Lesson 10-2 Solving Systems Using Substitution

BIG IDEA Substituting an expression that equals a single variable is an effective first step for solving some systems.

When equations for lines in a system are in $y = mx + b$ form, a method of solving called *substitution* can be very efficient. Example 1 illustrates this method.

Mental Math

Find the greatest common factor.

a. 15; 200

b. 1,500; 20,000

c. 14; 26; 53

d. 1,400; 2,600; 5,300

Example 1

Solve the system $\begin{cases} y = 7x + 25 \\ y = -5x - 11 \end{cases}$ **using substitution.**

Solution Because $7x + 25$ and $-5x - 11$ both equal y, they must equal each other. Substitute one of them for y in the other equation.

$7x + 25 = -5x - 11$	Substitution
$12x + 25 = -11$	Add $5x$ to both sides.
$12x = -36$	Subtract 25 from both sides.
$x = -3$	Divide both sides by 12.

Now you know $x = -3$. However, you must still solve for y. You can substitute -3 for x into either of the original equations. We choose the first equation.

$y = 7x + 25$

$y = 7(-3) + 25$

$y = -21 + 25$

$y = 4$

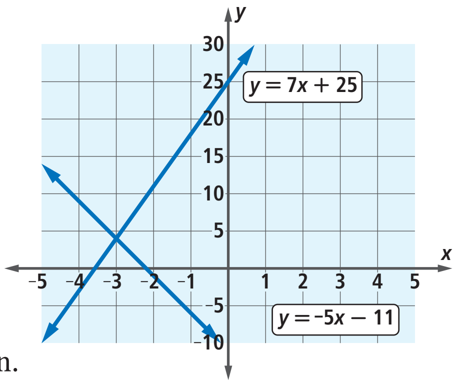
The solution is $x = -3$ and $y = 4$, or just $(-3, 4)$.

STOP QY

Check A graph shows that the lines with equations $y = 7x + 25$ and $y = -5x - 11$ intersect at $(-3, 4)$.

QY

Check that $(-3, 4)$ is a solution to $y = -5x - 11$.



Suppose two quantities are increasing or decreasing at different constant rates. Then each quantity can be described by an equation of the form $y = mx + b$. To find out when the quantities are equal, you can solve a system using substitution. Example 2 illustrates this idea.

Example 2

The Rapid Taxi Company charges \$2.15 for a taxi ride plus 20¢ for each $\frac{1}{10}$ mile traveled. A competitor, Carl's Cabs, charges \$1.50 for a taxi ride plus 25¢ for each $\frac{1}{10}$ mile traveled. For what distance do the rides cost the same?

Solution Let d = the distance of a cab ride in tenths of a mile.

Let C = the cost of a cab ride of distance d.
Rapid Taxi: $C = 2.15 + 0.20d$
Carl's Cabs: $C = 1.50 + 0.25d$

The rides cost the same when the values of *C* and *d* for Rapid Taxi equal the values for Carl's Cabs, so we need to solve the system formed by these two equations. Substitute $2.15 + 0.20d$ for *C* in the second equation.

$2.15 + 0.20d = 1.50 + 0.25d$

Now solve.

$0.65 = 0.05d$ Add −1.50 and −0.20*d* to both sides.

$d = 13$ Divide both sides by 0.05.

The two companies charge the same amount for a ride that is 13 tenths of a mile long, or 1.3 miles long.

Check Check to see if the cost will be the same for a ride of 13 tenths of a mile.

The cost for Rapid Taxi is $2.15 + 0.20 \cdot 13 = 2.15 + 2.60 = 4.75$.
The cost for Carl's Cabs is $1.50 + 0.25 \cdot 13 = 1.50 + 3.25 = 4.75$.
The cost is \$4.75 from each company, so the answer checks.

The average taxi fare in New York in 2006 was \$9.65.

Source: MSNBC

In Example 2, Carl's Cabs is cheaper at first, but as the number of miles increases, the prices become closer. Eventually the price for Carl's Cabs catches up with Rapid Taxi's price, and then Carl's is more expensive than Rapid. The next example also involves "catching up."

GUIDED

Example 3

Bart was so confident that he could run faster than his little sister that he bragged, "I can beat you in a 50-meter race. I'm so sure that I'll give you a 10-meter head start!" Bart could run at a speed of 4 meters per second, while his sister could run 3 meters per second. Could Bart catch up to his sister before the end of the race?

Solution Let d be the distance that Bart and his sister have traveled after t seconds. Recall that distance = rate • time.

For Bart, $d = 4t$.

Because Bart gives his sister a 10-meter head start, $d = 10 + 3t$. To know the time t when Bart will catch up to his sister, solve the system.

$$\begin{cases} d = \underline{\ ?\ } \\ d = \underline{\ ?\ } \end{cases}$$

Substitute $4t$ for d in the second equation.

$$\underline{\ ?\ } = \underline{\ ?\ } + \underline{\ ?\ }$$

Solve this equation as you would any other.

$$t = \underline{\ ?\ }$$

This means that after 10 seconds, Bart and his sister are at the same point. However, is the race finished at 10 seconds? Substitute 10 for t to find the distance. In 10 seconds Bart has run 40 meters. Because the race is 50 meters long, the race is not over when Bart catches up to his sister. Therefore, Bart wins.

