Chapter 10 Summary and Vocabulary

- A **system** is a set of sentences that together describe a single situation. Situations that lead to linear equations can lead to a **linear system.** All that is needed is that more than one condition must be satisfied. This chapter discusses ways of solving systems in which the sentences are equations or inequalities in two variables.

- The **solution set to a system** is the set of all solutions common to all of the sentences in the system. A solution to a system of two linear equations is an ordered pair (x, y) that satisfies each equation. Systems of two linear equations may have zero, one, or infinitely many solutions. Other systems may have other numbers of solutions.

- One way to solve a system is by graphing. There are as many solutions as intersection points. Graphing is also a way to describe solutions of systems that have infinitely many solutions, for example, systems of **coincident lines** and **systems of linear inequalities,** with overlapping half-planes.

- However, graphing does not always yield exact solutions. In this chapter, four strategies are presented for finding exact solutions to systems of linear equations. 1. Substitution is a good method to use if at least one equation is given in $y = mx + b$ form. 2. Addition is appropriate if the same term has opposite signs in the two equations in the system. 3. Multiplication is a good method when both equations are in $Ax + By = C$ form. Each of these methods changes the system into an equivalent system whose solutions are the same as those of the original system. 4. With **matrices,** the system $\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$ becomes $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$. This matrix equation is of the form $AX = B$, where A is the coefficient matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, X is the variable matrix $\begin{bmatrix} x \\ y \end{bmatrix}$, and B is the constant matrix $\begin{bmatrix} e \\ f \end{bmatrix}$. Multiplying both sides of $AX = B$ by the **multiplicative inverse** A^{-1} of the matrix A results in $X = A^{-1}B$.

Theorems and Properties

Generalized Addition Property of Equality (p. 601)
Slopes and Parallel Lines Property (p. 616)

Vocabulary

10-1
system
solution to a system
empty set, null set

10-4
addition method for solving a system

10-5
equivalent systems
multiplication method for solving a system

10-6
coincident lines

10-7
matrix (matrices)
elements
dimensions
matrix form
2×2 identity matrix

10-8
inverse (of a matrix)

10-10
nonlinear system

Chapter 10 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

In 1–4, solve the system by indicated method.

1. $\begin{cases} y = x - 7 \\ y = 1.5x + 2 \end{cases}$ substitution

2. $\begin{cases} -4d + 9f = 3 \\ 4d - 5f = 9 \end{cases}$ addition

3. $\begin{cases} 7h + 3g = 4 \\ 2h - g = 2 \end{cases}$ multiplication

4. $\begin{cases} 3a - b = 6 \\ \frac{3}{5}b = 37 - 4a \end{cases}$ graphing

5. Solve $\begin{cases} y = x^2 + 3x - 5 \\ y = 6x - 7 \end{cases}$ by using any method.

6. Determine whether the system $\begin{cases} 3s = 2t - 5 \\ \frac{2}{3}t - \frac{1}{3}s = -5 \end{cases}$ has 0, 1, or infinitely many solutions.

In 7 and 8, multiply.

7. $\begin{bmatrix} 2 & 7 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 8 \\ 1 & 7 \end{bmatrix}$

9. Solve the system $\begin{cases} 3p + 5q = 5 \\ p - q = 7 \end{cases}$ using matrices. Use a calculator to find the inverse of the matrix.

10. An electronics store receives two large orders. The first order is for 6 high-definition televisions and 3 DVD players, and totals $6,795. The second order is for 4 high-definition televisions and 4 DVD players, and totals $4,860. What is the cost of one high-definition television and what is the cost of one DVD player?

11. Give values for m, n, c, and d so that the system $\begin{cases} y = mx + c \\ y = nx + d \end{cases}$ has infinitely many solutions.

12. Rosie is buying posies and roses. She wants to spend no more than $40. Roses are $5 each, and posies are $2 each. Accurately graph all combinations of flowers that she can buy.

13. Solve the system $\begin{cases} y = 3x + 50 \\ y = -2x + 70 \end{cases}$ by graphing.

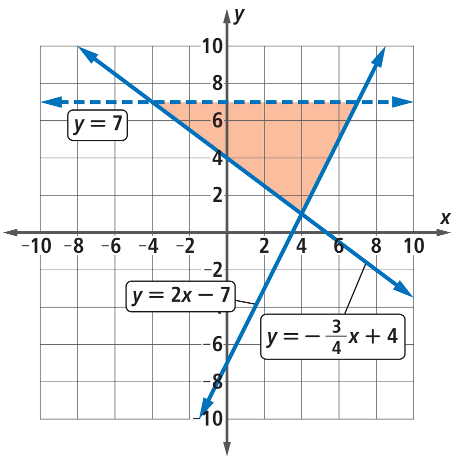
14. A passenger airplane took 2 hours to fly from St. Louis, Missouri, to Orlando, Florida, in the direction of the jet stream. On the return trip against the jet stream, the airplane took 2 hours and 30 minutes. If the distance between the two cities is about 1,000 miles, find the airplane's speed in still air and the speed of the jet stream.

