Lesson

10-1 An Introduction to Systems

Vocabulary

system

solution to a system

empty set, null set

BIG IDEA Solving a system of equations means finding all the solutions that are common to the equations.

Mental Math

a. How many quarters make \$10.50?

b. How many dimes make \$10.50?

c. How many nickels make \$10.50?

A **system** is a set of equations or inequalities joined by the word *and,* that together describe a single situation. The two equations at the beginning of this chapter describing the winning Olympic times of men and women in the 100-meter freestyle events are an example of a system of equations.

Systems are often signaled by using a single left-hand brace { in place of the word *and.* So we can write this system as $\begin{cases} y = -0.1627x + 372.99 \\ y = -0.269x + 589.83 \end{cases}$.

When you write a system in this way, it is helpful to align the equal signs under one another.

What Is a Solution to a System?

A **solution to a system** of equations with two variables is an ordered pair (x, y) that satisfies both equations in the system.

Swimmer Larsen Jensen of the United States celebrates his silver medal after finishing second in the 1500-meter freestyle during the 2004 Summer Olympic Games in Athens, Greece.

Example 1

Consider the system $\begin{cases} y = 3x - 7 \\ 2y - 2x = 10 \end{cases}$.

a. Verify that the ordered pair (6, 11) is a solution to the system.

b. Show that (1, −4) is *not* a solution to the system.

Solutions

a. In each equation, replace x with 6 and y with 11.

First equation: $y = 3x - 7$

Does $11 = 3 \cdot 6 - 7$?

$11 = 18 - 7$ Yes.

Second equation: $2y - 2x = 10$

Does $2 \cdot 11 - 2 \cdot 6 = 10$?

$22 - 12 = 10$ Yes.

(6, 11) is a solution because it satisfies both equations.

b. Substitute 1 for x and −4 for y in both equations. The pair (1, −4) is a solution to the first equation because $-4 = 3 \cdot 1 - 7$. However, $2 \cdot -4 - 2 \cdot 1 = -10 \neq 10$. So (1, -4) is not a solution to the system.

STOP QY

QY

Is the ordered pair (2, −1) a solution to the system in Example 1?

Solving Systems by Graphing

You can find the solutions to a system of equations with two variables by graphing each equation and finding the coordinates of the point(s) of intersection of the graphs.

Example 2

A second-grade class has 23 students. There are 5 more boys than girls. Solve a system of equations to determine how many boys and how many girls are in the class.

A total of 49.6 million children attended public and private schools in the United States in 2003.

Source: U.S. Census Bureau

Solution Translate the conditions into a system of two equations. Let x be the number of boys and y be the number of girls. Because there are 23 students, $x + y = 23$. Because there are 5 more boys than girls, $y + 5 = x$. The situation is described by the system $\begin{cases} x + y = 23 \\ y + 5 = x \end{cases}$.

Graph the equations and identify the point of intersection.

There are 14 boys and 9 girls in the class. The solution is (14, 9), as graphed on the next page.

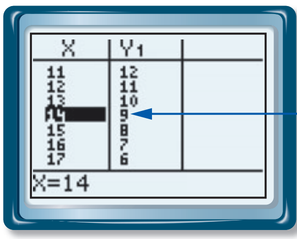
Check To check that (14, 9) is a solution, $x = 14$ and $y = 9$ must be checked in both equations.

Is (14, 9) a solution to $x + y = 23$? Does $14 + 9 = 23$? Yes.

Is (14, 9) a solution to $y + 5 = x$? Does $9 + 5 = 14$? Yes.

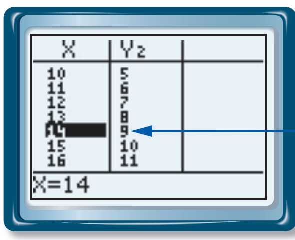
The two conditions about the numbers of boys and girls can be seen by looking at tables of solutions for each equation.

