

Lesson

10-3

More Uses of
Substitution

► **BIG IDEA** Substitution is a reasonable method to solve systems whenever you can easily solve for one variable in an equation.

In the previous lesson, you saw how to use substitution as a technique to solve systems of equations when the same variable was alone on one side of each equation. Substitution may also be used in other situations. Here is a typical situation that lends itself to substitution.

Example 1

A grandfather likes to play guessing games with his grandchildren. One day he tells them, “I have only dimes and quarters in my pocket. They are worth \$3.85. I have 14 fewer quarters than dimes. How many of each coin do I have?” Use a system of equations to answer his question.

Solution Translate each condition into an equation. Let D equal the number of dimes and Q equal the number of quarters. Dimes are worth \$0.10 each, so D dimes are worth $0.10D$. Quarters are worth \$0.25 each, so Q quarters are worth $0.25Q$. The total value of all the coins is \$3.85. So $0.10D + 0.25Q = 3.85$.

There are 14 fewer quarters than dimes. That leads to a second equation $Q = D - 14$. Together the two equations form a system.

$$\begin{cases} 0.10D + 0.25Q = 3.85 \\ Q = D - 14 \end{cases}$$

Because $Q = D - 14$, substitute $D - 14$ for Q in the first equation.

$$0.10D + 0.25(D - 14) = 3.85 \quad \text{Substitution}$$

$$0.10D + 0.25D - 0.25(14) = 3.85 \quad \text{Distributive Property}$$

$$0.10D + 0.25D - 3.5 = 3.85 \quad \text{Arithmetic}$$

$$0.35D - 3.5 = 3.85 \quad \text{Collect like terms.}$$

$$0.35D = 7.35 \quad \text{Add 3.5 to both sides.}$$

$$D = 21 \quad \text{Divide both sides by 0.35.}$$

Mental Math

Estimate between two consecutive integers.

- $\sqrt{26}$
- $\sqrt{171}$
- $-\sqrt{171}$



a grandfather posing with his three grandchildren

To find Q , substitute 21 for D in either equation. We use the second equation because it is solved for Q . When $D = 21$, $Q = D - 14 = 21 - 14 = 7$. So $(D, Q) = (21, 7)$. The grandfather has 21 dimes and 7 quarters.

Check The 21 dimes are worth \$2.10 and the 7 quarters are worth \$1.75.

$$\$2.10 + \$1.75 = \$3.85$$

In Chapter 6, you used the slope and y -intercept to find an equation of a line that passes through two given points. A different method for finding an equation through two points makes use of a system of equations.

Example 2

Find an equation of the line that passes through the points $(3, 26)$ and $(-2, 1)$.

Solution In slope-intercept form, the equation of the line is $y = mx + b$. If the values of m and b were known, each point on the line would make the equation true.

Substitute the coordinates of each given point for x and y to get two equations.

$$\text{Using } (3, 26), 26 = m \cdot 3 + b.$$

$$\text{Using } (-2, 1), 1 = m \cdot -2 + b.$$

$$\text{This gives the system } \begin{cases} 26 = 3m + b \\ 1 = -2m + b \end{cases}$$

Either equation can be solved for b . From the second equation, $b = 2m + 1$. Now substitute $2m + 1$ for b in the first equation.

$$26 = 3m + (2m + 1)$$

$$26 = 5m + 1$$

$$25 = 5m$$

$$5 = m$$

This is the slope of the line. To find b , substitute 5 for m in either of the original equations. We use the second equation.

$$1 = -2m + b$$

$$1 = -2 \cdot 5 + b$$

$$1 = -10 + b$$

$$11 = b$$

This is the y -intercept. Thus, an equation of the line through $(3, 26)$ and $(-2, 1)$ is $y = 5x + 11$.

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Check Does each ordered pair satisfy the equation of the line?

Does $26 = 5 \cdot 3 + 11$? *Yes, $26 = 15 + 11$.*

Does $(3, 26)$ satisfy $y = 5x + 11$? *Yes, $(3, 26)$ is on the line.*

Does $1 = 5 \cdot -2 + 11$? *Yes, $1 = -10 + 11$.*

Does $(-2, 1)$ satisfy $y = 5x + 11$? *Yes, $(-2, 1)$ is on the line.*

Some situations have been around for generations. Example 3 is taken from an 1881 algebra text; the prices are out of date, but the situation is not.

GUIDED

Example 3

A farmer purchased 100 acres of land for \$2,450. He paid \$20 per acre for part of it and \$30 per acre for the rest. How many acres were there in each part?

Solution You want to find two amounts, so use two variables.

Let x = the number of acres at \$20/acre,
and y = the number of acres at \$30/acre.

The farmer purchased a total of 100 acres, so $x + y = 100$.

The total cost is \$2,450. So $20x + 30y = 2,450$.

