Lesson

10-10 Nonlinear Systems

BIG IDEA The ideas used to solve systems of linear equations can be applied to solve some systems of nonlinear equations.

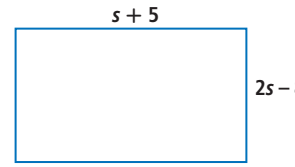
Previously in this chapter you have worked with systems of linear equations. In these systems, every graph involved is a line. In this lesson, we consider systems that involve curves. A **nonlinear system** is a system of equations or inequalities in which at least one of the equations or inequalities is nonlinear.

Vocabulary

nonlinear system

Mental Math

Refer to the rectangle.



a. What is the area of the rectangle?

b. What is the perimeter of the rectangle?

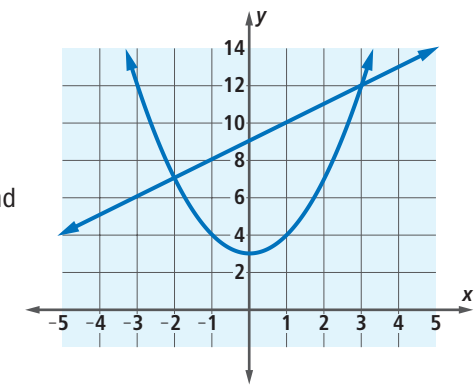
GUIDED

Example 1

Solve the system $\begin{cases} y = x^2 + 3 \\ y = x + 9 \end{cases}$.

Solution Look at the graphs at the right. The solutions occur at the points of intersection. These graphs intersect at (−2, ?) and (3, ?); therefore, the system has two solutions.

The solutions are (?) and (?).



STOP QY

Solving algebraically using substitution is also an option. You may need to use the Quadratic Formula.

Example 2

Solve the system $\begin{cases} y = -2x + 8 \\ y = -x^2 + 4x - 1 \end{cases}$.

Solution Because both expressions in x are equal to y, they are equal to each other at the point of intersection.

$-2x + 8 = -x^2 + 4x - 1$ Substitution

$8 = -x^2 + 6x - 1$ Add $2x$ to both sides.

$0 = -x^2 + 6x - 9$ Subtract 8 from both sides.

$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-9)}}{2(-1)}$ Quadratic Formula

$x = \frac{-6 \pm \sqrt{36 - 36}}{-2} = \frac{-6 \pm 0}{-2}$ or 3

QY

Check your answer to Guided Example 1 by substituting the coordinates of each point into both equations to verify that they are solutions to the system.

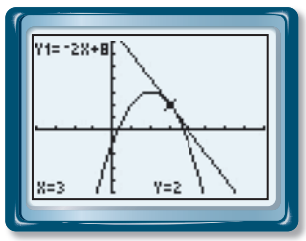
Now find the y-coordinate of the point of intersection.

$y = -2x + 8$

$y = -2(3) + 8 = -6 + 8 = 2$

The solution is (3, 2).

Check The graph reinforces that there is only one solution and that it is at (3, 2), as shown on the screen at the right.



Some nonlinear systems have no solutions.

Example 3

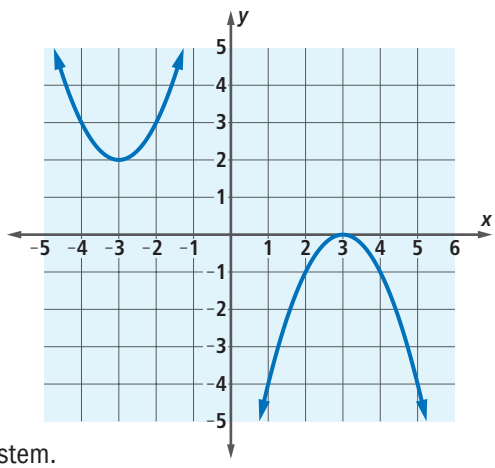
Solve the system $\begin{cases} y = x^2 + 6x + 11 \\ y = -x^2 + 6x - 9 \end{cases}$.

Solution 1 Use substitution.

$x^2 + 6x + 11 = -x^2 + 6x - 9$

$2x^2 + 6x + 11 = 6x - 9$	Add x^2.
$2x^2 + 11 = -9$	Subtract $6x$.
$2x^2 = -20$	Subtract 11.
$x^2 = -10$	Divide by 2.
$x = \pm\sqrt{-10}$	Take the square root.

The solutions $\sqrt{-10}$ and $-\sqrt{-10}$ are not real numbers, so there is no solution for this system in the set of real numbers.



Solution 2 Graph the equations.

The parabolas never intersect, so there is no solution to this system.

When you don't know how to solve a system of equations algebraically, you can use a graphing approach to make very good approximations.

Example 4

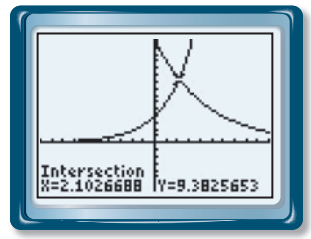
Jonas is working on a science fair project. He wants to study the result of combining equal populations (in weight) of two types of bacteria. The first bacteria (in grams) grow according to the function $f(x) = 15(0.8)^x$ where x = the time from now (in hours). The other bacteria growth (in grams) is modeled by $g(x) = 4(1.5)^x$, where x = the time from now (in hours). If Jonas wants to combine them when the two populations are equal, how long will he have to wait? How many grams of each type of bacteria will there be at that time?

(continued on next page)

Solution This situation can be represented by the following system.

$$\begin{cases} f(x) = 15(0.8)^x \\ g(x) = 4(1.5)^x \end{cases}$$

Graph the two functions on your calculator. Be sure to include any intersections in the viewing window. Use the INTERSECT command to find an accurate approximation for the solution.



