Lesson

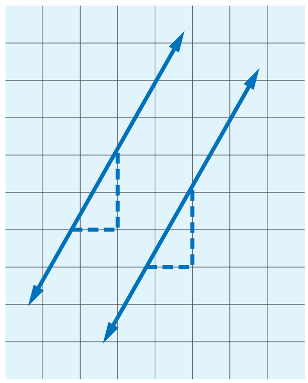
10-6 Systems and Parallel Lines

Vocabulary

coincident lines

BIG IDEA Systems having 0, 1, or infinitely many solutions correspond to lines having 0 or 1 point of intersection, or being coincident.

The idea behind parallel lines is that they "go in the same direction." So we call two lines parallel if and only if they are in the same plane and either are the same line or do not intersect. All vertical lines are parallel to each other. So are all horizontal lines. But not all oblique lines are parallel. For oblique lines to be parallel, they must have the same slope.



Mental Math

Evaluate.

a. $\frac{7 \cdot 6 \cdot 5}{3!}$

b. $\frac{9!}{9 \cdot 8 \cdot 7 \cdot 6}$

Slopes and Parallel Lines Property

If two lines have the same slope, then they are parallel.

Nonintersecting Parallel Lines

You have learned that when two lines intersect in exactly one point, the coordinates of the point of intersection can be found by solving a system. But what happens when the lines are parallel? Consider this linear system.

$$\begin{cases} 5x - 2y = 11 \\ 15x - 6y = -25 \end{cases}$$

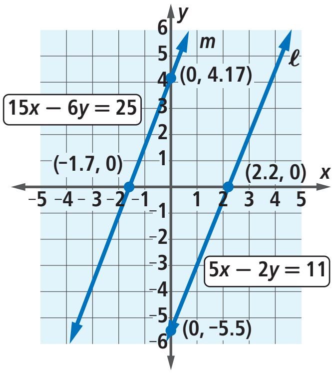
You can solve the system by multiplying the first equation by –3, and adding the result to the second equation.

$$\begin{array}{r} -15x + 6y = -33 \\ + \ 15x - 6y = -25 \\ \hline \end{array}$$

Notice that when you add you get 0 = –58.

This is impossible! When an equation with no solution (such as 0 = –58) results from correct applications of the addition and multiplication methods on a system of linear equations, the original conditions must also be impossible. There are no pairs of numbers that work in *both* equations.

Thus, the system has no solutions. The lines do not intersect. The graph of the system is two parallel nonintersecting lines, as shown at the right. As another check, rewrite the equations for the lines in slope-intercept form.



line ℓ:	line m:
$5x - 2y = 11$	$15x - 6y = -25$
$-2y = -5x + 11$	$-6y = -15x - 25$
$y = 2.5x - 5.5$	$y = 2.5x + 4.1\overline{6}$

Both lines ℓ and m have the same slope of 2.5, but different y-intercepts. Thus, they are parallel.

STOP **QY1**

> **QY1**
>
> Show that the system $\begin{cases} -4x + 2y = -16 \\ 6x - 3y = 18 \end{cases}$ has no solution.

Coincident Lines

Some systems have infinitely many solutions. They can be solved using any of the techniques you have studied in this chapter.

Example 1

Solve the system $\begin{cases} 4x + 2y = 6 \\ y = -2x + 3 \end{cases}$.

Solution 1 Rewrite the first equation in slope-intercept form.

$4x + 2y = 6$

$2y = -4x + 6$ Add $-4x$ to each side.

$y = -2x + 3$ Divide each side by 2.

Notice that this equation is identical to the second equation in the system. So, any ordered pair that is a solution to one equation is also a solution to the other equation. The graphs of the two equations are the same line.

Solution 2 Use substitution. Substitute $-2x + 3$ for y in the first equation.

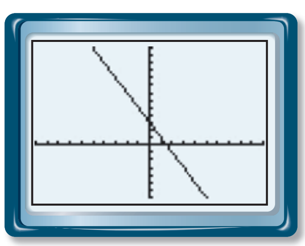
$4x + 2(-2x + 3) = 6$

$4x - 4x + 6 = 6$

$6 = 6$

The sentence $6 = 6$ is *always* true. So, any ordered pair that is a solution to one equation is also a solution to the other equation in the system. The graphs of the two equations are the same line.

