has the years as rows, and matrix $M2$ has the years as columns. Either matrix is an acceptable way to store the data.

$$\text{Matrix } M1: \begin{array}{l} 1980 \\ 1990 \\ 2000 \end{array} \begin{bmatrix} \text{Males} & \text{Females} \\ \hline ? & ? \\ \hline ? & ? \\ \hline ? & ? \end{bmatrix}$$

$$\text{Matrix } M2: \begin{array}{l} \text{Males} \\ \text{Females} \end{array} \begin{bmatrix} 1980 & 1990 & 2000 \\ \hline ? & ? & ? \\ \hline ? & ? & ? \end{bmatrix}$$

- Matrix $M1$ has ? rows and ? columns.
The dimensions of $M1$ are ?.
Matrix $M2$ has ? rows and ? columns.
The dimensions of $M2$ are ?.
- Use a CAS. Clear $M1$ or $M2$ before storing your matrix.



Although matrices $M1$ and $M2$ in the Example are both acceptable ways to store and represent the data, the two matrices are not considered equal. Matrices are **equal matrices** if and only if they have the same dimensions *and* their corresponding elements are equal.

Matrices and Geometry

Points and polygons can also be represented by matrices.

The ordered pair (x, y) is generally represented by the matrix $\begin{bmatrix} x \\ y \end{bmatrix}$.

This 2×1 matrix is called a **point matrix**. Notice that the element in the first row is the x -coordinate and the element in the second row is the y -coordinate. For instance, the point $(5, -1)$ is represented by the matrix $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$.

Similarly, polygons can be written as matrices. Each column of the matrix contains the coordinates of a vertex of the polygon in the order in which the polygon is named. The Activity illustrates this.

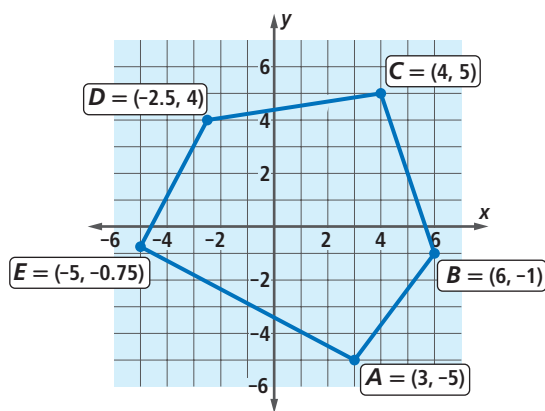
Activity

MATERIALS matrix polygon application

Step 1 Pentagon $ABCDE$ with vertices $A = (3, -5)$, $B = (6, -1)$, $C = (4, 5)$, $D = (-2.5, 4)$, and $E = (-5, -0.75)$, is shown at the right. Write a matrix representing the coordinates of the vertices of the pentagon, starting with point A .

Step 2 Write two other matrices representing the pentagon. (*Hint: Start with a different vertex.*)

Step 3 Verify that your matrices from Step 2 are correct by plotting the pentagon each matrix describes. Each picture should be the same as pentagon $ABCDE$. You can do this by using a matrix polygon application supplied by your teacher.

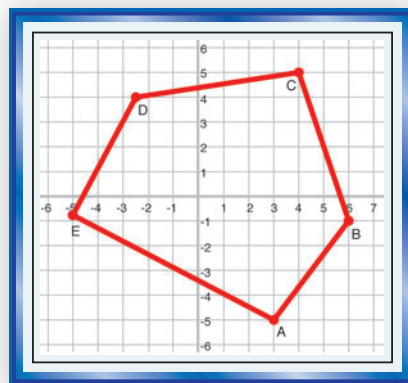


Enter New Polygon

How Many Points?

6	4	-2.5	-5	3
-1	5	4	-0.75	-5

Name:



Step 4 Plot $\begin{bmatrix} B & A & C & D & E \\ 6 & 3 & 4 & -2.5 & -5 \\ -1 & -5 & 5 & 4 & -0.75 \end{bmatrix}$.

Explain why $BACDE$ does not describe a pentagon.

QY

Are the four matrices in the Activity equal? Explain your answer.

Questions

COVERING THE IDEAS

1. What is a matrix?

In 2–4, refer to the matrix regarding coaching salaries at the beginning of the lesson.

2. a. What is the element in row 2, column 3?
b. What does this element represent?
3. Write instructions that someone could use to enter the matrix on your CAS.
4. What would be the dimensions of the matrix if *highest degrees* were rows, and *sports* were columns?
5. Refer to the Example. In 1985, there were 3,344,275 males and 1,807,121 females participating in high school athletic programs. Construct a 2×4 matrix that incorporates this new information with the old.

6. In the fall of 2008, mathematics classes at a local community college had a total enrollment of 2850, compared to 2241 in the fall of 2007. Additionally, English classes had total enrollments of 2620 and 2051, biology classes had enrollments of 1160 and 1572, and psychology classes had enrollments of 740 and 784, all respectively.

- a. Arrange the data into a matrix on a CAS or graphing calculator, representing years as rows.
- b. Now arrange the data into a matrix representing years as columns.

7. a. Write the ordered pair (x, y) as a matrix.
b. What is this matrix called?

