## Lesson

10-9 Systems of Inequalities

▶ **BIG IDEA** The graph of the solutions to a system of linear inequalities is a region bounded by lines.

In Lesson 6-9, you graphed linear inequalities like $y < 11$ and $y \geq 4x + 3$ on a coordinate plane. These inequalities describe half-planes. In this lesson, you will see how to graph regions described by a system of two or more inequalities. Solving a system of inequalities involves finding the common solutions of two or more inequalities.
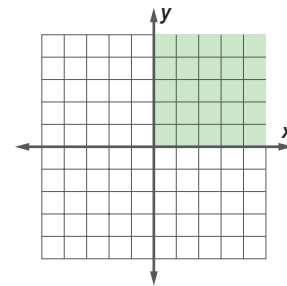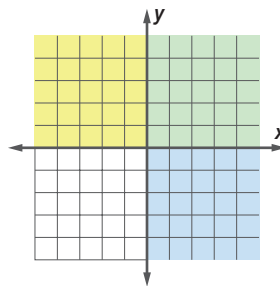
Mental Math

Refer to the figure.
Find *c* if

a. $a = 75°$ and $b = 40°$

b. $a = 75°$ and $b = 39°$

c. $a = b$

Example 1

Graph all solutions to the system $\begin{cases} x \geq 0 \\ y \geq 0 \end{cases}$.

Solution First graph the solution to $x \geq 0$. It is shown at the left below. Then graph the solution $y \geq 0$, shown at the right below.
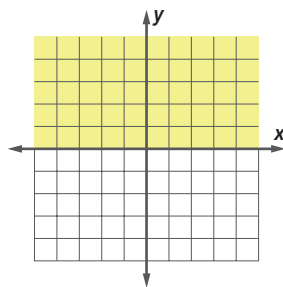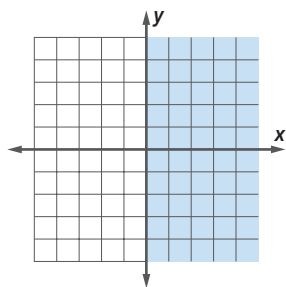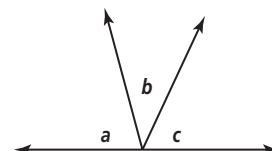

The solution to the system is the set of points common to both of the sets above. At the right we show the solutions to the two inequalities superimposed. At the far right is the solution to the system. The solution is the intersection of the two solution sets above shown in green. It consists of the first quadrant and the nonnegative parts of the *x*- and *y*-axes.

Recall that in general, the graph of $Ax + By < C$ is a half-plane, and that it lies on one side of the boundary line $Ax + By = C$.
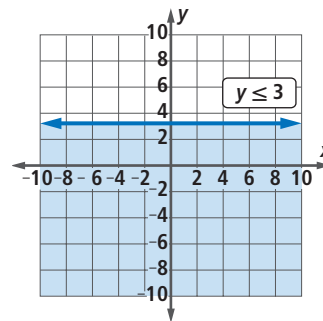
Example 2

Graph all solutions to the system $\begin{cases} y \le 3 \\ x > -2 \\ y > x - 1 \end{cases}$.

Solution The graph of the system is the set of points in common to all three half-planes. So graph all three inequalities.

Inequality 1 Graph $y \le 3$. First graph the boundary line $y = 3$. Graphically, $y \le 3$ consists of all points below (less than) or on (equal to) the solid boundary line. This is shaded in blue at the right.

Inequality 2 Graph $x > -2$. Graph the boundary line $x = -2$. This line is dashed since we want only values greater than -2. $x > -2$ tells us to shade to the right of the line in red where values are greater than -2.
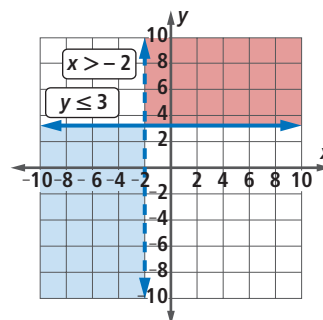
Inequality 3 Graph $y > x - 1$. Graph the dashed boundary line $y = x - 1$ and shade above the line as shown in yellow below.

The solution is the region shown below and includes one of the sides of the triangle. The graph at the far right shows what your final graph should look like.
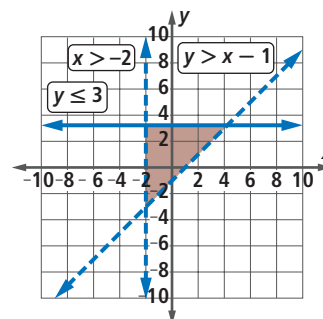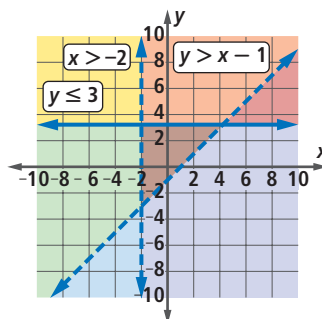
Check A partial check is to choose a point in the intersection region. We choose (1, 2). Substitute the point into all the inequalities.

Is $2 \le 3$? Yes.

Is $1 > -2$? Yes.

Is $2 > 1 - 1$? Yes.

When an intersection point cannot be found easily from a graph, you need to solve a system of equations to find it.

Example 3

Graph all solutions to the system $\begin{cases} y < 2x + 3 \\ y > 2x - 2 \end{cases}$.

Solution

1. The boundary lines $y = 2x + 3$ and $y = 2x - 2$ have the same slope. So they are ____?____ lines.

2. Graph the lines.

3. The solutions to $y < 2x + 3$ comprise the half-plane $\underset{\text{(above/below)}}{\underline{\quad?\quad}}$ the boundary line. The solutions to $y > 2x - 2$ make up the half-plane $\underset{\text{(above/below)}}{\underline{\quad?\quad}}$ the line.

