Chapter 11

Summary and Vocabulary

- A **monomial** is a product of terms. The **degree of a monomial** is the sum of the exponents of its variables. A **polynomial** is an expression that is either a monomial or a sum of monomials. The **degree of a polynomial** is taken to be the largest degree of its monomial terms. **Linear polynomials** are polynomials of degree 1. **Quadratic polynomials** are polynomials of degree 2.
- Polynomials emerge from a variety of situations. Our customary way of writing whole numbers in base 10 can be considered as a polynomial with 10 substituted for the variable. If different amounts of money are invested each year at a **scale factor** x, the total amount after several years is a **polynomial in x.** The number of permutations of n objects can be represented by a polynomial in n.
- Addition and subtraction of polynomials are based on adding like terms, one of the forms of the Distributive Property that you studied earlier in this book. Multiplication of polynomials is also justified by the Distributive Property. To multiply one polynomial by a second, multiply each term in the first polynomial by each term in the second polynomial, then add the products. For example:

  monomial by a polynomial: $a(x + y + z) = ax + ay + az$

  two polynomials: $(a + b + c)(x + y + z) =$
  $ax + ay + az + bx + by + bz + cx + cy + cz$

  two binomials: $(a + b)(c + d) = ac + ad + bc + bd$

  perfect square: $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

  difference of two squares: $(a + b)(a - b) = a^2 - b^2$
- If each of the terms of a polynomial has a common factor, then so does their sum. It can be factored out using the Distributive Property.
- The square of the difference of actual and expected values in an experiment, $(a - e)^2$, appears in the calculation of the **chi-square statistic.** This statistic can help you decide whether the assumptions that led to the expected values are correct.

Vocabulary

11-1
polynomial in x
standard form for a polynomial

11-2
monomial
polynomial
binomial
trinomial
degree of a monomial
degree of a polynomial
linear polynomial
quadratic polynomial

11-4
factoring
trivial factors
greatest common factor
factorization
prime polynomials
complete factorization

11-6
perfect square trinomials
difference of squares

11-7
permutation
$n!$, n factorial
circular permutation

11-8
expected number
deviation
chi-square statistic

Theorems and Properties

Unique Factorization Theorem for Polynomials (p. 677)
Extended Distributive Property (p. 680)
Perfect Squares of Binomials (p. 687)
Difference of Two Squares (p. 688)

Chapter 11 Self-Test

Take this test as you would take a test in class. You will need a calculator. Then use the Selected Answers section in the back of the book to check your work.

In 1–6, expand and simplify the expression.

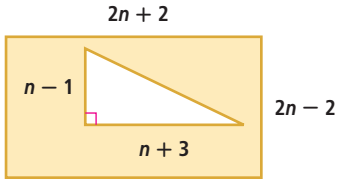
1. $3x(10 - 4x + x^3)$
2. $(2b - 5)^2$
3. $(8z + 3)(8z - 3)$
4. $6a(2a^2 + 9a - 1)$
5. $(5a^2 - a)(5a^2 - a)$
6. $(2 - 6c)(4 + 3c)$

In 7 and 8, consider the polynomial $8x^3 - 5 + 2x + 11x^3 - 9x^2$.

7. What is the degree of this polynomial?
8. Is the polynomial a *monomial*, *binomial*, *trinomial*, or *none of these*?
9. Factor completely: $12x^3y^2 - 24x^2y^3 + 30x^2y^4$.

In 10 and 11, write as a single polynomial.

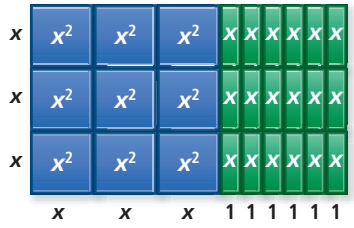
10. $(20n^2 - 8n - 12) + (16n^3 - 7n^2 + 5)$
11. $9p^4 + p^2 - 5 - p(3p^3 + p - 2)$
12. **True or False** The expression $3v^2 + 3v - 1$ is a trinomial of degree 3.
13. Simplify the fraction $\frac{28w^3 - 18w}{2w}$, assuming that $w \neq 0$.
14. Write the area of the shaded portion of the rectangular region as a polynomial in standard form.



15. A swim team has 7 swimmers available to race a 4-person relay. How many different orders of swimmers are possible?
16. Twenty-five art students entered a competition in which 1st, 2nd, and 3rd place prizes were to be awarded. How many permutations of students could receive these prizes?

In 17 and 18, consider the following. On Ashley's 16th birthday, she received $200. She received $150 on her 17th birthday, and $300 on each of her 18th and 19th birthdays.

17. If she invested all of this money each year in a savings account with a yearly scale factor x, how much money would she have on her 21st birthday?
18. Evaluate your answer to Question 17 if the savings account had an interest rate of 3%.
19. **a.** What is the area of the largest rectangle below?



b. What factorization of the rectangle's area is shown?

c. Is this a complete factorization? Explain your answer.