15. Akando bought 80 feet of chicken wire to make a coop on his farm. He needs the coop to be at least 10 feet wide and 15 feet long.
    - **a.** Draw a graph to show all possible dimensions (to the nearest foot) of the coop.
    - **b.** At most, how wide can the coop be?

16. Graph all solutions to the system $\begin{cases} y \le x + 3 \\ y \ge -2x + 4 \end{cases}$.

17. Kele paid for his lunch with 15 coins. He used only quarters and dimes. If his lunch cost \$2.40, how many of each coin did he use?

18. Write a system of inequalities to describe the shaded region below.



Chapter 10 Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A Solve systems using substitution. (Lessons 10-2, 10-3)

In 1 and 2, solve the system by using substitution.

1. $\begin{cases} m = n \\ 3m - 4 = n \end{cases}$

2. $\begin{cases} 2p = q + 5 \\ 2q = 4(p + 2) \end{cases}$

In 3 and 4, two lines have the given equations. Find the point of intersection, if any.

3. Line ℓ: $y = 2x + 5$;
   Line m: $y = -3x + 4$

4. Line p: $y = \frac{2}{3}x + \frac{1}{9}$;
   Line q: $y = \frac{1}{5}x - 4$

OBJECTIVE B Solve systems by addition and multiplication. (Lessons 10-4, 10-5)

In 5 and 6, solve the system by addition.

5. $\begin{cases} 3b + 4 = a \\ -3b - 5 = 2a \end{cases}$

6. $\begin{cases} 0.4x + 0.75y = 2.7 \\ 0.4x - 2y = 2.5 \end{cases}$

In 7–10, solve the system by multiplication.

7. $\begin{cases} 3f + g = 41 \\ f - 2g = 20 \end{cases}$

8. $\begin{cases} 3t - u = 5 \\ 6t + 3u = 7 \end{cases}$

9. $\begin{cases} 5w + 2v = 6 \\ 7 + 5v = w \end{cases}$

10. $\begin{cases} 3x - 5 = y \\ 2x - 7y = 9 \end{cases}$

OBJECTIVE C Multiply 2 × 2 matrices by 2 × 2 or 2 × 1 matrices. (Lesson 10-7)

In 11–14, multiply.

11. $\begin{bmatrix} 2 & -4 \\ 3 & 5 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

12. $\begin{bmatrix} 6 & -0.5 \\ 0.5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 12 \end{bmatrix}$

13. $\begin{bmatrix} -6 & 8 \\ -9 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

14. $\begin{bmatrix} 2 & 11 \\ 0 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & -4 \\ 5 & 9 \end{bmatrix}$

OBJECTIVE D Solve systems using matrices. (Lesson 10-8)

In 15–18, a system is given.

a. Write the system in matrix form.

b. Use technology to find the inverse.

c. Solve the system.

15. $\begin{cases} 3x + 2y = 7 \\ 5x + 7y = 9 \end{cases}$

16. $\begin{cases} 6m + 4d = 7 \\ 4m + 3d = 13 \end{cases}$

17. $\begin{cases} 5p - 7q = 20 \\ 4p - 8q = 14 \end{cases}$

18. $\begin{cases} w + 3z = 5 \\ 4z - 5w = 9 \end{cases}$

OBJECTIVE E **Solve nonlinear systems.** (Lesson 10-10)

In 19 and 20, solve by substitution.

19. $\begin{cases} y = -2x^2 \\ y + x^2 = -1 \end{cases}$

20. $\begin{cases} 2x^3 - 2y = x^2 - 5 \\ y = x^3 - 2x \end{cases}$

PROPERTIES Principles behind the mathematics

OBJECTIVE F **Determine whether a system has 0, 1, or infinitely many solutions.** (Lesson 10-6)

In 21–24, determine whether the given system has 0, 1, or infinitely many solutions.

21. $\begin{cases} 2y + 3x = 5 \\ 2y = 4 - 3x \end{cases}$

22. $\begin{cases} y + 3x = 7 \\ y = 3x + 7 \end{cases}$

23. $\begin{cases} 3p = 7q + 2 \\ 6p = 10q + 5 \end{cases}$

24. $\begin{cases} 4p + 5q = 7 \\ 2p = 3\frac{1}{2} - \frac{5}{2}q \end{cases}$

25. When will the system $\begin{cases} y = mx + a \\ y = mx + b \end{cases}$ have no solution? Explain your answer.

26. Can the given set of points be the intersection of two lines?
  a. exactly one point
  b. exactly two points
  c. infinitely many points
  d. no points

27. **Fill in the Blank** Two lines are parallel only if they have the same __?__.

28. **True or False** Two lines can intersect in more than one point if they have different y-intercepts.