The solution set consists of all ordered pairs on the line with equation $y = -2x + 3$, as shown at the right.



$-10 \le x \le 10, -7 \le y \le 13$

(continued on next page)

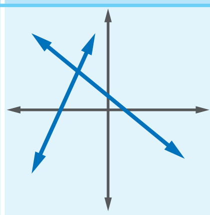
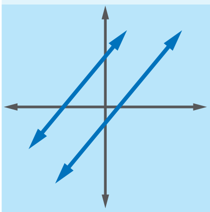
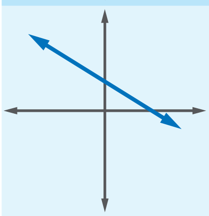
Check As a partial check, find an ordered pair that satisfies one of the equations of the original system. We use $y = -2x + 3$ to find the ordered pair (0, 3). Check that this ordered pair also satisfies the other equation.

Does $4 \cdot 0 + 2 \cdot 3 = 6$? Yes, $6 = 6$.

STOP QY2

Whenever a sentence that is always true (such as $0 = 0$ or $6 = 6$) occurs from correct work with a system of linear equations, the system has infinitely many solutions. We say that the lines *coincide* and that the graph of the system is two **coincident lines.**

You have now studied all the ways that two lines in the plane can be related, and all the types of solutions a system of two linear equations might have. The table below summarizes these relationships.

Description of System	Graph	Number of Solutions to System	Slopes of Lines
Two intersecting lines		1 (the point of intersection)	Different
Two parallel and nonintersecting lines		0	Equal
One line (parallel and coincident lines)		Infinitely many	Equal

QY2

Find a second ordered pair that satisfies one of the equations in Example 1 and show that it satisfies the other equation.

GUIDED

Example 2

Find all solutions to $\begin{cases} 12x - 10y = 2 \\ -18x + 15y = -3 \end{cases}$.

Solution

Step 1 Multiply both sides of the first equation by 3. __?__

Step 2 Multiply both sides of the second equation by 2. __?__

Step 3 Add the equations from Steps 1 and 2. __?__

The solution set consists of all ordered pairs on the line with equation $12x - 10y = 2$.

Questions

COVERING THE IDEAS

1. What is true about the slopes of parallel lines?

2. Which two lines among Parts a–d are parallel?
   a. $y = 8x + 500$
   b. $y = 2x + 500$
   c. $y = 8x + 600$
   d. $x = 2y + 500$

3. a. Graph the line with equation $y = \frac{1}{3}x + 5$.
   b. Draw the line parallel to it through the origin.
   c. What is an equation of the line you drew in Part b?

4. Give an example of a system with two nonintersecting lines.

5. Give an example of a system with two coincident lines.

In 6 and 7, a system is given.
a. Determine whether the system includes *nonintersecting* or *coincident* lines.
b. Check your answer to Part a by graphing.

6. $\begin{cases} 12a = 6b - 3 \\ 4a - 2b = -3 \end{cases}$

7. $\begin{cases} y - x = 5 \\ 3y - 3x = 15 \end{cases}$

8. **Matching** Match the description of the graph with the number of solutions to the system.
   a. lines intersect in one point
   b. lines do not intersect
   c. lines coincide
   i. no solution
   ii. infinitely many solutions
   iii. one solution

APPLYING THE MATHEMATICS

9. a. How many pairs of numbers M and N satisfy both conditions i and ii below?
      i. The sum of the numbers is –2.
      ii. The average of the numbers is 1.
   b. Explain your answer to Part a.

10. Could the situation described here have happened? Justify your answer by using a system of equations. A pizza parlor sold 36 pizzas and 21 gallons of soda for \$456. The next day, at the same prices, they sold 48 pizzas and 28 gallons of soda for \$608.

In 11–14, describe the graph of the system as two intersecting lines, two parallel nonintersecting lines, or coincident lines.