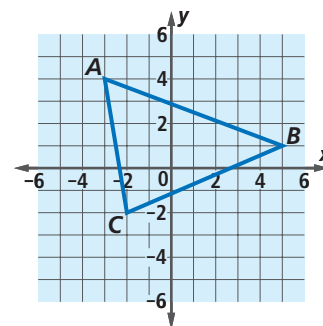
8. **Multiple Choice** Which matrix represents the point $(15\sqrt{2}, -7.3)$?

- | | |
|--|--|
| <p>A $\begin{bmatrix} 15\sqrt{2} & -7.3 \end{bmatrix}$</p> <p>C $\begin{bmatrix} -7.3 \\ 15\sqrt{2} \end{bmatrix}$</p> | <p>B $\begin{bmatrix} -7.3 & 15\sqrt{2} \end{bmatrix}$</p> <p>D $\begin{bmatrix} 15\sqrt{2} \\ -7.3 \end{bmatrix}$</p> |
|--|--|

9. Write $\triangle ABC$ at the right as a matrix.



Many colleges campuses have inner courtyards called quadrangles.



10. Refer to the Activity.
- The matrix at the right uses the same points as the Activity. Use a matrix polygon application to draw (or plot by hand) and connect the points in the matrix. Use the same window as in the Activity. Why is the picture different from pentagon $ABCDE$?
 - How many matrices can represent the pentagon $ABCDE$?
11. The matrix at the right gives the numbers of professional degrees earned in 2000 in four professions, separated by gender.
- What are the dimensions of this matrix?
 - What does the sum of the elements in row 3 represent?
 - What does the sum of the elements in column 2 represent?

$$\begin{matrix} A & B & C & E & D \\ \left[\begin{array}{ccccc} 3 & 6 & 4 & -5 & -2.5 \\ -5 & -1 & 5 & -0.75 & 4 \end{array} \right] \end{matrix}$$

	Males	Females
Medicine	8,759	6,527
Dentistry	2,546	1,704
Law	20,640	17,512
Theology	4,339	1,790

APPLYING THE MATHEMATICS

12. **Fill in the Blanks** If $\begin{bmatrix} -6 & 4.3 \\ \frac{1}{2} & w \end{bmatrix} = \begin{bmatrix} -6 & r \\ \frac{1}{2} & 0.9 \end{bmatrix}$, then $w = \underline{\quad}$ and $r = \underline{\quad}$.

13. **Fill in the Blanks** If $\begin{bmatrix} 2a - 3 \\ h + 0.4 \end{bmatrix} = \begin{bmatrix} -9 \\ \frac{1}{2} \end{bmatrix}$, then $a = \underline{\quad}$ and $h = \underline{\quad}$.

14. Recall on your CAS the matrix $M1$ from the Guided Example.

If $M1 = \begin{bmatrix} x & y \\ z - 1 & y \\ 3w & 2.8 \end{bmatrix}$, find w , x , y , and z .

15. In the English language, the vowels A, E, I, O, and U show up with frequencies among all letters of about 8%, 13%, 7%, 8%, and 3%, respectively. In the board game SCRABBLE®, these letters show up with frequencies 9%, 12%, 9%, 8%, and 4%, respectively.
- Arrange this information into a 2×5 matrix.
 - Explain how to enter this matrix into a CAS.



Many colleges graduations use different colored tassels or sashes to denote individual majors.

16. The endpoints of \overline{PA} on line m are given by the matrix $\begin{bmatrix} \frac{1}{2} & 2 \\ \frac{17}{2} & 13 \end{bmatrix}$.

The endpoints of \overline{LN} on line n are defined by the matrix $\begin{bmatrix} 4 & -1 \\ 8 & -7 \end{bmatrix}$.

Prove that lines m and n are parallel.

17. Use a matrix polygon application to draw (or plot by hand) the octagon $\begin{bmatrix} 0 & 1 & 5 & 1 & 0 & -1 & -5 & -1 \\ 5 & 1 & 0 & -1 & -5 & -1 & 0 & 1 \end{bmatrix}$. Sketch a picture of the output, and explain if the polygon is convex or nonconvex.

REVIEW

18. Evaluate the following expressions. (Lesson 3-9)
- a. $r[\pi] - r[\pi]$ b. $n[10] - n[10]$ c. $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$
d. $[-3.6] - [-3]$ e. $[56.63] - [-56.9]$
f. $[5] - [5.02]$ g. $[-4.5] + [9.7]$
19. Shelby put \$50 on her public transportation card. For every bus or train ride she takes, \$2 is deducted from her total. Express the total left on her card as an explicit formula of an arithmetic sequence dependent on the number of train or bus rides taken. (Lesson 3-8)
20. **Fill in the Blank** $Ax + By = C$ is the ___?___ form of an equation for a line. (Lesson 3-2)
21. If the volume of cube A is 27 times the volume of cube B , how do the lengths of their edges compare? (Lesson 2-3)
22. a. What does the Commutative Property of Addition say?
b. Is subtraction commutative? If so, explain. If not, give a counterexample. (Previous Course)

EXPLORATION

23. What is a dot-matrix printer? How is it related to the matrices discussed in this lesson?

QY ANSWER

No, corresponding elements are not equal.