20. Represent the product $(x + 2y)(5 + z)$ using areas of rectangles.

21. Jerry believes a die is weighted to favor certain numbers. The table below shows the outcome of 198 random tosses of the die.

Number	Outcomes
1	24
2	44
3	31
4	30
5	43
6	26

Use the chi-square statistic to provide evidence as to whether Jerry's view is correct. Justify your answer.

22. Represent the product $(x + 4)^2$ using areas of rectangles. Write your answer in standard form.

Chapter 11

Chapter Review

SKILLS
PROPERTIES
USES
REPRESENTATIONS

SKILLS Procedures used to get answers

OBJECTIVE A **Add and subtract polynomials.** **(Lessons 11-1, 11-2)**

In 1–4, simplify the expression and write the answer in standard form.

1. $(-k^3 + 2k^2 - 17k + 8) + (5k^3 - 2k^2 - 8)$
2. $\frac{4}{3}h^3 + 4 - \frac{2}{3}h^3 + \frac{1}{7}h^2 - 5$
3. $(5.4s^4 + 9.8s^2 - 8) - (-3.7s^3 - 4 + 5.2s)$
4. $(12w^3 - 3w^2 + -8w - 9) - (80w + 8)$

OBJECTIVE B **Multiply polynomials.** **(Lessons 11-3, 11-5, 11-6)**

5. **Fill in the Blank** $(-6x + 9) \cdot \left(\frac{5}{3}x + 6\right) = -10x^2 - \underline{?}\,x + 54$

In 6–16, write as a single polynomial in standard form.

6. $m(7m^2 - 13m + 12)$
7. $9p(p^3 + p^2 - 6p + 5)$
8. $-3x^2\left(12x^2 + \frac{2}{3}x\right)$
9. $-\frac{1}{7}q^8(-q^3 + 14q^2 - 112q + 5)$
10. $2(g + 3g^2 + 19g^3 - g^6) + g(-3g^5 + 2g + 6g^3)$
11. $(b - 3)(b + 3)$
12. $(8x - 2)(2x - 1)$
13. $2(1 + 6w)(1 - 6w)$
14. $3(a^2 + a - 1)(a + 1)$
15. $(n - 2)(n - 3)(n - 4)$
16. $(c^2 + 10c - 4)(2c^2 - 8c + 1)$
17. The length of one leg of a right triangle is $(3x - 1)$, and the length of the other leg is $(17x + 2)$. Express the area of the triangle as a polynomial in standard form.

OBJECTIVE C **Find common monomial factors of polynomials.** **(Lesson 11-4)**

In 18–21, factor the polynomial completely.

18. $9k^3 + 6k^2$
19. $u^2v - uv^2$
20. $-84y^3 - 18y^2 + 93y$
21. $45a^9b^5 + 60a^6b^4 - 15a^5b^3 + 420a^3b^2$
22. **Multiple Choice** Which is a complete factorization of $24y^7 + 18y^5 - 90y^3$?
    - **A** $y^3(24y^4 + 18y^2 - 90)$
    - **B** $3y^3(8y^7 + 6y^5 - 30y^3)$
    - **C** $6y^7(4 + 3y^{-2} + 15y^{-4})$
    - **D** $6y^3(4y^4 + 3y^2 - 15)$

OBJECTIVE D **Expand squares of binomials.** **(Lesson 11-6)**

In 23–26, expand and simplify the expression.

23. $(p + 6)^2$
24. $(-u - 5)^2$
25. $(11 - 45z)^2$
26. $(-w + 2)^2$
27. **Multiple Choice** The square of which binomial below is $64c^4 + -80c^3 + 25c^2$?
    - **A** $8c - 5$
    - **B** $8c^2 - 5c$
    - **C** $8c^2 + 5c$
    - **D** $8c^3 + 5c$

PROPERTIES The principles behind the mathematics

OBJECTIVE E **Classify polynomials by their degrees or number of terms.** **(Lesson 11-2)**

28. Give an example of a monomial of degree 5.

29. Give an example of a trinomial of degree 5.

In 30–33, consider the polynomials below.

a. $n^2 - 5$ b. $2m^2 + 4m - 7$

c. $12a^4 - 16a^2 + 3$ d. $8w^2y + 9wy$

30. Which are binomials?

31. Which are trinomials?

32. Which have degree 2?

33. Which have degree 4?

USES Applications of mathematics in real-world situations

OBJECTIVE F Translate investment situations into polynomials. (Lesson 11-1)

34. Flora decides to open a retirement account with an annual interest rate y. In the first year, Flora invests \$5,000. The second year, she invests \$3,000, and then in the third year she invests \$2,000. She keeps the money in the account at the same scale factor for five years after her last deposit.
    a. Write a polynomial that describes how much money she has in this account at the end of that time.
    b. If $y = 1.07$, how much money does Flora have in her retirement account after the five years?