Solve the system of these two equations.

$$\begin{cases} x + y = 100 \\ 20x + 30y = 2,450 \end{cases}$$

Although neither equation is solved for a variable, the first equation is equivalent to $y = \underline{\quad? \quad}$

$$20x + 30(\underline{\quad? \quad}) = 2,450 \quad \text{Substitute } \underline{\quad? \quad} \text{ for } y \text{ in the second equation.}$$

$$20x + \underline{\quad? \quad} - \underline{\quad? \quad} = 2,450 \quad \text{Distributive Property}$$

$$3,000 - \underline{\quad? \quad} = 2,450 \quad \text{Collect like terms.}$$

$$\underline{\quad? \quad} = -550 \quad \text{Add } -3,000 \text{ to both sides.}$$

$$x = 55 \quad \text{Divide both sides by } -10.$$

To find y , substitute 55 for x in either of the original equations. We use the first equation because it is simpler.

$$\begin{aligned} x + y &= 100 \\ 55 + y &= 100 \\ y &= 45 \end{aligned}$$

The farmer bought 55 acres at \$20/acre, and 45 acres at \$30/acre.

Check Substitute 55 for x and 45 for y into the second equation.

Does $20x + 30y = 2,450$?

Yes, $20(55) + 30(45) = 1,100 + 1,350 = 2,450$.

A CAS can be used to solve systems of equations even when neither equation has an isolated variable.

Example 4

Use a CAS to solve $\begin{cases} y = \frac{2}{3}x + \frac{10}{3} \\ 5x + 2y = 32 \end{cases}$.

Solution Since the first equation is already solved for y , simply substitute that expression for y into the second equation and solve the second equation for x . To do that quickly on a CAS, you can use the SOLVE command.

Step 1 Find the SOLVE command on your calculator and place it on the entry line. On some calculators you may find it in the Algebra menu.

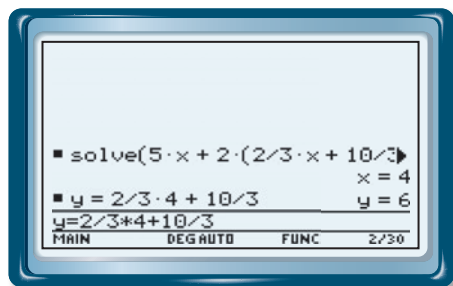
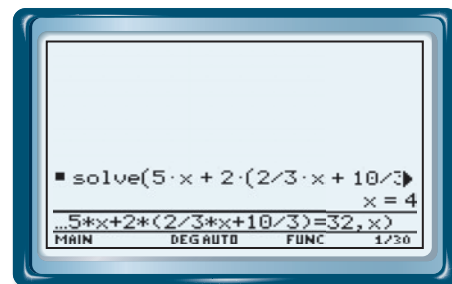
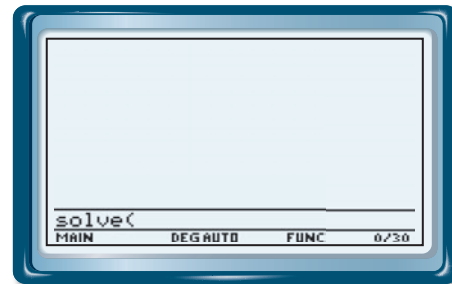
Step 2 Enter the second equation with $\left(\frac{2}{3}x + \frac{10}{3}\right)$ in place of y as you would if you were solving the equation by hand.

That is, enter

$$5x + 2\left(\frac{2}{3}x + \frac{10}{3}\right) = 32.$$

Step 3 The SOLVE command on many CAS requires you to specify the variable for which to solve. In this case it is x . You may need to enter a comma followed by x before you close the parentheses and hit $\boxed{\text{ENTER}}$. You should get $x = 4$.

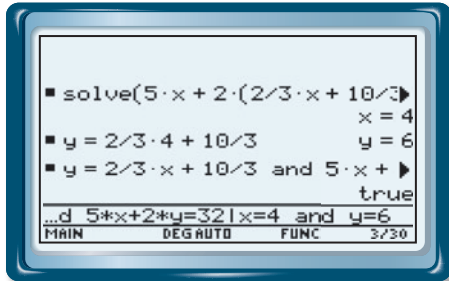
Step 4 Now substitute this x value into the first equation to get $y = 6$.



So, the solution to the system is $(4, 6)$.

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Check Determining if a point is a solution to a system is easy to do on a CAS. Enter the two original equations and the specific values for each variable. The symbol “|” on one CAS means “with” or “such that” and indicates these values. The response “true” indicates that (4, 6) checks.



STOP QY

QY

Below is the solution to a system found by using a CAS.

$$\text{solve}(4 \cdot x + 3 \cdot (2 - 5 \cdot x) = 17, x)$$

$$x = -1$$

$$y = 2 - 5 \cdot -1$$

$$y = 7$$

- Find the system of equations.
- Check the solution by hand.

Questions

COVERING THE IDEAS

- The owners of a carnival have found that twice as many children as adults come to the carnival. Solve a system to estimate the number of children and the number of adults at the carnival when 3,570 people attend.
- A jar of coins has only nickels and quarters, which are worth a total of \$9.40. There are 4 more quarters than nickels. How many nickels and quarters are in the jar?
- The Drama Club and Service Club held a charity car wash. There were four times as many Service Club members as Drama Club members working, so the Service Club earned four times as much money. The car wash raised \$280 in all. How much did each club earn for their charity?