USES Applications of mathematics in real-world situations

OBJECTIVE G **Use systems of linear equations to solve real-world problems.** (Lessons 10-2, 10-3, 10-4, 10-5, 10-6)

29. Austin drove four times as far as Antonio. Together, they drove 350 miles. How far did they each drive?

30. Car A costs $2,800 down and $100 per month. Car B costs $3,100 down and $50 per month. After how many months is the amount paid for the cars equal?

31. Good Job offers $30,000 per year, plus a $1,000 raise each year. Nice Job offers $32,000 per year, plus a $500 raise each year. When will you make more money per year with Good Job?

32. Tickets to see an orchestra cost $30 for adults and $15 for students. One night, the total number of tickets sold was 633. If they sold $15,945 worth of tickets, how many adults and how many students attended?

33. A chemist wishes to mix a 15% acid solution with 30% acid solution to make a 25% acid solution. If the chemist wants to make 8 pints of the solution, how many pints of each solution should the chemist use?

34. From 1990 to 2000, the population of Seattle, Washington, grew at a rate of about 4,700 people per year, to a population of about 565,000. Baltimore, Maryland, decreased at a rate of about 8,400 people per year, to a population of about 650,000. If these rates continue, in about how many years will Seattle and Baltimore have the same population?

OBJECTIVE H **Use systems of linear inequalities to solve real-world problems.** (Lesson 10-9)

35. A chef has 10 ducks and wants to make Peking duck and duck salad. It takes 1 duck to make Peking duck, and 2 ducks to make duck salad. Make a graph to show all the combinations of duck dishes that the chef can make.

36. Suppose 40 students want to play 2 games. If each game must have at least 10 students and at most 25 students, make a graph of the number of ways the students could divide up to play the games.

37. Jackie is running around a rectangular track with a perimeter of at most 500 feet. The track is at least 80 feet wide and 100 feet long.

a. Draw a graph to show all possible dimensions (to the nearest foot) of the track.

b. What is the maximum length of the track?

c. What is the maximum width of the track?

38. Nihad won \$300 in a soccer all-stars contest. She wants to buy soccer balls for \$25 and pairs of soccer shoes for \$30. She wants to buy at least two balls and at least two pairs of shoes.

a. Graph all the combinations she could buy.

b. What is the maximum number of pairs of shoes she can buy?

c. What is the maximum number of balls she can buy?

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE I **Find solutions to systems of equations by graphing.** (Lessons 10-1, 10-6, 10-10)

In 39–43, solve the system by graphing. Round your answers to the nearest tenth.

39. $\begin{cases} 5x + 4y = 7 \\ 3x + 2y = 6 \end{cases}$

40. $\begin{cases} 16x - 16y = 16 \\ \frac{1}{2}x + \frac{1}{2}y = -2 \end{cases}$

41. $\begin{cases} 0.5x - 0.4y = 0.8 \\ x - 1.6 = 0.8y \end{cases}$

42. $\begin{cases} y = 2^x \\ y = 1.32x + 2.67 \end{cases}$

43. $\begin{cases} 2y - 3x = 7 \\ x^2 + y = 15 \end{cases}$

OBJECTIVE J **Graphically represent solutions to systems of linear inequalities.** (Lesson 10-9)

In 44–47, graph all solutions to the system.

44. $\begin{cases} y \leq 3x \\ y \geq 2x + 1 \end{cases}$

45. $\begin{cases} x + 4 > 7 + 2y \\ x - 4 < y + 2 \end{cases}$

46. $\begin{cases} y > -2 \\ x + y < 0 \\ x - y \geq 1 \end{cases}$

47. $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y < 9 \end{cases}$

In 48 and 49, accurately graph the set of points that satisfies the situation.

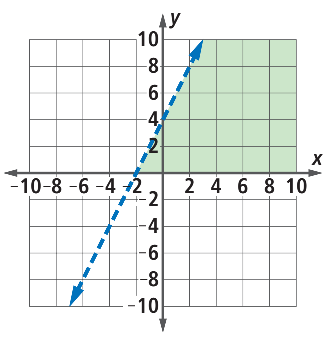
48. An elephant can eat 70 pounds of food in a meal. If the elephant eats P sacks of peanuts averaging 7 pounds each and L bunches of leaves weighing 5 pounds each, how many sacks and bunches can the elephant eat?

49. Conan wants to watch t television shows and m movies. Each show lasts 30 minutes, and each movie lasts 90 minutes. If he has 5 hours of viewing time available, how many full shows and movies can he watch?

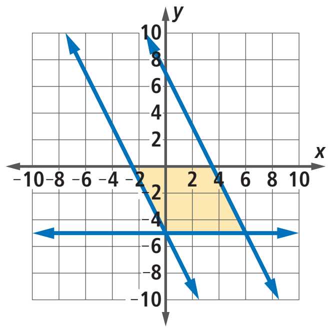
OBJECTIVE K Write a system of inequalities given a graph. (Lesson 10-9)

In 50–52, write a system of inequalities to describe the shaded region.

50.



51.



52.