$x + y = 23$



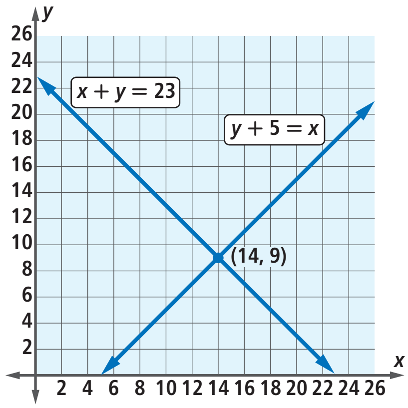
All these pairs add to 23, but only in this pair is the first number 5 greater than the second.

$y + 5 = x$



The first number in each pair is 5 greater than the second number, but only this pair has a sum of 23.

In general, there are four ways to indicate the solution to a system. They are shown below using the solution to the system in Example 2.



As an ordered pair	(14, 9)
As an ordered pair identifying the variables	$(x, y) = (14, 9)$
By naming the variables individually	$x = 14$ and $y = 9$
As a set of ordered pairs	{(14, 9)}

Systems with No Solutions

When the sentences in a system have no solutions in common, we say that there is no solution to the system. We cannot write the solution as an ordered pair or by listing the elements. The solution set is the set with no elements { }, written with the special symbol Ø. This set is called the **empty set** or **null set.**

GUIDED

Example 3

Find all solutions to the system $\begin{cases} y = -x + 4 \\ y = -x - 2 \end{cases}$.

Solution Graph both equations using a graphing calculator.

1. How many times do the lines appear to intersect? ___?___
2. What is the slope of each line? ___?___
3. If two lines in a plane do not intersect, they are called ___?___ lines.

Now look at the table of values on the graphing calculator.

4. Is there an ordered pair common to both lines? ___?___

This is an example of a system of equations for which there is no solution. The solution set is Ø.

Cost and Revenue Equations

In manufacturing, a *cost equation* describes the cost y of making x products. *Fixed costs* are things like rent and employee salaries, which must be paid regardless of the number of products made. *Variable costs* include materials and shipping, which depend upon how many products are made. The total of fixed and variable costs is the amount of money the business pays out each month.

A *revenue equation* describes the amount y that a business earns by selling x products. The *break-even point* is the point at which the revenue and total costs are the same. This point tells the manufacturer how many items must be sold in order to make a profit.

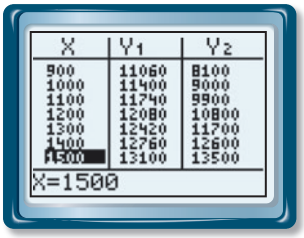
Example 4

A manufacturer of T-shirts has monthly fixed costs of \$8,000, and the cost to produce each shirt is \$3.40. Therefore, the cost y to produce x shirts is given by $y = 8{,}000 + 3.40x$. The business sells shirts to stores for \$9 each. So the revenue equation is $y = 9x$. Find the break-even point for the shirt manufacturer.

Children are shown silk-screening T-shirts. Silk-screening is a process in which color is forced into material like fabric or paper through a silk screen.

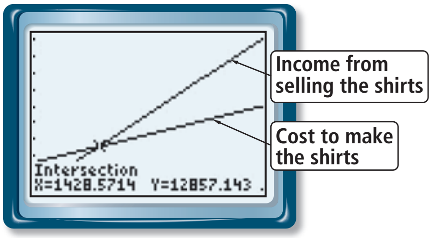
Solution The break-even point can be found by solving the system $\begin{cases} y = 8{,}000 + 3.40x \\ y = 9x \end{cases}$.

Make a table to help you find reasonable x and y values to use in setting the window. The table below shows that the intersection point will be seen in the window $1{,}400 \le x \le 1{,}500$, $12{,}600 \le y \le 13{,}500$.



X	Y1	Y2
900	11060	8100
1000	11400	9000
1100	11740	9900
1200	12080	10800
1300	12420	11700
1400	12760	12600
1500	13100	13500

X=1500



The INTERSECT command on a calculator shows that the point (1,429, 12,858) is an approximate solution. At the break-even point, 1,429 shirts are manufactured and sold. It costs about the same amount to make the shirts as the manufacturer earns from selling them. If more than 1,429 shirts are produced and sold, the business will earn a profit.