In 4–7, a system is given.

a. Solve each system of equations by substitution.

b. Check your answer.

$$4. \begin{cases} y = 2x \\ 3x + 2y = 21 \end{cases}$$

$$5. \begin{cases} n + 5w = 6 \\ n = -8w \end{cases}$$

$$6. \begin{cases} a - b = 2 \\ a + 5b = 20 \end{cases}$$

$$7. \begin{cases} y = x - 1 \\ 4x - y = 19 \end{cases}$$

- Here is another problem from the 1881 algebra textbook. A farmer bought 100 acres of land, part at \$37 per acre and part at \$45 per acre, at a total cost of \$4,220. How much land was there in each part?



Americans gave a total of \$260.28 billion in contributions to charities in 2005.

Source: Giving USA Foundation

APPLYING THE MATHEMATICS

In 9 and 10,

- solve each system by substitution.
- check your answer by graphing the system.

9.
$$\begin{cases} x + y = 8 \\ y = -3x \end{cases}$$

10.
$$\begin{cases} x = 2y - 10 \\ 5x + 3 = 15 \end{cases}$$

11. Solve the system $\begin{cases} b = -1.36a + 4.4 \\ 1.2a + 4.58b = -181 \end{cases}$ using a CAS.
12. Angles P and Q are complementary. If $m\angle P = 10x$ and $m\angle Q = 15x$, find x , $m\angle P$, and $m\angle Q$.
13. A business made \$120,000 more this year than it did last year. This was an increase of 16% over last year's earnings. If T and L are the earnings (in dollars) for this year and last year, respectively, then $\begin{cases} T = L + 120,000 \\ T = 1.16L \end{cases}$. Find the profits for this year and last year.
14. Mrs. Rodriguez leaves money to her two favorite charities in her will. Charity A is to get 2.5 times as much money as Charity B. The total amount of money donated in the will is \$28,000.
- Write a system of equations describing this situation.
 - Solve to find the amount of money each charity will get.
15. Anica received her results for mathematics and verbal achievement tests. Her mathematics score is 40 points higher than her verbal score. Her total score for the two parts is 1,230.
- Let v = Anica's verbal score, and m = her mathematics score. Write a system of equations for this situation.
 - Find Anica's two scores.

REVIEW

In 16 and 17 solve the system and check. (Lessons 10-2, 10-1)

16.
$$\begin{cases} y = x - 5 \\ y = -4x + 10 \end{cases}$$

17.
$$\begin{cases} y = 6x + 6 \\ y = 6x - 2 \end{cases}$$

18. One hot-air balloon takes off from Albuquerque, New Mexico, and rises at a rate of 110 feet per minute. At the same time, another balloon takes off from Santa Fe, New Mexico, and rises at a rate of 80 feet per minute. The altitude of Albuquerque is 4,958 feet and the altitude of Santa Fe is 6,950 feet. (Lesson 10-2)
- When are the two balloons at the same altitude?
 - What is their altitude at that time?
19. If $a^3b^{-4}c$ is equal to the reciprocal of $\frac{a^{-5}b^2c^3}{a^{-2}b^xc^4}$, find x . (Lessons 8-4, 8-3)



Hot-air balloons hold from 19,000 to 211,000 cubic feet of air and are from 50 to 90 feet tall.

Source: hotairballoons.com

In 20 and 21, graph the solution set

- on a number line.
- in the coordinate plane. (Lessons 6-9, 3-6)

20. $x < 6$

21. $-4y + 2 < 6$

22. In 2002, India ended its Police Pigeon Service. This is a system in which trained pigeons transport messages. The service was used when traditional communication broke down during natural disasters. Suppose a trained pigeon flies 41.3 mph in still air. (Lesson 5-3)
- How far can it fly in m minutes in still air?
 - How fast can it fly *with* the wind if the wind speed is s mph?
 - How fast can it fly *against* the wind if the wind speed is s mph?
 - If the pigeon is flying down a highway that has a speed limit of 65 mph and there is a 21.9 mph tailwind, would you give it a speeding ticket?

EXPLORATION

23. Here is a nursery rhyme whose earliest traceable publication date is around 1730 in *Folklore*, now in the library of the British Museum. (St. Ives is a village in England.)

As I was going to St. Ives,
I met a man with seven wives.
Each wife had seven sacks,
Each sack had seven cats,
Each cat had seven kits:
Kits, cats, sacks, and wives,
How many were going to St. Ives?

- Let W = the number of wives, S = the number of sacks, C = the number of cats, and K = the number of kits. Write three equations that relate two of these variables to each other.
- Find the value of $K + C + S + W$.
- What is an answer to the riddle?

QY ANSWERS

a.
$$\begin{cases} y = 2 - 5x \\ 4x + 3y = 17 \end{cases}$$

b. $4 \cdot -1 + 3 \cdot 7 =$
 $-4 + 21 = 17$ and
 $2 - 5 \cdot -1 = 2 + 5 = 7.$

So $(-1, 7)$ checks.