Questions

COVERING THE IDEAS

In 1–5, a system is given.

a. Use substitution to find the solution.

b. Check your answer.

1. $\begin{cases} y = 3x - 4 \\ y = 5x - 10 \end{cases}$

2. $\begin{cases} b = 48 + a \\ b = 60 - a \end{cases}$

3. $\begin{cases} y = -\frac{1}{9}x + 6 \\ y = \frac{5}{3}x + 38 \end{cases}$

4. $\begin{cases} x = \frac{2}{3}y - 8 \\ x = -12.5y + 150 \end{cases}$

5. $\begin{cases} m = 8n + 33 \\ m = 3n - 78 \end{cases}$

6. Suppose that in Freeport, a taxi ride costs \$2.50 plus 15¢ for each $\frac{1}{10}$ mile traveled. In Geneva, a taxi ride costs \$1.70 plus 20¢ for each $\frac{1}{10}$ mile traveled. Write a system of equations and solve it to find the distance for which the costs are the same.

7. Recall from Example 3 that Bart ran 4 meters per second and his sister ran 3 meters per second. Bart's sister said to him, "You'll beat me if I have only a 10-meter head start. I'll race you if you give me a 15-meter head start." Solve a system of equations to find out if she would then beat him in a 50-meter race.

8. A tomato canning company has fixed monthly costs of $4,200. There are additional costs of $2.35 to produce each case of canned tomatoes. The company sells tomatoes to grocery stores for $5.85 per case.
   a. Write a system of equations to describe this situation.
   b. How many cases must the company sell to break even?
   c. Check your solution.

Approximately 124,900 acres of tomatoes were harvested in the United States in 2002.

Source: U.S. Department of Agriculture

APPLYING THE MATHEMATICS

9. A car leaves a gas station traveling at 60 mph. The driver has accidentally left his credit card at the gas station. Six minutes later, his friend leaves the station with the credit card, traveling at 65 mph to catch up to him.
   a. Write two equations to indicate the distance d that each car is from the gas station t hours after the first car leaves.
   b. Solve the system to determine when the second car will catch up to the first car.
   c. How far will they have traveled from the gas station when they meet?

10. Cameron has $450 and saves $12 a week. Sean has only $290, but is saving $20 a week.
   a. After how many weeks will they each have the same amount of money?
   b. How much money will each person have then?

11. In 2000, the metropolitan area of Dallas had about 5,200,000 people and was growing at about 120,000 people a year. In 2000, the metropolitan area of Boston had about 4,400,000 people and was growing at about 25,000 people a year.
   a. If these trends had been this way for quite some time, in what year did Dallas and Boston have the same population?
   b. What was this population?

12. In July 2005, Philadelphia approved taxi fares with an initial charge of $2.30 and an additional charge of $0.30 for each $\frac{1}{7}$ mile. If $P(x)$ is the cost for taking a taxi x miles, then $P(x) = 2.30 + 0.30 \cdot 7x$. In October 2005, Atlanta established new taxi fares with an initial charge of $2.50 and an additional charge of $0.25 for each $\frac{1}{8}$ mile. If $A(x)$ is the cost of taking a taxi x miles in Atlanta, then $A(x) = 2.50 + 0.25 \cdot 8x$. Solve a system to approximate at what distance the fares for Philadelphia and Atlanta are the same.

13. One plumbing company charges \$55 for the first half hour of work and \$25 for each additional half hour. Another company charges \$35 for the first half hour and then \$30 for each additional half hour. For how many hours of work will the cost of each company be the same?

In 14 and 15, a system that involves a quadratic equation is given. Each system has two solutions.

a. Solve the system by substitution.

b. Check your answers.

14. $\begin{cases} y = \frac{1}{9}x^2 \\ y = 4x \end{cases}$

15. $\begin{cases} y = 2x^2 + 5x - 3 \\ y = x^2 - 2x + 5 \end{cases}$

REVIEW

16. Consider the system $\begin{cases} y = 20x + 8 \\ 24x - y = -6 \end{cases}$. Verify that $\left(\frac{1}{2}, 18\right)$ is a solution to the system, but that (1, 20) is not. **(Lesson 10-1)**

In 17 and 18, solve the system of equations by graphing. (Lesson 10-1)

17. the system in Question 3

18. the system in Question 4

19. **a.** Simplify $y(y - 9) + 4y + 1$.

    b. Solve $y(y - 9) = 4y + 1$. **(Lesson 9-5)**

20. ***Skill Sequence*** Solve each equation. **(Lessons 9-1, 8-6)**

    a. $n^2 = 16$ **b.** $\sqrt{n} = 16$ **c.** $\sqrt{n^2} = 16$

21. What is the cost of x basketballs at \$18 each and y footballs at \$25 each? **(Lessons 5-3, 1-2)**

EXPLORATION

22. Find the taxi rates where you live or in a nearby community. Graph the rates to show how they compare to those in Question 12.

QY ANSWER

Does $4 = -5(-3) - 11$?

$4 = 15 - 11$

$4 = 4$ Yes, it checks.