Check When $x = \$1{,}429$ in the cost equation, $y = \$8{,}000 + 3.40 \cdot \$1{,}429 = \$12{,}858.60$. When $x = 1{,}429$ in the revenue equation, $y = 9 \cdot \$1{,}429 = \$12{,}861$. These values are close enough to make 1,429 the first coordinate of the break-even point.

In Example 4, the solution is an approximation. When solutions do not have integer coordinates, it is likely that reading a graph will give you only an estimate. But graphs can be created quickly. In the next few lessons, you will learn algebraic techniques to find exact solutions to systems.

Questions

COVERING THE IDEAS

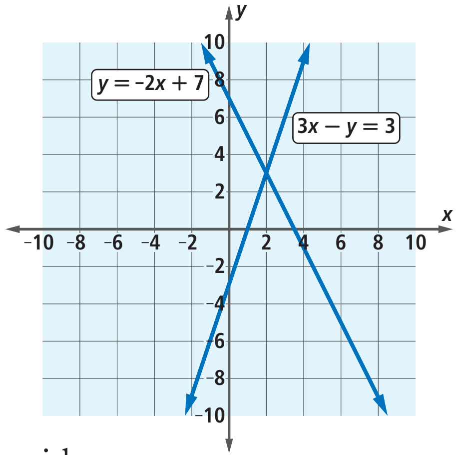
1. **True or False** When a system has two variables, each solution to the system is an ordered pair.

2. What does the brace { represent in a system?

3. a. Verify that (8, –2) is a solution to the system $\begin{cases} -x = 4y \\ 2x + 3y = 10 \end{cases}$.
   b. Write this solution in two other ways.

4. Show that (11, 8) is *not* a solution to $\begin{cases} y = x - 3 \\ x + 5y = 50 \end{cases}$.

5. Refer to the graph at the right.
   a. What system is represented?
   b. What is the solution to the system?
   c. Verify your answer to Part b.



In 6 and 7, a system is given.
a. Solve the system by graphing.
b. Check your solution.

6. $\begin{cases} y = -\frac{1}{2}x + 3 \\ y = 2x - 7 \end{cases}$

7. $\begin{cases} y = x \\ 4x - 2y = 12 \end{cases}$

8. An elementary school has 518 students. There are 4 more girls than boys.
   a. If g is the number of girls and b is the number of boys, translate the given information into two equations.
   b. Letting $x = g$ and $y = b$, graph the equations on a calculator.
   c. Using the graph from Part b, use the INTERSECT command to find the number of boys and girls in the school.

9. A small business makes wooden toy trains. The business has fixed expenses of $3,800 each month. In addition to this, the production of each train costs $4.25. The business sells the trains to stores for $12.50 each.
   a. Write cost and revenue equations as a system.
   b. Use a calculator table to find a good window to display the graph. What window did you use?
   c. Find the break-even point.
   d. Last month the business made and sold 518 trains. Did the business earn a profit?

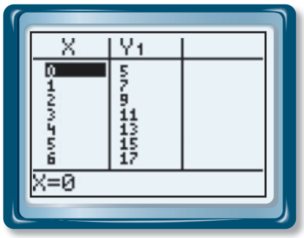
10. Find all solutions to the system $\begin{cases} y = 3x - 5 \\ y = 3x - 1 \end{cases}$.

Archaeologists have discovered jointed wooden dolls, carved horses, chariots, and even a crocodile with moveable jaws that date back to the year 1100 BCE.

Source: *TDmonthly*

11. Consider the system $\begin{cases} y_1 = 2x + 5 \\ y_2 = 3x \end{cases}$.

The screen at the right shows solutions to $y_1 = 2x + 5$. Make a column for y_2 and use it to find the ordered pair that also satisfies $y_2 = 3x$, and therefore is a solution to the system.