35. Jeffrey is saving money during his high school years to go on a trip to Egypt after he graduates. The trip costs \$3,000. At the end of his freshman year, he deposits \$1,200 in a savings account with an annual scale factor of x. At the end of his sophomore year, he deposits \$700 in the same account. At the end of his junior year, he deposits \$500. It is now the end of his senior year.
    a. Write an expression that shows how much money Jeffrey has in his account.
    b. If the savings account pays 4% annual interest, how much more money does he need to afford the trip?

OBJECTIVE G Determine numbers of permutations. (Lesson 11-7)

In 36–38, Beth has 7 different blouses in her closet.

36. How many different ways can she select a blouse for each of the 7 days of the week?

37 How many different ways can she select a blouse for each of the 5 school days of the week?

38. Suppose Beth wants to wear her favorite blouse on Monday. How many different ways can she select a blouse for each of the remaining 4 school days of the week?

In 39–41, Kyle has recently opened a bank account and is asked to set the 4-digit PIN (Personal Identification Number) for his ATM card, in which each digit can be any number from 0 to 9.

39. How many different ways can Kyle select a PIN?

40. If the digits of the PIN must all be different, how many ways can Kyle select the PIN?

41. If the first digit of the PIN is *not* allowed to be zero, how many different ways can Kyle a select PIN, assuming the digits cannot be repeated?

42. Six guests arrive for a dinner party and are to be seated around a large circular table. How many different ways can the host of the party seat the guests around the table?

43. During a track meet, 9 runners are racing in the 100-yard dash, but only the top 3 runners receive an award. How many different ways are there for the top 3 runners to place?

OBJECTIVE H Use a chi-square statistic to determine whether or not statistics support a conclusion. (Lesson 11-8)

44. Sixty people were surveyed in a taste test of two types of chocolate. 26 people preferred chocolate A and 34 people preferred chocolate B.
    a. If the chocolates were equally tasty, what would be the expected numbers of preference for each chocolate?
    b. Calculate the chi-square statistic for this situation using the actual numbers and the expected numbers from Part a.
    c. Use the chi-square table on page 699. Does the evidence support the fact that chocolate B is preferred to chocolate A? Explain why or why not.

45. A company was open only Monday through Friday. Because it was not open Saturday or Sunday, it expected that it would get about the same amount of mail each day Tuesday through Friday, but three times this amount on Monday. However, some people thought there was too much mail coming on Monday. When the numbers of pieces of mail for each day for a few weeks were totaled, here were the numbers on each day.

Day	Mon.	Tues.	Wed.	Thurs.	Fri.
Pieces of Mail	122	30	41	35	27

    a. How many pieces of mail did the company expect each day?
    b. Calculate the chi-square statistic for this situation using the actual numbers and the expected numbers from Part a.
    c. Use the chi-square table on page 699. Does the evidence support the company's expectations on how much mail to expect? Justify your answer.

46. A large factory believes that its floor manager is becoming careless towards the end of his shift each day. Below is a table that shows the hour of his shift and the number of accidents that occurred during that hour.

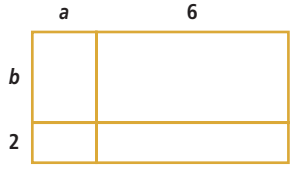
Hour	1	2	3	4	5	6	7	8
Accidents	5	4	4	7	9	8	11	10

    a. If the factory were to have an equal number of accidents each hour, find the expected number.
    b. Calculate the chi-square statistic for this situation using the actual numbers and the expected number from Part a.
    c. Use the chi-square table on page 699. Does the evidence support the factory's belief about the floor manager? Justify your answer.

REPRESENTATIONS Pictures, graphs, or objects that illustrate concepts

OBJECTIVE I Represent polynomials by areas. (Lessons 11-3, 11-5, 11-6)

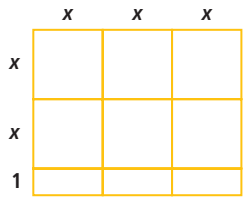
47. a. Write the area of the largest rectangle below as the sum of 4 terms.



    b. Write the area of the largest rectangle as the product of 2 binomials.
    c. Are the answers to Parts a and b equal?

48. Represent $(n + m)(p + q)$ using areas of rectangles.

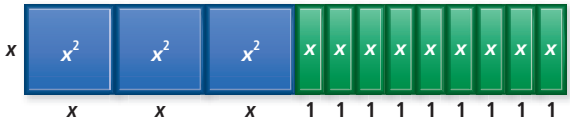
49. a. What polynomial multiplication is represented by the area of the largest rectangle below?



b. What is the product in standard form?

50. Show the product $(3x + 1)^2$ using areas of rectangles. Write your answer in standard form.

51. Show a factorization of $3x^2 + 9x$ by rearranging these tiles into a different rectangle.



52. Show $4x^2 + 4x + 1 = (2x + 1)^2$ by rearranging these tiles into a square with sides $2x + 1$.