   You saw in Lesson 10-6 that a system whose graph is made up of nonintersecting and parallel lines has no solutions. Is this necessarily true for a system of inequalities whose boundary lines are nonintersecting and parallel? Explain. $\underline{\quad?\quad}$

4. Graph the intersection of the two half-planes in Step 3.

Questions

COVERING THE IDEAS

1. The graph of all solutions to $\begin{cases} x > 0 \\ y < 0 \end{cases}$ consists of all points in which quadrant?

2. Graph the solutions to the system $\begin{cases} x < 0 \\ y > 0 \end{cases}$.

3. Consider the system $\begin{cases} y \le 4x + 1 \\ y > 2x + 1 \end{cases}$.

   a. How is the graph of all solutions to the system related to the graphs of $y \le 4x + 1$ and $y > 2x + 1$?

   b. **Fill in the Blank** The graph of $y > 2x + 1$ is a $\underline{\quad?\quad}$.

   c. Why does the graph of $y \le 4x + 1$ include its boundary line?

   d. Is $(2, 5)$ a solution to this system? How can you tell?

In 4 and 5, graph the solution to the system.

4. $\begin{cases} x > 0 \\ y > 0 \\ 3x + y < 10 \end{cases}$

5. $\begin{cases} x \ge -3 \\ y \le 4 \\ y \ge 2x + 1 \end{cases}$

6. Consider the system $\begin{cases} x > 0 \\ y > 0 \\ x + y < 30 \\ x + y > 20 \end{cases}$.

   a. The graph of all solutions to this system is the interior of a quadrilateral with what vertices?

   b. Name two points that are solutions to the system.

   c. Is $(9, 10)$ a solution to the system? Why or why not?

7. The solution to a system of equations that involves different parallel lines is Ø (empty set). You saw in Example 3 this is not necessarily true of a system of inequalities that involves parallel lines. Is it possible to have a system of inequalities whose solution is Ø? Why or why not?

APPLYING THE MATHEMATICS

In 8 and 9, describe the shaded region with a system of inequalities.

8.

$2x - y = 10$

9.

$3x + 7y = 10$
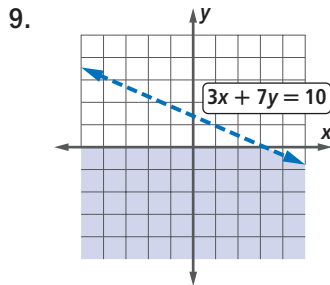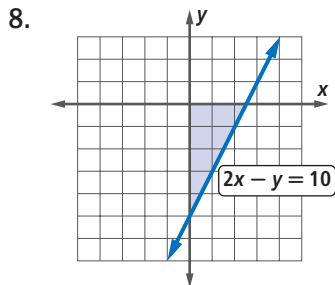
10. Write the system of inequalities whose solution is the interior of the rectangle with vertices $(-3, 10)$, $(5, 10)$, $(5, -1)$, and $(-3, -1)$.

11. It takes Tippie about 10 minutes to type a letter of moderate length and about 8 minutes to type a normal double-spaced page.
    a. Write a system of inequalities that describes the total number of letters L and pages P Tippie can type in an hour or less. Assume the number of letters typed is greater than or equal to zero, as is the number of pages.
    b. Accurately graph the set of points that satisfies the system.

12. An actress is paid $300 per day to understudy a part and $750 per day to perform the role before an audience. During one run, an actress earned between $4,000 and $7,000.
    a. At most, how many times might she have performed the role?
    b. What is the maximum number of times she might have been an understudy?
    c. Graph all possible ways she might have earned her salary.

13. A hockey team is scheduled to play 14 games during a season. Its coach estimates that it needs at least 20 points to make the playoffs. A win is worth 2 points and a tie is worth 1 point.
    a. Make a graph of all the combinations of wins w and ties t that will get the team into the playoffs.
    b. How many ways are there for the team to make the playoffs?

Actors' Equity Association, founded in 1913, is the labor union that represents more than 45,000 actors and stage managers in the United States.

Source: Actors' Equity Association

REVIEW

In 14 and 15, a system of equations is given.
   a. Write the system in matrix form.
   b. Use technology to find the inverse of the coefficient matrix.
   c. Solve the system. (Lesson 10-8)

14. $\begin{cases} y = 3x - 8 \\ 4x + 4y = 12 \end{cases}$ **15.** $\begin{cases} 6x + 4y = 14 \\ -2x - 3y = -18 \end{cases}$

16. Let ℓ be the line with equation $y = -4x + 9$. Write an equation of a line that (Lesson 10-6)

   a. coincides with ℓ.   **b.** does not intersect ℓ.

17. Solve the system $\begin{cases} 2.5x + y = 6 \\ 9x + 4y = 17.5 \end{cases}$ using any method.

   (Lessons 10-5, 10-4, 10-2, 10-1)

In 18–20, use the formula $h = -16t^2 + 94t + 2$ for the height h in feet of a model rocket t seconds after being fired straight up from a stand 2 feet off the ground. (Lessons 9-5, 9-4, 9-3)

18. a. Graph the equation.
   b. Use the graph to find the maximum height of the rocket.

19. a. Find the height of the rocket after 6 seconds.
   b. Interpret your answer to Part a.

20. Use the Quadratic Formula to calculate at what time(s) the rocket is at a height of 100 feet.

EXPLORATION

21. Find a system of linear inequalities whose solution on a coordinate plane is a region like the one shown at the right.