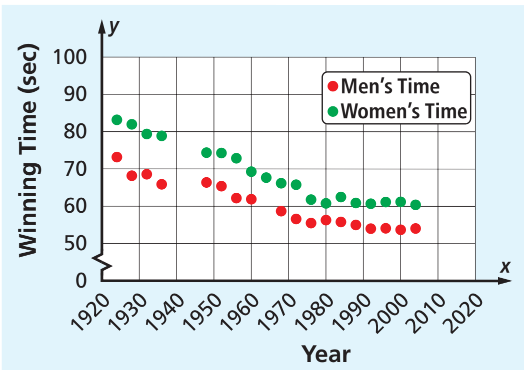
APPLYING THE MATHEMATICS

12. The sum of two numbers is –19 and their difference is –5. Write a system of equations and solve it using a graph.

13. Below are a table and graph of the winning times in seconds for the Olympic men's and women's 100-meter backstroke events.

Year	Men's Time	Women's Time	Year	Men's Time	Women's Time
1924	73.2	83.2	1972	56.58	65.78
1928	68.2	82.0	1976	55.49	61.83
1932	68.6	79.4	1980	56.33	60.86
1936	65.9	78.9	1984	55.79	62.55
1948	66.4	74.4	1988	55.05	60.89
1952	65.4	74.3	1992	53.98	60.68
1956	62.2	72.9	1996	54.10	61.19
1960	61.9	69.3	2000	53.72	61.21
1964	NA	67.7	2004	54.06	60.37
1968	58.7	66.2			

Source: International Olympic Committee



In Lesson 4-4, you were asked to estimate when the women's time might equal the men's time. Now repeat this question by finding equations for lines of best fit for the men's and women's times. Graph the two equations. According to these lines, will the women's winning time ever equal the men's winning time in the 100-meter backstroke? If yes, estimate the year when this will happen. If no, explain why not.

14. Buffy is hosting a meeting and plans to serve 4 dozen muffins. She wants to have twice as many blueberry muffins as plain muffins. The table at the right shows some of the possible ways to order 4 dozen muffins.
   a. Using x for the number of plain muffins and y for the number of blueberry muffins, write a system of equations to describe this situation.
   b. Graph your equations from Part a to find how many of each kind Buffy should order.

Number of Plain Muffins	Number of Blueberry Muffins
0	48
10	38
20	28
30	18
40	8
48	0

REVIEW

15. ***Skill Sequence*** Solve each equation. **(Lessons 9-5, 4-4, 3-4)**
   a. $5x + 6 = 3$
   b. $5x + 6 = 2x + 3$
   c. $5x + 6 = 2x^2 + 3$
   d. $5x + 6 = 2x(x + 1)$

In 16–18, simplify the expression. (Lessons 8-4, 8-3, 8-2)

16. $m^2 \cdot n^3 \cdot m \cdot n^4$
17. $(-5x^7y^9)^4$
18. $\frac{18r^2s^3}{6rs^4}$

19. Graph $\{(x, y): 4x - 8y < 2\}$. **(Lesson 6-9)**

20. Find the values of the variables so that the given point lies on the graph of $10x - 4y = 20$. **(Lessons 6-8, 4-7)**
   a. $(5, p)$
   b. $(q, -2)$
   c. $(r, 0)$

21. Two workers can dig a 20-foot well in 2 days. How long will it take 6 workers to dig a 90-foot well, assuming that each of these 6 workers dig at the same rate as each of the 2 workers? **(Lessons 5-4, 5-3)**

Two men stand above a well that serves as an extractor of gold.

EXPLORATION

22. Some experts believe that even though the women's swim times are decreasing faster than the men's, it is the ratio of the times that is the key to predictions.
   a. Compute the ratio of the men's time to the women's time for the 100-meter freestyle for each Olympic year in Question 13.
   b. Graph your results.
   c. What do you think the ratio will be in the year 2020? Does this agree with the prediction in Question 13?

QY ANSWER

no