11. $\begin{cases} a = b \\ b - a = 0 \end{cases}$

12. $\begin{cases} y = 5 - 3x \\ 6x + 2y - 10 = 0 \end{cases}$

13. $\begin{cases} 10x + 20y = 30 \\ y + 2x = 3 \end{cases}$

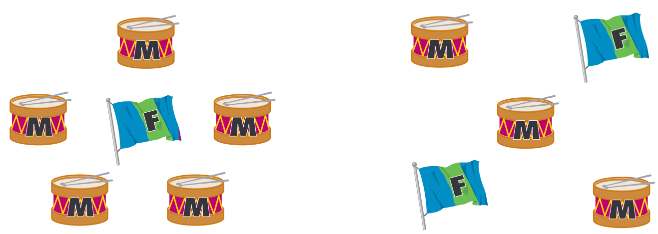
14. $\begin{cases} \frac{4}{5}c - \frac{3}{5}d = 3.6 \\ 8c = 6d + 72 \end{cases}$

15. Melissa is the costume manager for a theater company and is supposed to receive a 15% professional discount from a fabric store. Last week, she bought 40 yards of a red material and 35 yards of a black fabric and paid $435.63. Two friends of hers went to the same store the following week. One bought 20 yards of red material and 10 yards of black fabric for $185, and the other bought 15 yards of red material and 40 yards of black fabric for $447.50. Did Melissa receive a discount? If so, was it the correct percentage?

A dancer is performing in a stage production.

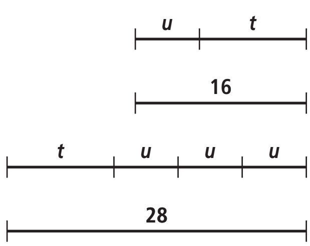
REVIEW

16. A band has 59 musicians (M) with an additional 24 flag bearers (F). They plan to form pentagons and squares with one person in the middle, as shown below.



Let p be the number of pentagons formed and s be the number of squares formed. Can all musicians and flag bearers be accommodated into these formations? Why or why not? **(Lesson 10-5)**

17. Each diagram at the right represents an equation involving lengths t and u. **(Lesson 10-4)**



a. Write a pair of equations describing these relationships.

b. Use either your equations or the diagrams to find the lengths of t and u. Explain your reasoning.

c. Check your work.

18. Find the x-intercepts of the graph of $y = x^2 + 9x - 5$ using each method. **(Lessons 9-5, 9-3)**

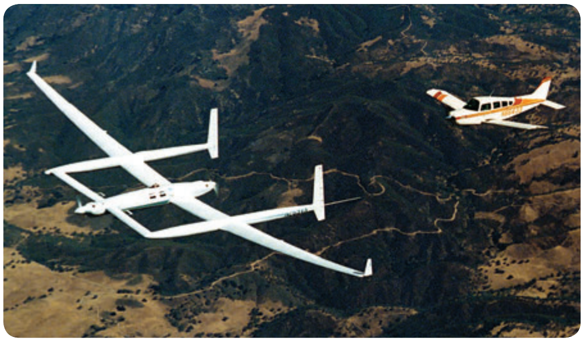
a. Let $y = 0$ and use the Quadratic Formula.

b. Use a graphing calculator and zoom in on the intercepts.

19. a. Simplify $\sqrt{d^8 + 3d^8}$.

b. Check your answer from Part a by testing the special case where $d = 2$. **(Lesson 8-9)**

20. In December of 1986, Dick Rutan and Jeana Yeager flew the *Voyager* airplane nonstop around Earth without refueling, the first flight of its kind. The average rate for the 24,987-mile trip was 116 mph. How many days long was this flight? **(Lesson 5-3)**



A chase plane follows the *Voyager* as it flies over Southern California.

21. a. Identify an equation of the vertical line through the point (–6, 18).

b. Identify an equation of the horizontal line through the point (–6, 18). **(Lesson 4-2)**

EXPLORATION

22. Consider the general system $\begin{cases} ax + by = e \\ cx + dy = f \end{cases}$ of two linear equations in two variables, x and y.

a. Find y by multiplying the first equation by c and the second equation by $-a$ and then adding.

b. Find x by multiplying the first equation by d and the second equation by $-b$ and then adding.

c. Your answers to Parts a and b should be fractions. Write them with the denominator $ad - bc$ if they are not already in that form.

d. Use a CAS to solve this system and compare the CAS solution with what you found by hand.

QY ANSWERS

1. Multiply the top equation by 3 and the bottom equation by 2, then add the equations. You should get $0 = -48 + 36 = -12$. This is never true, so the system has no solution.

2. Answers vary. Sample answer: (1, 1) satisfies $4x + 2y = 6$. Since $-2(1) + 3 = 1$, (1, 1) also satisfies $y = -2x + 3$.