Jonas should combine the bacteria in approximately 2.10 hours (or 2 hours and 6 minutes). There will be about 9.4 grams of bacteria in each population at that time.

Questions

COVERING THE IDEAS

In 1 and 2, solve the system using substitution.

1. $\begin{cases} y = 2x^2 + 12x + 17 \\ y = -x^2 - 6x - 10 \end{cases}$

2. $\begin{cases} y = x^2 + 4x + 3 \\ y = 3x + 1 \end{cases}$

3. Solve the system $\begin{cases} y = 3x^2 \\ y = -4x^2 \end{cases}$ by picturing the graph in your head.

In 4–6, solve the system.

4. $\begin{cases} y = x^2 - 6x + 1 \\ y = -x^2 + 4x + 1 \end{cases}$

5. $\begin{cases} p = 9q + 25 \\ p = 2q^2 + 7q + 1 \end{cases}$

6. $\begin{cases} y = x^2 + 3 \\ y = -x + 1 \end{cases}$

In 7 and 8, solve the system by graphing. Round solutions to the nearest hundredth.

7. $\begin{cases} y = 5x^2 + 5 \\ y = 0.5x^2 \end{cases}$

8. $\begin{cases} y = 2.718^x \\ y = 3.14(0.25)^x \end{cases}$

APPLYING THE MATHEMATICS

In 9 and 10, solve the system by graphing.

9. $\begin{cases} y = |x - 3| \\ y = -2x^2 + 12x - 18 \end{cases}$

10. $\begin{cases} y = x^2 \\ y = 2^x \end{cases}$

11. Solve the system of Example 3 by addition.

12. A quarterback passes a football to a receiver downfield. The path of the ball is described by the equation $h = -0.025x^2 + x + 6$, where $x =$ the horizontal distance (in yards) of the ball from the quarterback and h is the height of the ball (in feet) above the ground. The receiver catches the ball 40 yards downfield. A bird flies over the quarterback in the same direction as the ball. Its flight is described by the equation $y = \frac{7}{60}x + 10$, where $x =$ the distance from the quarterback (in yards) and y is the bird's height (in feet).
    a. When the quarterback first threw the ball, how high was it off the ground?
    b. How high was the bird when it was right over the quarterback?
    c. How many times did the path of the bird and the football cross?
    d. Give these points of intersection.
    e. Notice that the time when the bird or the football is moving is not given. Explain why this means the bird and ball might not have hit each other.

Philadelphia Eagles quarterback Donovan McNabb throws a pass against the Tampa Bay Buccaneers in the first quarter of the NFC championship game at Veterans Stadium in Philadelphia, January 19, 2003.

Source: Associated Press

13. During a spring training game in 2001, baseball pitching great Randy Johnson threw a fastball. A bird flew in the ball's path and was accidentally struck by the ball. This is the only time in Major League Baseball that a bird has been struck by a pitch. A possible equation for the height of Randy Johnson's pitch is $h = -5.56t^2 - 1.39t + 4.25$, where t is the time in seconds since he threw the ball and h is the ball's height in feet above the ground. A possible equation for the bird's height is $h = -24.42t^2 + 3.94t + 5.14$, where t is the time in seconds since Randy's pitch and h is the bird's height above the ground in feet.
    a. For the ball and the bird to collide, what must be true of the system $\begin{cases} h = -5.56t^2 - 1.39t + 4.25 \\ h = -24.42t^2 + 3.94t + 5.14 \end{cases}$?
    b. Find how long after the pitch was thrown that it struck the bird.
    c. What was the height of the bird when it got struck by the ball?
    d. Does the graph of the parabola $h = -5.56t^2 - 1.39t + 4.25$ represent the flight of the ball? Explain.

14. Company One's stock values during the month of January can be represented by $V = 40(0.9)^x$, where x is time in days since the beginning of the month. Company Two's stock values can be represented by $V = |2x - 9|$ for the same time period.
    a. Approximate when Companies One and Two had the same stock value and give that value.
    b. Describe how well each company's stock values changed over the month (from $x = 0$ to $x = 31$).
    c. When would have been the best time to buy Company Two's stock? Explain your answer.

REVIEW

15. Solve $\begin{bmatrix} 1 & -2 \\ 3 & 1.5 \end{bmatrix} \cdot \begin{bmatrix} \frac{-3}{2} \\ n \end{bmatrix} = \begin{bmatrix} -11.5 \\ 3 \end{bmatrix}$ for n. **(Lesson 10-8)**

16. Solve $\begin{cases} x - 1.5y = 11 \\ 5x + 9y = 11 \end{cases}$ by any method. **(Lessons 10-5, 10-4, 10-3)**

17. Suppose Rodney bought a car 32 years ago for $8,000. It lost 6% of its value each year for the first 15 years. Then its value stayed the same for 3 years. When it was 18 years old, the car became a collector's item and its value increased 21% each year. Find the value of the car now. **(Lessons 7-3, 7-2)**

18. Graph the solutions to the inequality $-4.5y - 18x + 3 < 12$ on a coordinate grid. **(Lesson 6-9)**

EXPLORATION

19. An equation for a circle with radius 2 centered at the origin is $x^2 + y^2 = 4$. Use your calculator to find a pair of values of m and b so that the number of solutions to the system $\begin{cases} x^2 + y^2 = 4 \\ y = mx + b \end{cases}$ is
    a. 2. b. 1. c. 0.

QY ANSWER

For (−2, 7):
Does $7 = (-2)^2 + 3$? Yes.
Does $7 = -2 + 9$? Yes.

For (3, 12):
Does $12 = 3^2 + 3$? Yes.
Does $12 = 3 + 9$? Yes.
Both solutions check.